

The Senior Transportation Problem

Chang Liu, Dionne M. Aleman, J. Christopher Beck

Department of Mechanical & Industrial Engineering
University of Toronto, Toronto, Ontario, Canada
{cliu,aleman,jcb}@mie.utoronto.ca

Introduction

The senior transportation problem (STP) is an optimization problem in which a fixed fleet of volunteer-operated, heterogeneous vehicles from multiple depots must satisfy as many elderly door-to-door transportation requests as possible within a fixed time horizon. All requests consist of a one-to-one pickup and delivery with time windows. Moreover, as the clients are seniors, a maximum ride time is enforced to minimize user discomfort and inconvenience. The problem is similar to the classical dial-a-ride Problem (DARP) or the pickup and delivery problem with time windows (PDPTW). However, due to the limited resources, not all requests can be met within the time horizon and, therefore, the problem is to select a subset of requests such that the total weight of all served requests is maximized. Additionally, all drivers operate on a volunteer basis, thus aspects such as multi-depots, heterogeneous vehicles, and time windows on vehicles also need to be considered. This document formally defines the STP and presents related work and a mixed integer programming formulation for the STP.

Problem Definition

The STP is described as follows. Let $G = (\mathcal{V}, \mathcal{A})$ be a directed graph with vertex set $\mathcal{V} = \mathcal{D} \cup \mathcal{C}$ where \mathcal{D} represents the depot vertices and \mathcal{C} represents the client vertices. The set \mathcal{D} is partitioned into $\mathcal{D}^+ = \{1, \dots, M\}$ (starting depot vertices) and $\mathcal{D}^- = \{M + 1, \dots, 2M\}$ (ending depot vertices) where M is the number of vehicles. The set \mathcal{C} is partitioned into $\mathcal{C}^+ = \{2M + 1, \dots, 2M + N\}$ (pickup vertices) and $\mathcal{C}^- = \{2M + N + 1, \dots, 2M + 2N\}$ (delivery vertices) where N is the number of requests. Each vertex $i \in \mathcal{V}$ is associated with a time window $[E_i, L_i]$ and a service duration, S_i indicating how much time needs to be spent at the location. Each arc $(i, j) \in \mathcal{A}$ has a non-negative routing time $T_{i,j}$ satisfying the triangular inequality.

Vehicles and Depots

Let \mathcal{K} represent the set of vehicles. Each vehicle $k \in \mathcal{K}$ is associated with a starting depot $i_{k+} \in \mathcal{D}^+$ and an ending depot $i_{k-} \in \mathcal{D}^-$. Each vehicle must start and end at its associated depots. Multiple vehicles can share the same geographical

location for depots, however, relocation of vehicles between depots is not allowed. Each vehicle also specifies its available time windows. If a vehicle has non-overlapping multiple time windows, then it is considered as multiple vehicles. Furthermore, vehicles are heterogeneous and differ in capacity, thus each vehicle $k \in \mathcal{K}$ is associated with a maximum capacity P_k .

Therefore, each vehicle has known depot locations (i_{k+}, i_{k-}) with $S_k = 0$, a capacity P_k , and a time window $[E_k, L_k]$ in which the vehicle must leave the starting depot, perform all pickup and delivery requests assigned to it and travel to its ending depot.

Pickup and Delivery Requests

Let \mathcal{R} represent the set of requests. Each request, r , is paired with a positive weight, W_r , denoting the importance of the request. The total weight of served requests contributes to the objective function. Each request also specifies the size of its load, Q_r , that represents the number of people traveling or any accompanying mobility aid. A request $r \in \mathcal{R}$ has an associated pickup location $i_{r+} \in \mathcal{C}^+$ and delivery location $i_{r-} \in \mathcal{C}^-$. Requests are divided into two categories: *outbound* and *inbound* trips. In an outbound trip, the client is typically travelling from his/her home location to a destination location and in an inbound trip, the client requests a return trip to their home location. In the context of the STP, the client only imposes a time window on the delivery location of an outbound trip and on the pickup location of an inbound trip. In addition, all clients are restricted to a maximum ride time, F , on any vehicle.

Let Z be the end of the time horizon. For an outbound trip r , the time window associated with its pickup location is $[0, Z]$, whereas the time window of an inbound trip's delivery location is $[0, Z]$. The load size is positive for a pickup location vertex and negative for a delivery vertex, $Q_{i_{r+}} = -Q_{i_{r-}}, \forall r \in \mathcal{R}^+$.

Routing Plan

A *route* for vehicle k is a sequence of vertices, $[i_{k+}, \dots, i_{k-}]$. A request is *served* when it is part of a route. The set of routes must satisfy the following constraints for the served requests:

1. The pickup and delivery vertices of a request must be on the same route;

2. The pickup vertex must precede the delivery vertex;
3. A vertex is visited by at most one vehicle;
4. The load of a vehicle k cannot exceed its maximum capacity P_k at any point;
5. A route must start and end within the vehicle's availability window;
6. No subtours are allowed in any route;
7. The ride time of a client cannot exceed the maximum ride time F ;
8. All pickup and delivery vertices must be served within their specified time windows.

Related Work

There are three levels of decisions in the STP: the selection of requests, the assignment of requests to vehicles, and the routing of all vehicles. Subsets of these decisions form well-studied optimization problems.

Selectivity and routing arise in the Team Orienteering Problem (TOP), extensively reviewed in Gunawan et al.'s survey paper (Gunawan, Lau, and Vansteenwegen 2016). The assignment and routing of requests are seen in PDPTW and DARP. Parragh et al. (Parragh, Doerner, and Hartl 2008a; 2008b) surveyed multiple variations of the pickup-and-delivery problem while Cordeau & Laporte presented a survey of solution methods for the DARP (Cordeau and Laporte 2007). Though in the classical DARP the objective is to minimize the travel costs, the authors recognize that there can be other objectives such as maximizing satisfied demand or quality of service. However, no formulations or references to such problems are provided. With the current state-of-the-art algorithms, PDPTW has been solved to optimality for problems with up to 209 requests. The DARP, however, has only been solved to optimality for problems with 96 requests. Most approaches in the literature that have been applied to these two problems are heuristic-based methods.

To our knowledge, these three decisions have been looked at together by two groups. Baklagis et al. (Baklagis, Dikas, and Minis 2016) proposed a branch-and-price framework while Qiu et al. (Qiu, Feuerriegel, and Neumann 2016) developed graph search and maximum set packing formulations to tackle this problem. However, these two works neglect three characteristics of the STP: the existence of multiple depots, the maximum ride time of a client, and heterogeneous fleets. As our problem involves volunteer drivers, these three constraints are crucial.

A MIP Formulation

To provide a formal definition of the STP, we present a mixed integer programming (MIP) formulation adapted from that of Ropke & Cordeau (Ropke and Cordeau 2009). The objective function has been modified to reflect the selective nature of the STP. Constraints (2) and (3) have been added to model the multi-depots aspect and that not all vehicles have to be used in the final solution. Big M constraints are introduced in (7), (8), and (9) to model the optionality of location visits and the sequencing. Constraint (11) is a new constraint to represent the maximum ride time.

Decision Variables

Our MIP formulation uses three variables: a binary variable $x_{k,i,j}$ and two continuous variables $y_{k,i}$ and $u_{k,i}$. $x_{k,i,j} = 1$ if vehicle k visits location j immediately after visiting location i and 0 otherwise. $x_{k,i,j}$ is only instantiated for $(i,j) \in \mathcal{A}' \subseteq \mathcal{A}$ where $(i,j) \in \mathcal{A}'$ if one of the following is true: 1. $i \in \mathcal{D}^+$ and $j \in \mathcal{C}^+$, 2. both i and $j \in \mathcal{C}$, 3. $i \in \mathcal{C}^-$ and $j \in \mathcal{D}^-$, or 4. $i \in \mathcal{C}^+$ and $j \in \mathcal{C}^-$. $y_{k,i}$ indicates the load of the vehicle k after visiting location $i \in \mathcal{V}$. It is non-negative and is less than or equal to the vehicle capacity. $u_{k,i}$ indicates the time when vehicle k leaves location $i \in \mathcal{V}$. It is non-negative and less than or equal to the maximum time horizon Z .

MIP Formulation

$$\max \sum_{k \in \mathcal{K}} \sum_{r \in \mathcal{R}} \sum_{j \in \mathcal{V}} (W_r \times x_{k,i_r+,j}) \quad (1)$$

$$\text{s.t.} \quad \sum_{r \in \mathcal{R}} x_{k,i_k+,i_r+} \leq 1 \quad \forall k \in \mathcal{K} \quad (2)$$

$$\sum_{r \in \mathcal{R}} x_{k,i_r-,i_k-} \leq 1 \quad \forall k \in \mathcal{K} \quad (3)$$

$$\sum_{k \in \mathcal{K}} \sum_{j \in \mathcal{V}} x_{k,i_r+,j} \leq 1 \quad \forall r \in \mathcal{R} \quad (4)$$

$$\sum_{j \in \mathcal{V}} (x_{k,i,j} - x_{k,j,i}) = 0 \quad \forall k \in \mathcal{K}, i \in \mathcal{C} \quad (5)$$

$$\sum_{j \in \mathcal{V}} (x_{k,i_r+,j} - x_{k,j,i_r-}) = 0 \quad \forall k \in \mathcal{K}, r \in \mathcal{R} \quad (6)$$

$$u_{k,j} \geq (u_{k,i} + T_{i,j} + S_j) - M \times (1 - x_{k,i,j}) \quad \forall k \in \mathcal{K}, i, j \in \mathcal{V} \quad (7)$$

$$u_{k,i} \geq E_i - M \times \left(1 - \sum_{j \in \mathcal{V}} x_{k,i,j}\right) \quad \forall k \in \mathcal{K}, i \in \mathcal{V} \quad (8)$$

$$u_{k,i} \leq L_i - S_i + M \times \left(1 - \sum_{j \in \mathcal{V}} x_{k,i,j}\right) \quad \forall k \in \mathcal{K}, i \in \mathcal{V} \quad (9)$$

$$u_{k,i_r+} \leq u_{k,i_r-} \quad \forall k \in \mathcal{K}, r \in \mathcal{R} \quad (10)$$

$$(u_{k,i_r-} - u_{k,i_r+}) \leq F \quad \forall k \in \mathcal{K}, r \in \mathcal{R} \quad (11)$$

$$y_{k,j} \geq (y_{k,i} + Q_i) - M \times (1 - x_{k,i,j}) \quad \forall k \in \mathcal{K}, i, j \in \mathcal{V} \quad (12)$$

$$x_{k,i,j} \in \{0, 1\} \quad \forall k \in \mathcal{K}, (i, j) \in \mathcal{A}' \quad (13)$$

$$x_{k,i,j} = 0 \quad \forall k \in \mathcal{K}, (i, j) \notin \mathcal{A}' \quad (14)$$

$$0 \leq u_{k,i} \leq Z \quad \forall k \in \mathcal{K}, i \in \mathcal{V} \quad (15)$$

$$0 \leq y_{k,i} \leq P_k \quad \forall k \in \mathcal{K}, i \in \mathcal{V} \quad (16)$$

The objective function (1) maximizes the sum of weights of served requests. Constraints (2) and (3) ensure that each vehicle leaves from its starting depot and ends at its ending depot. Constraint (4) allows for the selectivity of requests. Constant flow is enforced with Constraint (5). Both the pickup and delivery locations must be visited by the same vehicle as enforced through Constraint (6).

In Constraint (7), the travel time and service time of visited nodes are enforced. Constraints (8) and (9) make sure that each node that is visited must be visited within its time window. Constraint (10) imposes that pickup nodes must precede delivery nodes. Constraint (11) enforces that each ride does not exceed the maximum ride time. The capacity Constraint (12) keeps track of the load of each vehicle after visiting the node. Constraints (13), (14), (15) and (16) bound the variables x , u and y .

References

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