Integrating Maintenance Planning and Production Scheduling: Making Operational Decisions with a Strategic Perspective

by

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Abstract

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In today's competitive environment, the importance of continuous production, quality improvement, and fast delivery has forced production and delivery processes to become highly reliable. Keeping equipment in good condition through maintenance activities can ensure a more reliable system. However, maintenance leads to temporary reduction in capacity that could otherwise be utilized for production. Therefore, the coordination of maintenance and production is important to guarantee good system performance. The central thesis of this dissertation is that integrating maintenance and production decisions increases efficiency by ensuring high quality production, effective resource utilization, and on-time deliveries.

Firstly, we study the problem of integrated maintenance and production planning where machines are preventively maintained in the context of a periodic review production system with uncertain yield. Our goal is to provide insight into the optimal maintenance policy, increasing the number of finished products. Specifically, we prove the conditions that guarantee the optimal maintenance policy has a threshold type.

Secondly, we address the problem of integrated maintenance planning and production scheduling where machines are correctively maintained in the context of a dynamic aircraft repair shop. To solve the problem, we view the dynamic repair shop as successive static repair scheduling sub-problems over shorter periods. Our results show that the approach that uses logic-based Benders decomposition to solve the static sub-problems, schedules over longer horizon, and quickly adjusts the schedule increases the utilization of aircraft in the long term.

Finally, we tackle the problem of integrated maintenance planning and production scheduling where machines are preventively maintained in the context of a multi-machine production system. Depending on the deterioration process of machines, we design decomposed techniques that deal with the stochastic

and combinatorial challenges in different, coupled stages. Our results demonstrate that the integrated approaches decrease the total maintenance and lost production cost, maximizing the on-time deliveries. We also prove sufficient conditions that guarantee the monotonicity of the optimal maintenance policy in both machine state and the number of customer orders.

Within these three contexts, this dissertation demonstrates that the integrated maintenance and production decision-making increases the process efficiency to produce high quality products in a timely manner.

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Chapter 1

Introduction

In many industries the conditions of the systems used to produce goods or deliver services are major determinants of the efficiency of the production or service delivery process (Wang, 2002; Sloan, 2008). For example, delayed repair of an expensive asset like a fighter aircraft can translate to costly under-use of a valuable resource, a dull drill-bit in manufacturing can significantly slow down production, and contaminated equipment in the pharmaceutical industry can dramatically increase the number of defective products. In these three examples, keeping excess inventory of aircraft, finished products, or drugs on hand is not a practical approach to maintain high system performance due to economic pressure, rapid technological advancements, highly customized products, and/or regulations.

An alternative strategy is to ensure a reliable system where equipment always operates at the highest speed, never breaks-down, and never produces defective products (Waeyenbergh et al., 2000). While such an ideal system is not achievable in reality, investment in properly performed maintenance can result in a more reliable system with less variance in machine speed, fewer breakdowns, and higher yield. However, since maintenance results in temporary production interruptions, treating maintenance as a function separate from production does not guarantee good system performance, especially the ability to produce the required quantity of high quality products in a timely manner. Instead, one should seek to harmonize maintenance with production to ensure process efficiency.

> The central thesis of this dissertation is that integrating maintenance and production decisions increases efficiency by ensuring high quality production, effective resource utilization, and on-time deliveries.

The challenges for coordination of maintenance and production depend on the type of maintenance strategy. The general maintenance strategy of a production system can be one of the following:

- Corrective maintenance where there is no control over machine conditions and maintenance is carried out only after machine failure. This maintenance strategy is appropriate if the machine failure behaviour is independent of its state, for example, its age, or if precautionary maintenance is not beneficial due to economic considerations.

- Preventive maintenance where machine conditions can be partially controlled by performing maintenance both before and at failures to decrease the number of breakdowns. This maintenance strategy is applicable if the frequency of machine failure changes depending on its state or there is a measurable condition which can signal incipient failures.

When the maintenance plan is corrective, there is no explicit decision on when to schedule maintenance and machines are maintained reactively upon failure. The only decision is about production. Since each machine failure interrupts the system, ignoring the possibility of machine breakdowns in determining production decisions leads to an imprecise estimate of the production capacity, and the production plan and schedule[1] will likely be inaccurate. It is desirable to make planning and scheduling decisions that are optimal for the particular machine failures that are actually going to happen. Clearly, this ideal situation is not achievable since information on machine breakdowns and, consequently, the production unavailability periods are not known in advance. Therefore, a realistic research challenge is to construct a production plan and schedule which perform well for the majority of scenarios of machine breakdowns and are flexible enough to be adjusted as new information on unavailable production periods becomes known.

In the case of a preventive maintenance strategy, both maintenance and production decisions are relevant. Determining maintenance decisions individually based only on the state of machines, such as their age and failure characteristics, results in a static rule (Dijkhuizen and Harten, 1998): Machine X should be maintained after Y hours of operations. However, static rules are indifferent to fluctuations that might happen over time in a production system. For example, if the production system is heavily loaded, there is an opportunity for significant financial gains by delaying maintenance (Kaufman and Lewis, 2007). As another example, if there is large inventory on hand, it intuitively makes sense to benefit from the reduced need for production by scheduling maintenance earlier. Therefore, incorporating the operational state of the production system such as inventory, workload, and due dates into maintenance decisions leads to better allocation of resources to maintenance and production. Furthermore, in a system with partial control over machine conditions, performing preventive maintenance, though decreasing the number of breakdowns, will result in planned periods of process unavailability that could be otherwise allocated to production. Thus, the desirable production plan and schedule are not only hedged against various unplanned interventions; they also include the minimum number of planned unavailability periods while ensuring a highly reliable system. The research challenge in this case is to utilize both the available information on machine conditions and the operational state of the process to simultaneously make maintenance and production decisions.

This dissertation develops models and optimization techniques that integrate maintenance and production decisions, addressing the challenges noted above. To create a framework capturing possible interdependencies between production problems and maintenance strategies, we divide the production problems into planning and scheduling and the maintenance strategies into corrective and preventive.

[1] In this dissertation we distinguish between production plans and schedules as follows. A production plan evaluates capacity needs and determines the optimal production quantities (Nahmias, 2005). A production schedule allocates the available capacity to competing customer orders over time (Pinedo, 2005).

The investigation of our thesis focuses on three areas.

1. Integrated maintenance and production planning with partial control over machine conditions in the context of a periodic review production system. The goal is to determine the optimal amount of investment in maintenance for a given production quantity considering the inventory on hand.

2. Integrated maintenance planning and production scheduling with no control over machine conditions in the context of a dynamic military aircraft repair shop. The goal is to create the optimal production schedule considering the uncertainty on machine conditions and due dates of products.

3. Integrated maintenance planning and production scheduling with partial control over machine conditions in the context of a multi-machine production system. The goal is to simultaneously find the maintenance plan and determine the optimal maintenance and production schedules considering the available information on machine conditions, planned production interruptions, and product due dates.

In the first area of our study, we use stochastic optimization techniques to make integrated maintenance and production planning decisions since both decisions are long-term and are based on stochastic and aggregate information. For example, the production process is abstracted as a single-stage process and all customer orders are considered similar, requiring the same production capacity and due at the same time. In the last two areas of our study, to make integrated maintenance and production scheduling decisions, both stochastic and complex combinatorial properties must be properly modeled. Production scheduling decisions are addressed by combinatorial optimization tools since, in contrast to maintenance decisions, they are short-term and are based on combinatorial information. The existence of different customer orders with various requirements and multiple machines with complex interdependencies is, for example, taken into account. For making integrated maintenance and scheduling decisions, we develop decomposition techniques that deal with stochastic and combinatorial challenges in different, coupled stages. These techniques combine the ideas of stochastic optimization tools such as dynamic programming (Puterman, 1994) with those of combinatorial optimization techniques such as mixed integer programming (Queyranne and Schulz, 1994), constraint programming (Baptiste et al., 2006), and logic-based Benders decomposition (Hooker and Yan, 1995; Hooker and Ottosson, 2003).

1.1 Dissertation Outline

Chapter 2 provides a review of the literature on the interdependencies of maintenance and production problems. It presents a novel framework with three axes: the type of production problem, the maintenance strategy, and the length of the decision horizon. Production problems are divided into planning and scheduling, maintenance strategies into corrective and preventive, and the length of the decision horizons into long- and short-term. The combinations of different problems on the three axes indicate the areas where maintenance and production can be integrated. A thorough review of the formulations and the solution methodologies in each area is provided. The review forms a foundation where possible

relationships between maintenance and production are presented in a principled and intuitive structure. Finally, the chapter identifies the dissertation's three main areas of investigation: integrated maintenance and production planning with partial control over machine conditions, integrated maintenance planning and production scheduling with no control over machine conditions, and integrated maintenance planning and production scheduling with partial control over machine conditions.

Chapter 3 addresses the integrated maintenance and production planning problem with partial control over machine conditions in the context of a periodic review production system. It considers a firm that produces a single product in a single-stage process. Due to machine deterioration and unexpected failures, the quantity produced (yield) is random and the firm decides to invest in preventive maintenance to increase the number of finished high quality products. The problem at the beginning of each period is to simultaneously determine the production quantity and the amount of investment in maintenance. We use stochastic dynamic programming to identify the optimal policy such that the discounted expected total cost is minimized over multiple periods. However, due to the non-convexity of the cost function, we analyze a simpler problem where the production quantity is fixed. Our goal is to characterize the structure of the optimal maintenance policy. The results show that the optimal maintenance investment policy is a threshold policy if the yield linearly changes in the amount of money invested and there is a strong condition on the expected value of yield such that the marginal yield is always positive. If investment does not always increase the yield, by using Chebyshev's other inequality, we provide insight into the optimal threshold investment policy. Finally, we provide several managerial insights by comparing different problem parameters.

Assuming that the production quantity and the amount of maintenance investment are known, Chapter 4 studies the relationship between maintenance and production from an operational perspective. The problem of integrated maintenance planning and production scheduling where machines are only correctively maintained is investigated in the context of a dynamic military aircraft repair shop. The set of production activities (flights) is already scheduled, with each flight requiring a certain number and type of machines (aircraft). The machines might unexpectedly break-down, limiting their availability for production. Maintenance on failed machines must be scheduled to ensure that the production activities are carried out as scheduled. To solve the problem, we view the dynamic repair shop as successive static repair scheduling sub-problems over shorter time periods where the uncertainty about machine failures is incorporated in the repair schedule. We propose a complete approach based on the logic-based Benders decomposition to solve the static sub-problems and design different rescheduling policies to schedule the dynamic repair shop. Computational experiments demonstrate that the Benders model is able to find and prove optimal solutions on average four times faster than a mixed integer programming model. The rescheduling approach that can schedule over a longer horizon and quickly adjust the schedule increases the number of machines available for production in the long term by 10% over the approaches using only one aspect.

Continuing the study of integration of maintenance planning and production scheduling, Chapter 5 assumes that machines can be maintained both before and at failure. A multi-machine production system over multiple periods is considered where different customer orders must be processed on each

machine and are due at different times. Each machine deteriorates as it is used for production, and the production capacity decreases as a result. Maintaining machines before failure improves their conditions but interrupts production. The challenge is to simultaneously determine the allocation of maintenance to machines and time periods and to schedule maintenance and production in each period, utilizing the available information on machine conditions. The deterioration of each machine is modeled assuming that its speed decreases deterministically as the number of time periods since maintenance increases. To solve the problem, motivated by logic-based Benders decomposition, we design an integrated two-stage algorithm. The first stage assigns maintenance to machines and time periods, abstracting the scheduling problem, while the second stage creates a schedule for the current time period. The first stage is then re-solved using feedback from the schedule. This iteration between maintenance planning and scheduling continues until the solution costs in two stages converge. Our results demonstrate that the benefit of integrated decision making increases when maintenance is less expensive relative to lost production cost and that a longer horizon for maintenance planning is beneficial when maintenance cost increases.

Chapter 6 addresses the same problem as Chapter 5, using a more sophisticated and realistic model of stochastic machine deterioration. A set of discrete states represents different machine conditions and the deterioration process follows a continuous time Markov chain. Similar to Chapter 5, we design a two-stage algorithm to solve the problem. In the first stage, we formulate a Markov decision process model to determine the maintenance policy where the scheduling constraints are abstracted. The maintenance policy defines a decision rule for performing maintenance. We also derive sufficient conditions guaranteeing the monotonicity of the maintenance policy in both machine state and demand. In the second stage, we formulate a mixed integer programming model to find the maintenance and the production schedule in the current period incorporating all scheduling combinatorics. Our computational results demonstrate that exploiting machine condition information in maintenance and production scheduling decisions leads to 21% cost savings on average. Furthermore, the benefit of integrating maintenance reasoning in production scheduling decisions is higher for high discount factors and for industries with medium mean time to failure.

Chapter 7 outlines future work extending the problems studied in Chapters 3 to 6 and suggests two general directions to better model realistic problems which are both stochastic and combinatorially complex. Finally, it discusses the relevance of our approach to other integrated decisions in supply chain management.

Chapter 8 summarizes the contributions of the dissertation and provides a conclusion.

Appendix A includes the proofs of the propositions in Chapter 3. Several structural properties of the production scheduling problem in Chapter 5 are given in Appendix B. The exact method for calculating the average production rate used in the production scheduling problem and the detailed experimental setup of Chapter 6 are presented in Appendices C and D, respectively.

1.2 Summary of Contributions

The main contributions of this dissertation are summarized below.

- We are the first to theoretically identify the set of conditions that guarantee the existence of an optimal threshold type maintenance policy in a periodic review production system with random yield. The results will help managers to decide how much money should be invested to improve the state of the production system.

- We provide several managerial insights, including how the amount of investment in maintenance changes as the inventory or the total available budget increases. Understanding these relationships is useful to coordinate maintenance and production planning decisions.

- We are the first to develop optimization techniques that can effectively reason about both stochastic and combinatorial challenges in the context of maintenance and production scheduling decisions over a long-time horizon. Our techniques are all based on the idea of decomposition where the stochastic and the combinatorial challenges are addressed in different, coupled stages.

- We design an integrated technique to create a repair schedule for a dynamic military aircraft repair shop problem and show that adjusting the repair schedule as new short-term information becomes known significantly increases flight coverage. The integrated technique is based on a novel logic-based Benders decomposition approach which is four times faster than a novel mixed integer programming model on average which in turn is two orders of magnitude faster than an existing mixed integer programming model in the literature.

- We are the first to explicitly model the effect of machine deterioration and restoration on the processing times of customer orders in integrated maintenance and scheduling decisions.

- To precisely model the production capacity as a function of both machine state and the operational state of the system in a multi-machine production environment, we design appropriate solution techniques that depend on the deterioration process of machines. More specifically,

  - if machines deteriorate as the number of time periods since maintenance increases, we design a coupled two-stage integrated approach inspired by the idea of logic-based Benders decomposition; and
  - if machines deteriorate following a continuous Markov chain, we design a two-stage decomposed approach combining Markov decision process and mixed integer programming.

- We are the first to prove the conditions guaranteeing the monotonicity of a maintenance policy on both machine state and the number of customer orders when the effect of performing preventive maintenance on the production is not certain. More specifically, we consider preventive maintenance does not necessarily make the machine as good as new.

Chapter 2

Integrated Maintenance & Production: A Literature Review

Changing trends in production including the introduction of "just-in-time" inventory management in the past few decades has increased the importance of timely and continuous production (Dekker et al., 1997; Waeyenbergh et al., 2000). Keeping large inventories of finished products does not ensure customer satisfactions anymore because of the fast technological changes (Waeyenbergh et al., 2000). As a consequence, to cope with the current competitive environment where customers expect higher product quality, on-time deliveries, and a higher degree of customization, a production system is forced to be highly reliable (Waeyenbergh et al., 2000). However, a real-world production system is dynamic and uncertain where machine breakdowns make the production capacity unavailable and imperfect processes produce defective products. Deteriorating machine condition is one of the main sources of uncertainty in both machine breakdowns and imperfect processes. Maintenance, on the contrary, improves machine conditions, but it uses the potential production time that could be otherwise allocated to production. Therefore, the interdependency between maintenance and production and their conflicts in the short term have necessitated developing models and techniques that integrate maintenance reasoning into production problems. The goal of the integrated techniques is to guarantee the continuous functionality of the production system to produce the required quantity of the products with the required quality in a timely manner (Ben-Daya and Rahim, 2001; Pintelon and Parodi-Herz, 2008).

In this chapter, we review the literature addressing the interdependency between maintenance and production. We first define our framework for classification of this relationship, we then provide an overview of each area of our classification in detail and state the connection between the relevant literature and the corresponding contributions of this dissertation.

2.1 Classification Scheme

A production system deals with two different problems:

- Production planning: Production planning determines the optimal production quantities, also

known as lot-sizing, and evaluates the required production capacity (Nahmias, 2005).

- Production sequencing/scheduling: Production sequencing/scheduling addresses the problem of allocating the available production capacity and assigning start times to production jobs (Pinedo, 2002).

Maintenance reasoning in a production system is mainly addressed in two situations in which there is either no control or partial control over machine conditions. The former situation includes corrective maintenance, i.e., machines are maintained only at failure while the latter includes both corrective and preventive maintenance (Wang, 2002), machines can be maintained to prevent incipient failures. The combination of two different production problems and two different maintenance situations defines different problems where maintenance and production decisions are interdependent.

We further classify the literature addressing the interdependency between maintenance and production into two categories based on the length of the decision horizon.

The first category includes the literature that considers a long-term decision horizon, optimizing strategic goals which are mainly aligned with maintenance literature objectives. A graphical organization of this literature can be seen in Figure 2.1 where the x-axis and the y-axis represent the production problems and the maintenance situations, respectively. As illustrated, the integration of production planning problem with two different maintenance situations defines the Random Yield literature without and with Maintenance. The integration of production sequencing/scheduling problem with maintenance where machines can be partially controlled is the concern of Maintenance & Production Sequencing/Scheduling literature, while its integration with maintenance where there is no control over machine conditions in the long term can be seen as a queuing problem with unreliable servers (Wang, 1990; Wang and Kuo, 1997; Chakravarthy and Agarwal, 2003).

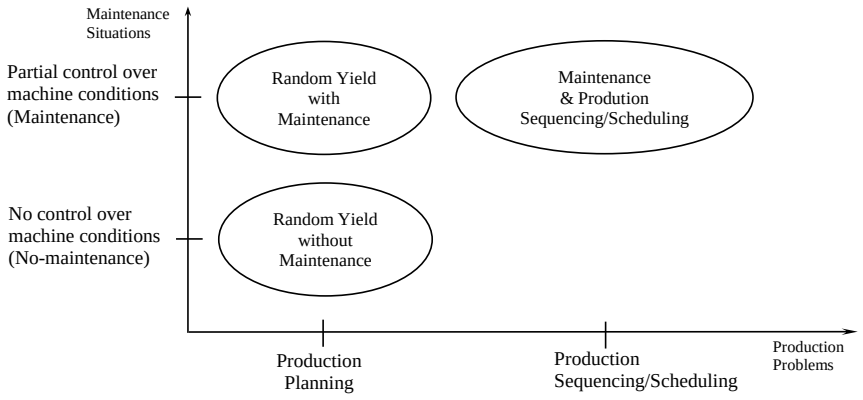


Figure 2.1: Different literature on addressing the relation between maintenance and production taking a long-term perspective.

The second category consists of problems studied in the scheduling literature which frequently optimizes the short-term and operational goals of the production system. Figure 2.2 illustrates the themes in this literature. Since the production planning is a long-term strategic decision, its integration with

maintenance is not a concern in short run and, consequently, there is no literature addressing this relationship. The interdependency between production sequencing/scheduling with a maintenance situation of no control and partial control over machine conditions is addressed in three separate literatures. Stochastic Sequencing/Scheduling and Dynamic Sequencing/Scheduling both study the former and Sequencing/Scheduling with Availability Constraints studies the latter.

In the following sections, we provide a detailed description of each category.

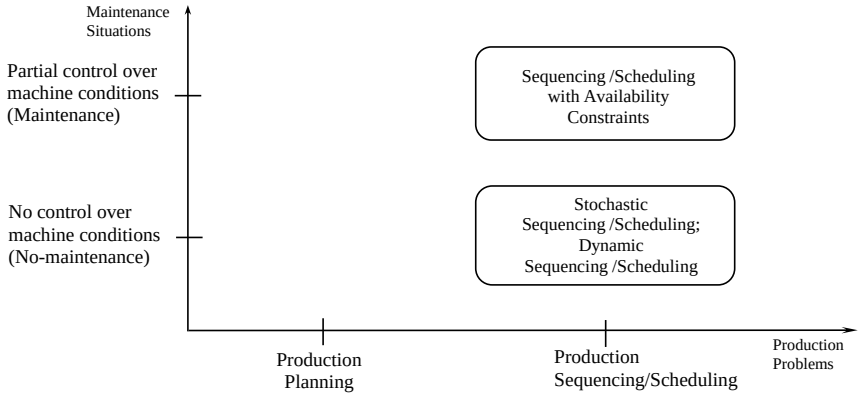


Figure 2.2: Different literature on addressing the relation between maintenance and production taking a short-term perspective.

2.2 Long-term Perspective

Taking a long-term strategic view, the main goal of a production system is to continuously produce the required number of products at the right moments. However, the dynamic characteristics of a real-world production environment such as deteriorating machine conditions, imperfect processes, and uncertain arrivals of customer orders constrain a production system from consistently meeting this goal. Maintenance, though resulting in temporary production capacity reduction, slows the degradation of the production process, increasing the production capacity in the long term. Therefore, utilizing the available information about the production process and machine conditions to simultaneously plan for maintenance and for production might be a potential strategy for improving the performance of the production system.

In this section, we first review the fundamentals of maintenance optimization problems. We then provide an overview of the literature integrating maintenance optimization and production where the objective is to guarantee the long-term efficiency of the production system by finding integrated production and maintenance policies.

2.2.1 Maintenance Fundamentals

Maintenance is the set of all actions keeping a system composed of multiple units or machines[1] in or restoring it to an appropriate condition to fulfill its defined function (Geraerds, 1985). To optimize

[1] In this chapter, a system refers to a production system and units and machines are used interchangeably.

maintenance, a mathematical model in which both costs and benefits of maintenance are quantified is constructed and an optimum balance is achieved (Dekker et al., 1996). The cost and benefit of maintenance are dependent on the maintenance policy, a mapping from the system states (breakdown, age, etc.) to maintenance actions (inspection, repair, replacement) (McCall, 1965; Dekker et al., 1996). One example of a maintenance policy is the age replacement policy where a unit is replaced at age T or at failure, whichever occurs first. Maintenance optimization models usually assume that the maintenance policy is known in advance, for example, it is the age replacement policy. Their focus is then on determining the optimal values for the policy parameters, i.e., finding the optimal value of T in the age replacement policy.

Since the literature on maintenance optimization problems is extensive and independent of production problems, we only review its main concepts to provide the reader with understanding of the maintenance terms used in this dissertation. Interested readers are referred to the books by Ebeling (1997), Jardine and Tsang (2006), and Nakagawa (2005; 2010; 2011). A detailed survey on the maintenance models can be found in papers by Cho and Parlar (1991), Dekker et al. (1997), Wang (2002), and Nicolai and Dekker (2008).

In this section, we first formally define the most important quantity of the maintenance theory, *failure rate*, we then provide an explanation for the most general maintenance action, *repair*, and recapitulate the four most widely used maintenance policies in their simplest form (Pintelon and Parodi-Herz, 2008). We finally briefly describe the common techniques for solving the maintenance optimization problems.

2.2.1.1 Failure rate

Failure rate is a measurement of how a unit improves or deteriorates with age. Suppose that a non-negative random variable, X, denoting the failure time of the unit, has the probability distribution $F(t) = \Pr(X \leq t)$ and the probability density function $f(t)$. The failure rate is defined as:

$$\lambda(t) = \frac{f(t)}{1 - F(t)} = \frac{\frac{dF(t)}{dt}}{1 - F(t)}$$

which physically means that $\lambda(t)\Delta t \approx \Pr(t < X \leq t+\Delta t | X > t)$, representing the instantaneous probability of failure: the probability that the unit with age t will fail in the interval $(t, t + \Delta t]$ for a small Δt, given it has survived until t (Nakagawa, 2005).

If $\lambda(t)$ is constant, i.e., $\lambda(t) = \lambda$, the failure behavior of the unit is random and does not depend on its age. However, if $\lambda(t)$ is increasing in t, the probability that the unit fails increases with its age.

2.2.1.2 Repair

The general maintenance action is repair, classified into three categories based on the degree of restoration: perfect, minimal, and imperfect (Wang, 2002). Assuming that $\lambda(t_b)$ and $\lambda(t_a)$ indicate the failure rate of the unit right before and right after the repair, the three different types are defined as follows:

1. If repair makes the unit as good as new, it is called renewal, perfect (complete) repair, or replace-

ment. Specifically, the failure rate of the unit after repair equals its failure rate at age 0, i.e., $\lambda(t_a) = \lambda(0)$.

2. If repair makes the unit as good as right before it fails, it is called minimal repair. In this case, $\lambda(t_a) = \lambda(t_b)$.

3. If repair makes the unit better than right before its failure, but worse than new, it is called imperfect (incomplete) repair. In this case, $\lambda(0) < \lambda(t_a) < \lambda(t_b)$.

2.2.1.3 Maintenance Policies

The four policies that we review are: failure-based maintenance (FBM), preventive-based maintenance (PBM), condition-based maintenance (CBM), and opportunity-based maintenance (OBM) (Pintelon and Parodi-Herz, 2008).

Failure-based Maintenance: Maintenance is carried out only after breakdowns. This policy is typically used in case of constant failure rate (random failure behavior) and low breakdown cost. For example, general machine-repairman problem has a set of workers and a set of machines that are subject to failures and therefore need repair. As the number of workers is typically less than the number of machines, several optimization problems can be defined in order to minimize the average cost per time unit: optimizing the repair rate, i.e., number of workers and number of repair facilities; optimizing the number of spare machines; and optimizing the repair scheduling policy.[2] A detailed survey of the first two optimization problems is provided by Haque and Armstrong (2007) and Cho and Parlar (1991), while the latter is comprehensively reviewed by Iravani et al. (2007).

The maintenance policy used in Chapter 4 is a failure-based maintenance policy where aircraft are minimally repaired upon failure.

Preventive-based Maintenance: Maintenance is carried out after a specified amount of time. The main assumptions of this policy are:

1. A unit fails gradually with time, i.e., its failure rate is increasing in time. Therefore, performing maintenance can change the failure time distribution of the unit, decreasing the expected number of failures in future.

2. The cost of preventive maintenance, repairing the unit before it fails is less than the cost of corrective maintenance, repairing the unit at failure. This assumption is essential to make the problem non-trivial; otherwise, the optimal decision is always to let the system operate until failure.

Three examples of PBM policies are as follows:

- Age replacement: The age replacement policy completely repairs (or replaces) a unit at age T or at failure, whichever occurs first. The age of a unit corresponds to its total up-time. The decision

[2]The problem of finding the repair scheduling policy can be seen as a problem of integrated production sequencing/scheduling with maintenance where there is no control over machines with a long-term perspective. This problem is addressed in queuing theory literature (Wang, 1990; Wang and Kuo, 1997; Chakravarthy and Agarwal, 2003).

is to find the optimal T. This policy is appropriate for a unit with catastrophic failure mode, where its failure is very serious and incurs a significant loss and, therefore, replacement at each failure is required (Nakagawa, 2005).

- Periodic replacement: Under this policy, a unit is completely repaired (replaced) at periodic times, kT, $k \in \{1, 2, ...\}$, independent of its age and failure history and is minimally repaired at its failures. If the unit is also replaced at failure times, the periodic replacement is called block replacement. The main advantage of this policy is that there is no necessity to record the age of the unit. This policy is reasonably applicable for units in complex systems when it is costly to replace a unit in operation (Wang, 2002; Nakagawa, 2005).

- Repair number counting: This policy replaces a unit at its k-th failure and the first $(k-1)$ failures are fixed by minimal repair. This policy finds the optimal k and its main difference with the age and the periodic replacement policies is that the time of performing preventive maintenance is random, being equal to the time of the k-th failure (Wang, 2002).

The maintenance model studied in Chapter 5 is a PBM policy where the number of time periods between consecutive maintenance on a machine is optimized.

Condition-based Maintenance: Maintenance is carried out after the values of one or several system parameters exceed predetermined values. As in PBM, the cost of preventive maintenance is assumed to be less than the cost of corrective maintenance, but the failure rate of the unit does not necessarily increase in time. CBM is becoming a common approach in industries because the investment in underlying techniques such as vibration analysis and oil spectrometry is economically justified (Pintelon and Parodi-Herz, 2008).

In Chapter 6, a CBM policy is developed to perform maintenance on a machine each time its degradation level exceeds a threshold value.

Opportunity-based Maintenance: The OBM policy is defined for a multi-unit system with dependency among its units. The optimal maintenance policy of each unit, therefore, depends on the state of the other units: the failure of one unit results in a potential opportunity to perform maintenance on some of the other units (Wang, 2002). One of the main interactions between components is economic where the cost of joint maintenance of a group of components is not equal to the total cost of individual maintenance of each component: it can be either lower (positive economic dependence) or higher (negative economic dependence) (Nicolai and Dekker, 2008).[3]

The maintenance policies in Chapters 5 and 6 are developed for a multi-machine production system where there is negative economic dependency between machines. More specifically, there is a limit on the available maintenance capacity, implying that more than a specified number of machines cannot be maintained at the same time.

[3] An opportunity-based maintenance policy is also applicable for a single-unit system with different failure modes where breakdown as a result of one failure mode is an opportunity for performing maintenance to decrease the probability of failure due to other failure modes (Jhang and Sheu, 1999).

2.2.1.4 Solution Techniques

There are two typical forms of maintenance optimization problems. The first form assumes that the maintenance policy is known and finds the optimal values of the given policy parameters. For example, with a periodic replacement policy, the maintenance optimization problem is to find the optimal T to perform complete repair at periodic times of $kT, k \in \{1, 2, ...\}$. Renewal theory is the usual solution technique for this form of maintenance optimization problems with the typical objective of minimizing the total expected cost per time unit. The second form aims to find the optimal maintenance policy, i.e., the optimal mapping between system states and maintenance actions and its primary solution technique is dynamic programming (McCall, 1965; Dohi et al., 2000).

In this section, we briefly describe each solution technique.

Renewal Theory: The renewal process can be formally defined as follows (Nakagawa, 2005; Ross, 2010):

Renewal Process: Consider a sequence of independent and non-negative random variables $\{X_1, X_2, ...\}$ where $\Pr(X_i = 0) < 1, \ \forall i$, to avoid triviality. Suppose that $X_2, X_3, ...$ have an identical distribution of $F(t)$ with finite mean μ and X_1 possibly has a different distribution of $F_1(t)$ with mean μ_1. Three different renewal processes can be defined depending on the following types of $F_1(t)$.

1. If $F_1(t) = F(t)$, i.e., all random variables are identically distributed, the process is called an *ordinary renewal process* or a *renewal process*.[4]

2. If $F_1(t)$ and $F(t)$ are not the same, the process is called a *modified* or a *delayed renewal process*.

3. If $F_1(t)$ is defined as $F_1(t) = \frac{\int_0^t [1-F(u)]du}{\mu}$, the process is an *equilibrium* or a *stationary process*.

The following example makes the above definitions clearer. Consider a unit with the maintenance policy of replacing it with a new one upon failure. Further assume that the time of replacing the unit is negligible such that the unit starts operating immediately after replacement. Assume that X_1 and $X_i, \ \forall i > 1$, are random variables representing the time to the first failure and the time between the $(i-1)$- and the i-th failures with distributions $F_1(t)$ and $F(t)$, respectively. If the unit is installed at time $t = 0$, then all the failure times have the same distribution representing an ordinary renewal process. If the unit is in use at time $t = 0$, X_1 is the residual time to the first failure and could have a different distribution from the failure time of a new unit. The sequence of time to failures, therefore, represents a delayed renewal process. If the observed time origin is sufficiently long after the installation of the unit and X_1 has the distribution of $F_1(t) = \frac{\int_0^t [1-F(u)]du}{\mu}$, the sequence of time to failures then denotes an equilibrium (a stationary) renewal process. Under both ordinary and stationary renewal processes, the expected number of failures in the interval of $[t_1, t_2]$ depends on the length of the interval, $(t_2 - t_1)$, and the mean time to failure for a new unit, μ, being equal to $\frac{t_2-t_1}{\mu}$ (Barlow and Proschan, 1996).

The fundamental theorem in minimizing the cost function in the first forms of maintenance optimization problems is the *renewal reward theorem*. Before stating the theorem, we define some notation.

[4]The ordinary renewal process is the most common process used in the maintenance optimization models.

Since at each failure, the unit is renewed and X_i corresponds to the time between the $(i-1)$- and the i-th renewals, $S_n = \sum_{i=1}^{n} X_i$ represents the time of the n-th renewal and $N(t) = \max\{n : S_n \leq t\}$ denotes the number of renewals during $(0, t]$. Assuming that R_i is the reward earned at the i-th renewal, $R(t) = \sum_{i=1}^{N(t)} R_i$ represents the total reward during $(0, t]$. Now, we state the theorem below (Nakagawa, 2005; Ross, 2010).

Renewal Reward Theorem: Assuming that $\mathbb{E}[R] = \mathbb{E}[R_i]$ and $\mathbb{E}[X] = \mathbb{E}[X_i]$

a) with probability 1, $\lim_{t\to\infty} \frac{R(t)}{t} = \frac{\mathbb{E}[R]}{\mathbb{E}[X]}$

b) $\lim_{t\to\infty} \frac{\mathbb{E}[R(t)]}{t} = \frac{\mathbb{E}[R]}{\mathbb{E}[X]}$

Letting the time between renewals represent one cycle, the renewal reward theorem shows that the expected reward per unit of time for an infinite time span equals the expected reward per one cycle divided by the mean time of one cycle.

To show the application of the renewal reward theorem in maintenance models, consider the age replacement policy as defined earlier where the goal is to find the optimal T such that the expected maintenance cost per time unit is minimized. We define C_r and C_p as the cost of replacing the unit at failure and the cost of replacing the unit at time T before failure. Under the age replacement policy, the unit is replaced at the first failure denoted by random variable X with failure distribution $F(x)$ or at time T, whichever occurs first. Therefore, the length of each renewal cycle equals $\min(X, T)$ with the expected value $\mathbb{E}[\min(X, T)]$. The cost of each renewal cycle is equal to $C_r I(X \leq T) + C_p I(X > T)$ where I is an indicator function. The expected cost per cycle is then equal to $C_r F(T) + C_p(1 - F(T))$. Using the renewal reward theorem, the optimal T minimizes the expected cost per cycle divided by the expected length of the cycle, i.e., $\frac{C_r F(T)+C_p(1-F(T))}{\mathbb{E}[\min(X,T)]}$.

Dynamic Programming: The dynamic programming framework provides an opportunity for the decision maker to influence the behavior of a probabilistic system through choosing a sequence of actions which causes the system to perform optimally with respect to some predetermined performance criterion (Puterman, 1994). There are five elements to almost any dynamic programming model: decision epochs, states, actions, rewards, and transition probabilities. Each element is briefly described below accompanied with maintenance related examples. Unless otherwise indicated, the details of the following are from the book by Puterman (1994).

Decision Epochs: Decisions are made at points of time called decision epochs. The set of decision epochs, T, can be classified as either a discrete or a continuous set and as either a finite or an infinite set. In a discrete time problem, time is divided into periods and the decision epochs correspond to the beginning of each period denoted as $t \in \{1, 2, ...\}$. In continuous time problems, the decision time points are random points of time denoted as $t \in [0, \infty)$; for example, the breakdown of a machine or the completion time of a repair corresponds to a potential continuous decision epoch.

States: At each decision epoch, the probabilistic system is in a state. The set of all possible system states is denoted by $\mathcal{S}$ where $S_t = s$ corresponds to the system state at decision epoch t. The set $\mathcal{S}$ can have several dimensions and can be finite or infinite. For example, the age of a machine can represent the

machine state at every decision time epoch. However, the most common approach in the maintenance literature is to represent the state of the machine using a finite discrete set, i.e., $\{0, 1, ..., N\}$ where 0 and N indicate the new and the failed conditions of the machine and the middle states represent intermediate levels of machine health.

Actions: At each decision time epoch, t, after observing the state of the system, $S_t = s$, the decision maker chooses an action a_t from the non-empty set of all possible actions at state s, $a_t \in A_s$ where $\mathcal{A} = \cup_{s\in\mathcal{S}} A_s$ is the action space. The common actions in maintenance models at each decision time epoch, independent of the state of the system, are "no repair" and "repair".

Rewards: As a result of performing action a_t at state S_t at decision time point t, the decision maker pays cost $C_t(S_t, a_t)$. For example, if the chosen action is to perform repair, the decision maker pays the cost of repair plus the other related operational costs such as lost production cost.

Transition Probabilities: The system state at next decision epoch is determined by the probability distribution $p_t(.|S_t, a_t)$. For example, let's divide the time into equal time periods with length $\mathcal{T}$ where the beginning of each period represents a decision epoch. If the decision maker replaces the machine with a new one at the beginning of the n-th period, the transition probability that the machine is in a failed state at the next decision epoch, the beginning of the $(n+1)$-th period, equals $\Pr(X \leq \mathcal{T}) = F(\mathcal{T})$ where $F(x)$ is the failure distribution of the new unit.

Given the above five elements, the main goal of a dynamic programming model is to find a decision rule, or a policy, prescribing a procedure for choosing the actions at each state of the system at any decision time point such that a predetermined objective, for example, the expected sum of costs over finite decision epochs $t \in T$, is minimized. Formally, the Markov[5] policy π, chooses action $A_t^\pi(S_t)$ at time t if the system state is S_t. If the selection of actions does not depend on time, the policy is a stationary policy. The goal is then to find the optimal policy $\pi \in \Pi$, minimizing the expected sum of costs over the finite set of decision epochs $t \in T$ denoted as $\min_{\pi\in\Pi} \mathbb{E}[\sum_{t\in T} C(S_t, A_t^\pi(S_t))]$. For example, representing the possible states of the machine as $\{0, 1, ..., N\}$ and the possible actions at each state as "no repair" and "repair", the stationary policy π divides the state space into two sub-sets I and II: sub-set I includes the states in which it is optimal to perform repair and sub-set II includes the remaining states where it is always optimal not to perform repair.

The stochastic dynamic programming methods such as policy iteration, value iteration, and modified policy iteration are the common algorithms used to find the optimal policy (Heyman and Sobel, 1984; Puterman, 1994). However, the majority of literature has focused on characterizing the structure of the optimal maintenance policy. Interested readers are referred to the books by Bertsekas (2007) and Puterman (1994) for an extensive coverage on dynamic programming models and their applications.

In Chapter 6, we use Markov decision process to find the optimal maintenance policy.

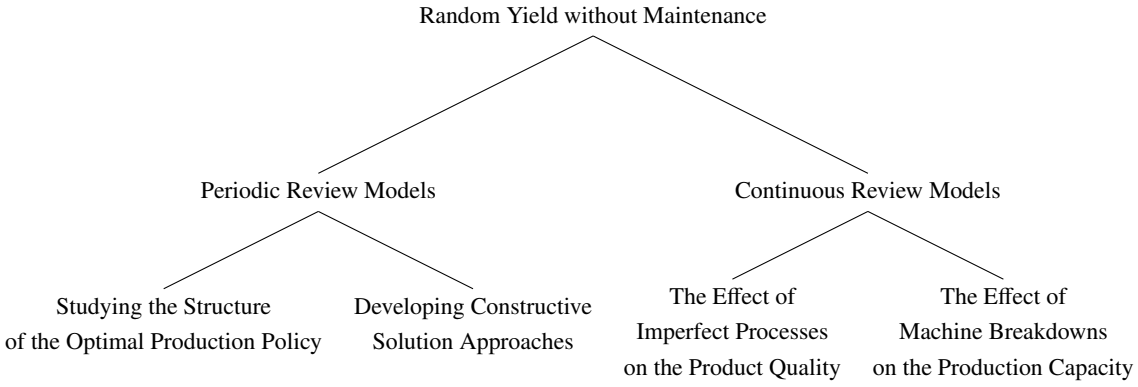
In the next three sub-sections, we review the literature that integrate maintenance and production taking a long-term decision horizon.

[5]The policy is Markovian since it depends on the previous system states and actions only through the current state of the system (Puterman, 1994).

2.2.2 Random Yield without Maintenance

This literature studies the problem of determining the production quantities where the yield (quantity produced) or the production capacity is random due to machine deterioration or imperfect processes. The main challenge in this literature is that the production output quantity is uncertain, i.e., it might differ from the input quantity (Yano and Lee, 1995) and since it is assumed that there is no control over machines, the only decision is to determine the production input quantity. Although in this literature there is no explicit connection between maintenance and production, we can simply assume that the maintenance policy is a failure-based policy and the cost of maintenance is reflected in decreasing the output quantities by representing the yield as a random variable with a certain distribution.

This literature can be divided into several categories as illustrated below. We briefly review work in each category.



2.2.2.1 Periodic Review Models

A classical periodic review model is a discrete model where the decision horizon is divided into time periods, the demand occurs at the end of each time period, the inventory level is periodically reviewed, and the decision to produce or not is made only at the beginning of the review periods. However, in this chapter, a model is classified as a periodic model if the decision time points are discrete even if some of the other classical assumptions do not hold. This sub-division of the random yield models without maintenance has been developed in two main directions: studying the structure of the optimal production policy and developing constructive solution approaches. A comprehensive review of the former is provided by Yano and Lee (1995) and the latter is thoroughly reviewed by Bollapragada and Morton (1999).

Studying the structure of the optimal production policy: Gerchak et al. (1988) studied a periodic review model with uncertain demand. They showed that the optimal production policy for the single-period problem is of threshold type where it is always optimal to produce if the amount of inventory on hand is below the re-order point and not to produce, otherwise. They further showed that the re-order point is not dependent on the distribution of the yield and that the policy of "order-up-to" is not optimal. That is, the optimal input production quantity is not the difference between the re-order point and the current inventory level. For the general multi-period problem, when yield is constant, it is known that

the optimal policy is myopic, i.e., each period's decision problem can be solved as if it were the last period, with appropriate modification of the parameters (Heyman and Sobel, 1984). However, Gerchak et al. (1988) showed that the optimal policy for the multi-period problem is not myopic when the yield is random. Henig and Gerchak (1990) later provided more insight into the properties of the order quantity (the input production quantity) and derived an easy approximation to find it.

The problem of lot-sizing with random demand was then extended to consider other sources of uncertainty such as random capacity (Wang and Gerchak, 1996); multi-product production systems (Hsu and Bassok, 1999); and costs that are dependent on the realized yield (Kazaz, 2004). Wang and Gerchak (1996) studied the problem of production planning in the presence of both random yield and supply disruption, i.e., random capacity. Although the cost function of the finite-horizon problem is proved to be quasi-convex, they showed that the optimal production policy still has a threshold type. Their work is a generalization of the work by Ciarallo et al. (1994) where the lot-sizing problem with random demand and random capacity is addressed. It is shown that the optimal production policy considering only the random capacity is an order-up-to policy distinct from the random yield and the random demand model. Hsu and Bassok (1999) addressed the problem of random yield with downward substitution. They assumed that there is one raw material as production input, producing N different products and that the demands and yields for the products are random and different. Downward substitution implies that one unit of a product class may be used to satisfy the demand for certain other product classes. They developed three different solution approaches, a stochastic linear program, a decomposition approach, and a greedy heuristic to determine the optimal production input and the allocation of the products to satisfy demands. Kazaz (2004) considered the problem of random yield in the olive industry where the sale price and the purchasing cost, though exogenous, are dependent on the realized yield.

While the majority of the models assume a single-stage production process, some attempts have been made considering multi-stage problems. Lee and Yano (1988) studied the problem of determining the optimal input production quantity at each stage of a series system where the yield in each stage is random and the demand is constant. Gerchak et al. (1994) and Gurnani et al. (2000) also investigated the problem of choosing the optimal lot-sizes in assembly systems. Since the presence of random demand makes the analysis of the multi-stage problems intractable, they are mainly studied in the context of Multiple Lot-sizing in Production to Order (MLPO) problems with a constant demand. Interested readers are referred to the survey paper by Grosfeld-Nir and Gerchak (2004).

In all the above models, yield is considered as an exogenous parameter and the focus is on determining the optimal lot-size. Although there are a few works in the continuous review system jointly optimizing the yield and the production policy (see Section 2.2.2.2 below), such a problem has not been studied in the periodic review system. To the best of our knowledge, there is only one work in the context of the periodic review system: Gupta and Cooper (2005) assumed that product- and process-improvement projects can increase the yield and studied the optimal direction of change in the yield distribution. They show that changing the yield so that it is stochastically larger does not guarantee the higher expected profit. The expected profit increases if the yield is smaller in the convex order, meaning that its expected value does not change and its variance decreases. From a managerial view point, know-

ing the optimal direction of change in yield is not enough for initiating process improvement projects such as maintenance. A manager needs to know the optimal amount of money that should be allocated to maintenance, and our work in Chapter 3 is a step in this direction.

Developing Constructive Solution Approaches: Since the structure of the optimal production policy with random yield is not myopic, the literature on developing efficient solution approaches has focused on when a myopic policy can be a good approximation to the optimal policy and has devised efficient algorithms to find the optimal production quantity at the beginning of each review period. Baker and Erhardt (1995) proposed a heuristic policy in the form of order-up-to policy where the demand and the yield are random. Their simulation study showed that ignoring the random yield does not result in a significant increase in the cost unless the service level is high or the process is extremely random. Bollapragada and Morton (1999) considered the problem with random yield where the demand is random but not stationary. Three heuristics are developed to solve the problem based on the position of the inventory at the end of the time period. The solutions of the heuristics are then compared with the optimal solution of a dynamic programming approach and it is experimentally shown that the best heuristic has the worst-case error of 3% and 5% for the infinite and finite cases. Li et al. (2008) based their work on the papers by Henig and Gerchak (1990) and by Bollapragada and Morton (1999) to find more insight to the values of order quantity and the re-order point. They derived upper and lower bounds for both the optimal order quantity and the order threshold and used the derived bounds to construct efficient heuristics.

2.2.2.2 Continuous Review Models

The classical continuous review model is the economic manufacturing quantity (EMQ) problem where the goal is to find the production up-time or alternatively the production quantity that minimizes the production and inventory costs assuming that the production and the demand rates are constant, that there is a single machine processing the products, that the inventory level is continuously monitored, that the product quality is always acceptable, and that the production capacity is always at its maximum limit (Nahmias, 2005). Relaxing the last two assumptions to explicitly model the effect of machine conditions on the product quality or on the production capacity is the focus of the random yield literature without or with maintenance in continuous review models. Continuous review models have been mainly used for the joint optimization of maintenance and production policies, described in Section 2.2.3.2. However, they were initially used to extend the EMQ models to address the uncertainty in the production process without explicitly considering maintenance.

To address the effect of imperfect processes on the product quality, Rosenblatt and Lee (1986) studied the effect of an imperfect production process assuming two states: "in-control" and "out-of-control". The time that the process shifts between two states is exponentially distributed and the product quality is determined only after production. They showed that the production run is shorter than that of the classical EMQ formula and that it decreases as the defective rate or the cost of defective items increases. Their analysis is further extended to incorporate the dynamic nature of deterioration processes, i.e., the proportion of defective items is not constant. Porteus (1986) studied the same problem assuming that the system incurs an extra cost for rework of each defective piece that it produces. Thus, there is an in-

centive to produce smaller lots, and have a smaller fraction of defective units. He also introduced three options for investing in quality improvements: (i) reducing the probability that the process moves out of control; (ii) reducing setup cost; and (iii) simultaneously using the two previous options. By assuming a specific form of the investment cost function for each option, he explicitly obtained the optimal investment strategy. Porteus (1986) motivated later work on jointly optimizing the yield variability and lot-sizing: Gerchak and Parlar (1990) where the yield variability can be affected by investment and Lin and Hou (2005) where the set-up cost and yield variability can be reduced through capital investment are two examples.

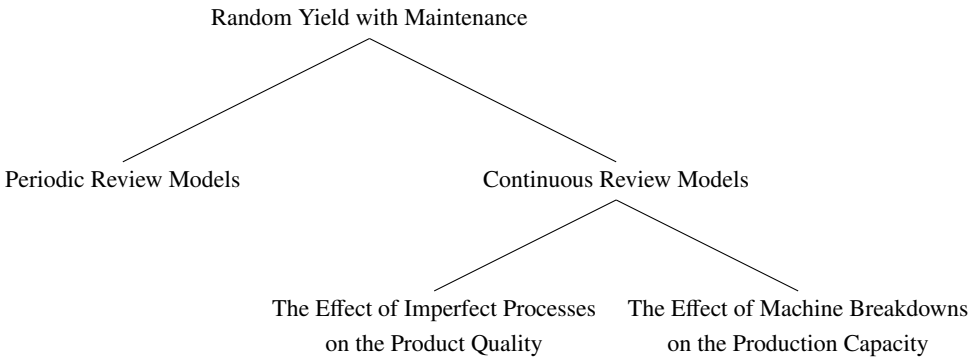
Groenevelt et al. (1992b) addressed the effect of machine breakdowns on the production capacity in the EMQ model. They studied the problem under two production policies: no-resumption and resume (abort). Under the no-resumption policy, the production of the interrupted lot is not resumed after a breakdown. The on-hand inventory is used before a new cycle is initiated. Under the resume policy, production is immediately resumed after a breakdown if the current on-hand inventory is below a certain threshold level. They showed that under both policies, the optimal lot-size will be always bigger than the one in the corresponding deterministic cases, and that the optimal lot-size increases with the failure rate. This paper has initiated a large amount of work discussed in Section 2.2.3.2.

To address both effects of an imperfect production process on the product quality and the machine breakdowns on the production capacity, Boon et al. (2000) used the no-resumption policy of Groenevelt et al. (1992b) such that two different probability distributions for time to transition from the in-control state to the out-of-control state and time to breakdown are considered.

2.2.3 Random Yield with Maintenance

The main assumptions of the Random Yield with Maintenance literature are similar to those in the literature reviewed in Section 2.2.2. The key difference is that maintenance and production decisions are both addressed because machine conditions can be partially controlled. Therefore, it is generally assumed that the production process deteriorates with time, i.e., the distribution of time to failure is considered to result in an increasing failure rate.

As shown below, we review this literature in two main categories of periodic review and continuous review models similar to the previous section. Budai et al. (2006) have classified the integrated maintenance and production models where both decisions are relevant into four categories: conceptual and process design models, economic manufacturing quantity models, production systems with buffer capacity models, and production and maintenance rate optimization models. Their classification can be considered as an alternative classification of our Random Yield with Maintenance category where our periodic review and continuous review sub-categories include the production systems with buffer capacity and the economic manufacturing quantity models of Budai et al. (2006), respectively. Our classification allows a better description of the similarities and the differences in Random Yield literature with and without maintenance.



2.2.3.1 Periodic Review Models

In periodic review models, the decision time points are discrete, though they are not necessary fixed. The main focus of this body of the literature, similar to the work reviewed in Section 2.2.2.1, is on characterizing the structure of the optimal joint maintenance and production policy. However, since finding the joint optimal policy is analytically hard, the problem has been usually reduced to finding the optimal maintenance policy assuming a fixed production policy and more specifically to determining sufficient conditions which guarantee a threshold optimal maintenance policy. The main solution techniques for proving such conditions are either Markov decision processes or semi-Markov decision processes depending on whether the decision time points are fixed or random, respectively.

Van der Duyn Schouten and Vanneste (1995) did early work where a single-stage production process with constant production rate (p) and constant demand rate (d) is considered. The process is subject to costly failures which result in production shutdowns. Three possible options are, therefore, considered to decrease the interruption of the production capacity: a buffer with fixed capacity of K, corrective maintenance, and preventive maintenance. The state of the system, composed of the number of time periods since previous preventive maintenance and the size of the buffer, is observed at discrete time periods. The number of time periods since previous maintenance is a discrete indication of the age of the machine. These time epochs are the only opportunity to stop production and start maintenance in order to minimize the average lost demand of the production process per time unit. Since the structure of the optimal policy is hard to characterize, a sub-optimal policy of (n, N, k) is developed to perform preventive maintenance if the system age is n and the buffer is full or if the system age is N, $(N \geq n)$ and there are at least k finished products in the buffer. The main two assumptions of this model are: (i) a stationary deterioration process and (ii) constant demand and production rates. Relaxing each of these two assumptions has initiated a body of work.

Addressing a non-stationary deterioration process, Kyriakidis and Dimitrakos (2006) considered the same problem as Van der Duyn Schouten and Vanneste (1995) where the state of the system is composed of three values: the number of time periods since maintenance, the buffer size, and the age of the single-stage production process. Adding the latter to the state representation allows consideration of situations where the transition probabilities between states might change depending on time. The non-stationary deterioration process model is then extended assuming that the production process remains idle from completion time of maintenance until the buffer size reaches zero (Karamatsoukis and Kyriakidis, 2009)

and that the repair time is increasing in the number of previous repairs (Dehayem Nodem et al., 2011).

To address random demand and production rates, Das and Sarkar (1999) did the early work studying a problem with an application in discrete manufacturing industries. The production time of each product is generally distributed, the production process is prone to failures where the time to failure and the time to repair are generally distributed, and the inventory system is controlled using a (S, s) policy. Under this inventory policy, the production stops when the buffer inventory reaches S and the production resumes when the inventory drops to s. Decision epochs in this model are random, equivalent to completion times of products. When processing of a product is finished, the state of the system, a vector of the number of products produced since previous maintenance and the buffer size, is observed. The decision on whether to continue production or to stop it for maintenance is then made. Since decision epochs are random times, a semi-Markov decision process framework is developed to solve the problem using a numerical search technique.

While in the model of Das and Sarkar (1999) the production policy is defined as a (S, s) policy, Iravani and Duenyas (2002) focused on the joint characterization of maintenance and production policies where the general distributions of time to produce, time to failure, and time to repair in the model by Das and Sarkar (1999) are replaced by exponential distributions. However, using a semi-Markov decision process, the structure of the optimal policy is shown to be complex. Utilizing several properties of the optimal joint policy, a heuristic policy with two threshold values for stopping the production and for undertaking maintenance is numerically investigated. Yao et al. (2005) further relaxed several other assumptions and under some conditions, such as high positive or negative inventory levels, showed that the maintenance policy has a control-limit structure.

In the models addressing the relationship between maintenance and production with random demand and production rates, it is assumed that time to produce a final product is random. However, Sloan (2004) took a different modeling approach and assumed that the number of acceptable final products, i.e., yield, produced in a given time period is random and has a binomial distribution. He showed that under some reasonable conditions regarding the yield and machine deterioration, threshold maintenance and threshold production policies exist, but the production policy is not monotone in the machine state. A numerical investigation resulted in 18% cost savings of the integrated solution approach over a sequential one where the maintenance and the production policies are independently determined.

Keeping inventory in stock to satisfy customer orders at process failures is considered as a strategy in all the previous models. Kaufman and Lewis (2007), however, considered the problem of integrated maintenance and production excluding this strategy. They studied the problem for a single server queue where the deterioration process of the server is described by decreasing service rates. The production policy is first-come, first-served and the maintenance policy is considered to be dependent on both the number of customers in the queue and the deterioration level of the server. It is shown that the maintenance policy has a switching curve structure[6] which is monotone in the service rate and is non-

[6]A deterministic stationary policy is a switching curve policy if it can be described by a curve in state space $\mathbb{X}$ that separates $\mathbb{X}$ into two connected regions. In one region the policy calls for action "no repair" to be used, while in the other region action "repair" is used. Furthermore, a switching curve policy is called monotone if the curve dividing $\mathbb{X}$ into two regions is monotone.

monotone in the number of customers in the queue.

In all the above models, it is assumed that the state of the machine is fully observable. However, to gain insight into conditions where it is beneficial to invest in sensor technologies, Gilbert and Bar (1999) addressed the problem of determining the optimal maintenance policy in a small lot production setting considering two different models. In the first model, they assumed that the technology is available to observe the true state of the machine and in the second model they assumed that the state of the machine can be inferred from the quality of its previous outputs. For each of the models, it is shown that the optimal maintenance policy is a threshold type. Furthermore, using a set of numerical studies, three parameters are identified, contributing to the value of information of the true state of the machine. The three parameters are the rate at which the machine goes out of control, the ratio between the cost of repairing the machine and the cost of producing a defective product, and the probability with which the machine produces a defective product.

While all the papers reviewed above are developed for a single-product system, there are several papers addressing a multi-product system. For example, Aghezzaf et al. (2007) and Najid et al. (2011) presented a non-linear mixed integer programming model to address the problem of simultaneous determination of production quantities and the maintenance schedule in a single machine, multi-product system. They defined the maintenance policy such that the system is periodically renewed and it is minimally repaired when failure happens. Aghezzaf and Najid (2008) later extended the model to address a parallel machine production system where a noncyclic preventive maintenance policy is allowed. The differences of these models with the previous models of single-product systems are: the maintenance policy is assumed fixed, the production policy is instead optimized, and the solution approach is changed from dynamic programming to mathematical programming where there is a finite decision horizon as opposed to infinite one.

2.2.3.2 Continuous Review Models

In this section, we review the relevant work in the context of EMQ models where both the production policy and the maintenance policy are to be determined. The common theme of this sub-category of the literature is that a preventive-based maintenance policy is defined and its parameters are jointly optimized with the production input quantity. The main features of this literature distinct from the periodic review models reviewed in Section 2.2.3.1 are: instead of a fixed production policy, a fixed maintenance policy is considered, and the main solution technique is changed from Markov decision processes to renewal theory.

Similar to Section 2.2.2.2, we divide this sub-category into two streams: the effect of imperfect processes on the product quality and the effect of machine breakdowns on the production capacity.

To model the effect of imperfect processes on the product quality, Lee and Rosenblatt (1987) extended their previous work (Rosenblatt and Lee, 1986) by relaxing the assumption that the product quality is determined at the end of the production process. They assumed that the product quality can be determined during production by inspection. The problem of joint control of production cycles or manufacturing quantities and maintenance by inspection is considered for the first time in their work.

When maintenance by inspection is adopted, it is shown that the optimal inspection schedule is equally-spaced throughout the production run. The problem is then solved by using an approximation of the cost function. This model has later extensively studied, generalizing some of its assumptions. Porteus (1990) considered that the inspection model is a delay model: whenever a product is inspected, there is a delay for the outcome of the inspection to be revealed. Tseng (1996) extended the model assuming that the process lifetime has a general distribution with an increasing failure rate rather than an exponential distribution. Wang and Sheu (2003) extended the model addressing imperfect periodic inspections and used a Markov chain to jointly determine the production cycle, process inspection intervals, and maintenance level. Wang (2006) extended his previous work to derive some structural properties for the optimal production and preventive maintenance policy under the assumption that the sufficient conditions for the optimality of the equal-interval preventive maintenance schedule hold.

Although the demand in EMQ models is usually considered constant, the second stream, modeling the effect of machine breakdowns on the production capacity, studies both constant and random demand models. The constant demand models were initiated by Groenevelt et al. (1992a) where the problem of selecting the economic lot-size for an unreliable manufacturing facility with a constant failure rate and general distributed repair times is studied. They assumed that during each production run, a certain fraction, β, of the products is diverted to a separate stock, called safety stock, while the rest of products are considered as running stock and are used to meet customer demand. The safety stocks are used after each breakdown to satisfy demand while the machine under goes corrective maintenance. However, at the end of each production cycle, when machine under goes preventive maintenance, the running stock is used to meet the demand. It is further assumed that lost sales occur when the machine is broken and safety stocks are depleted, regardless of the running stock level. A closed form expression is then derived to determine the optimal lot-size. Cheung and Hausman (1997) considered the same problem for a manufacturing facility with an increasing failure rate where the preventive maintenance is performed every m time periods. Dohi et al. (2001) later revised the model of Cheung and Hausman (1997) to relax the strong assumption that the production process does not fail if the amount of stock is less than s. While all the previous work assumes that the duration of preventive maintenance is constant, Chelbi and Ait-Kadi (2004) considered a random duration for preventive maintenance.

To address models with random demand, Srinivasan and Lee (1996) extended the EMQ model assuming a (S, s) production policy where demand occurs according to a Poisson process and the production facility deteriorates in time. The maintenance policy minimally repairs the facility upon failure during operations and initiates preventive maintenance as soon as the inventory level increases to a certain prespecified value, S. After the preventive maintenance operation, the facility is restored to as good as new condition and the production resumes when the inventory level drops down to another prespecified value, s. Under a cost structure that includes preventive maintenance cost, repair cost, setup cost, holding cost, and backorder cost, an expression for the expected cost per time unit is obtained for a given policy. Some properties of the cost functions are developed and on the basis of these properties, an efficient algorithm is presented to find the optimal values of the given policy.

Makis and Fung (1996) and later Chakraborty et al. (2009) presented a model for joint determination

of the lot-size, inspection intervals and preventive replacement time, addressing both machine failure and imperfect process.

2.2.4 Maintenance & Production Sequencing/Scheduling

This literature addresses the problem of production sequencing/scheduling when machines can be partially controlled. The majority of this literature is defined for a single-stage (or a single machine) production process and, therefore, the problem reduces to product sequencing since there is no decision regarding the allocation of products to machines. The differences between sequencing and scheduling are explained in detail in Section 2.3.1.

The work in this area assumes that the lot-size is already determined and addresses the problem of *which* product to process next given that the machine deteriorates over time. The quality of a product depends on the machine condition and maintenance actions can improve machine conditions. The problem is formulated as a periodic review model with discrete decision time points. In the simplest version of the problem, the sequence of the events at the beginning of each time period is as follows: the condition of the machine is observed, then the decision either "to maintain" or "to produce" is made. If the decision is made to do maintenance, it is assumed that the whole period is taken by maintenance and the production is 0. If the decision is "to produce", the single product to be produced during the time period is selected. In different problem variations, maintenance and production times may be random (Sloan, 2008) or constant (Sloan and Shanthikumar, 2000; Kazaz and Sloan, 2008); there might be one (Sloan and Shanthikumar, 2000, 2002) or multiple maintenance actions (Sloan, 2008); the effect of maintenance on machine conditions might be probabilistic (Sloan, 2008) or certain (Sloan and Shanthikumar, 2000; Kazaz and Sloan, 2008).

Markov decision processes are the main solution approaches used to find the optimal solution minimizing the expected maintenance and production costs over infinite horizon. The focus of the solution approaches, similar to the periodic models reviewed in earlier sections, is on determining when a particular type of policy such as a monotone policy is optimal rather than finding the optimal values for a predefined policy.

Sloan and Shanthikumar (2000) did early work in a single-stage production process where all products have the same processing times, there is one maintenance operation, making the machine as good as new, and the machine deterioration process is independent of the products produced. The machine condition, which deteriorates over time, is represented by a value drawn from a discrete set. The problem of integrated production and maintenance is formulated as a constrained Markov decision process model where product mix constraints require that $\gamma_k \times 100\%$ of the total production must consist of product k. Sloan and Shanthikumar (2000) derived analytical conditions to have a control-limit maintenance policy and used linear programming to compare their proposed integrated approach with traditional sequential approaches. They also experimentally showed that the integrated production and maintenance policy has a substantial gain over traditional first-come, first-served (FCFS) production policy and fixed-time maintenance policy such as periodic maintenance. Recently, Batun and Maillart (2012) reassessed the numerical results of Sloan and Shanthikumar (2000) and concluded that the previous work overestimates

the sub-optimality of FCFS and underestimates the benefit of simultaneously optimizing maintenance and production. The work of Sloan and Shanthikumar (2000) has been extended to a multi-stage production system where the dependency between the stages are not considered (Sloan and Shanthikumar, 2002) and to a case where the processing times and machine state transition probabilities depend on the product type (Kazaz and Sloan, 2008, 2013).

Random processing times for different products, random maintenance duration, multiple maintenance actions, and probabilistic effects of performing maintenance where the machine does not necessarily return to the best state are studied by Sloan (2008). The decision epochs are random corresponding to completion times of production activities and maintenance operations where the state of machine is observed and the decision maker has the option to produce one of K products or to stop production and perform one of M maintenance operations. Using a semi-Markov decision process framework, sufficient conditions are developed to guarantee the monotonicity of both production and maintenance policies.

In all the previous work except Sloan and Shanthikumar (2002), a single-stage problem is considered, while Sloan (2013) extended their previous work to a multi-product, multi-stage system. Lee and Ni (2013) also extended the work of Sloan (2000) to address the integrated problem of maintenance and production dispatching in a multi-stage production system. They considered each machine individually and using the model from Sloan (2000) found the optimal maintenance policy for each machine. Since the maintenance capacity limit is not considered in the long-term plan, using a mixed integer programming model, they prioritized the maintenance activities in the short term.

2.2.5 Summary

We surveyed the models integrating maintenance reasoning into production problems, optimizing the performance of the production system in the long term. First, we provided a brief background on the common maintenance concepts and solution approaches. We then divided the production problems into production planning and production sequencing/scheduling and maintenance situations into no control and partial control over machine conditions.

Sections 2.2.2 and 2.2.3 reviewed the literature addressing the interdependency between production planning and maintenance situations with no control and partial control over machine conditions, respectively. Each literature is reviewed in two categories of periodic review and continuous review models to characterize the optimal joint production and maintenance policy. Section 2.2.4 presented the literature on the integrated problem of production sequencing/scheduling and maintenance reasoning with partial control over machine conditions where the periodic review models are mainly used to formulate the problem. Since modeling the relation between production sequencing/scheduling and maintenance with no control over machine conditions falls in queuing theory framework, we have not provided its review in this chapter.

2.3 Short-term Perspective

Taking an operational view, after the production quantities and the preventive maintenance activities are determined, one of the main short-term goals of the production process is to optimize the time and sequence of maintenance and production operations. Because of the preventive maintenance activities and unexpected breakdowns, the resources are not available all the times. Therefore, the main challenge is to allocate the right resource to the right operation, either maintenance or production, such that the products are ready at the right time or completed as soon as possible. The scheduling literature is the area that deals with this challenge.

In this section, we first provide the necessary background on scheduling problems, particularly on their solution approaches. We then review the literature addressing the relationship between maintenance and production where the short-term performance measures are to be optimized. Recall that in our classification scheme, we have considered two different production problems, planning and sequencing/scheduling, and two maintenance situations, no control and partial control over machine conditions. Four different problems can be defined in different intersections of production and maintenance problems. However, since production planning is not a short-term decision, we review the literature addressing the integrated problem of production sequencing/scheduling with two different maintenance situations of no control and partial control over machine conditions. As shown in Figure 2.3 which is repeated from Section 2.1, there are two separate literatures studying the former relationship and one dealing with the latter interdependency.

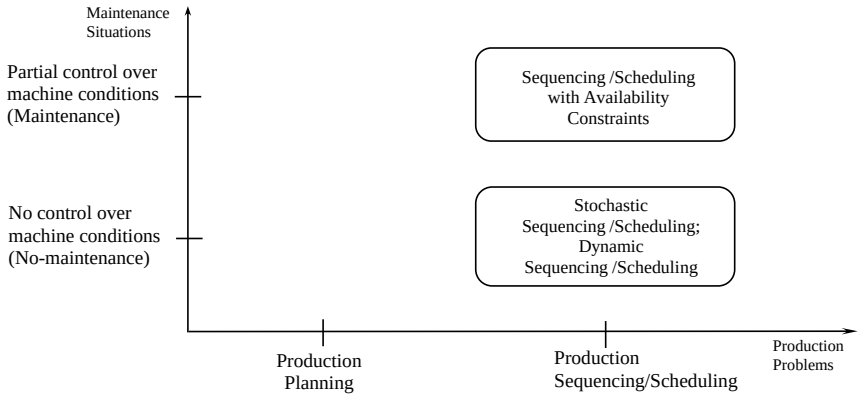


Figure 2.3: Different literature on addressing the relation between maintenance and production taking a short-term perspective.

2.3.1 Scheduling Fundamentals

Scheduling is the allocation of the available resources (machines) to competing tasks (jobs) over time with the goal of optimizing one or a set of predetermined objectives such as finishing all the jobs as early as possible or finishing as many jobs as possible within a given interval (Pinedo, 2002). Depending on the application, the machines and the jobs can refer to different things. In this dissertation, in Chapter 4 where a problem is studied in a military application, the aircraft and the flights correspond to the

machines and the jobs. While, in Chapters 5 and 6 where a production application is studied, the manufacturing machines and the production operations represent the machines and the jobs.

In scheduling problems, each job, j, is usually associated with four pieces of data: the processing time on each machine, for example for machine i, p_{ij}; the ready time, r_j, denoting the earliest time that processing on job j can start; the due date, d_j, indicating the latest time that processing of job j should finish without incurring a penalty; and the weight, w_j, representing the relative importance of job j compared to the other jobs (Pinedo, 2005).

A notation for describing a scheduling problem is a triple $\alpha|\beta|\gamma$ where α represents the machine environment, β describes the processing characteristics and constraints in detail, and γ denotes the objective function (Graham et al., 1979; Pinedo, 2002).

The machine environment usually states the number of machines and the relations among them. Some examples are (Pinedo, 2002):

1. A single machine denoted as **1** where there is only one machine processing the jobs.

2. Parallel machines denoted as Rm where there are m machines in parallel. Each job requires one operation and can be processed by any machine, though its processing time is dependent on the machine assigned. The scheduling problem studied in Chapter 4 can be considered as a special case of parallel machines.

3. A flowshop denoted as Fm where there are m machines in series. Each job has m operations and has to be processed on each machine in sequence. The scheduling problem studied in Chapters 5 and 6 is a flowshop problem.

4. A jobshop denoted as Jm where there are m machines and each job may have different number of operations with a specific route for processing on machines.

The second field states the constraints of the scheduling problems such as ready times, preemptions, precedence constraints, machine capacity, machine breakdown and workforce constraints. We briefly explain four of them since they are used in this dissertation (Pinedo, 2005).

1. Precedence constraints *(*prec): Precedence constraints imply that the processing of one or some jobs must be finished before the processing of another job is started.

2. Machine breakdowns *(*brkdwn): Machine breakdowns imply that machines are not continuously available to process the jobs.

3. Machine capacity: Machine capacity can be *unary* or *multi*. In case of unary, at most one job can be processed on a machine at any given time point. If the machine capacity is multi being equal to C, the capacity required by the jobs executing on the machine at any time must sum to less than or equal to C.

4. Resource constraints: Processing a job on a machine might require a specific operator, piece of machinery, fixture, or tool and a facility may have limited number of the specific resources. Therefore, the job might need to wait for the specific resource to become available.

The objective function of the scheduling problems, denoted in the third field, can be considered in two wide categories of completion-based and due date-based objectives. While both are functions of the completion time of the jobs, C_j, the latter is also a function of the jobs' due dates. In this dissertation, the following objective is used, belonging to the latter category.

Weighted number of tardy jobs ($\sum w_j U_j$): U_j is a binary variable that equals 0 if the processing of job j finishes before its due date, i.e., $C_j \leq d_j$ and it equals 1, otherwise.

In the scheduling literature, there is a distinction between a *sequence* and a *schedule*. A sequence usually determines the order of the execution of jobs and a schedule refers to allocation of jobs within a more complex setting typically involving the assignment of a start time to each job (Pinedo, 2002).

Similar to the maintenance literature, the scheduling literature is separate from the production literature and is enormous. For more details, interested readers are referred to the books written by Pinedo (2002; 2009), Leung (2004), and Baker and Trietsch (2009). In the balance of this section, we focus on three common approaches for solving scheduling problems: Mixed integer programming (MIP), Constraint programming (CP), and Hybrid optimization methods. We briefly explain each approach and then provide the formulation of the following scheduling problem.

Scheduling Example: There are a set of jobs $\mathcal{J}$ and a set of machines $\mathcal{I}$. Each job $j \in \mathcal{J}$ has the ready time r_j and due date d_j and each machine $i \in \mathcal{I}$ has the capacity C_i. Processing job j on machine i costs f_{ij}, requiring p_{ij} units of processing times and consumes c_{ij} amount of machine capacity. The goal is to assign each job to exactly one machine and to schedule it within its time window such that the total cost of assignment is minimized and the sum of capacity consumptions on each machine, i, does not exceed its total capacity, C_i, at any time (Hooker, 2005; Heinz and Beck, 2012).

2.3.1.1 Mixed integer Programming

Mixed integer programming (MIP) is the default solution approach for many scheduling problems (Heinz and Beck, 2012). In a MIP formulation, the constraints are represented in the form of linear equalities and/or inequalities and polyhedral theory and linear programming techniques such as relaxation and cutting planes embedded in the state-of-the-art MIP solvers are applied to solve the problem (Queyranne and Schulz, 1994; Heinz and Beck, 2012). Queyranne and Schulz (1994) provided four different formulations for scheduling problems based on the choice of decision variables: time-indexed variables; linear ordering, start time and completion time variables;[7] assignment and positional date variables; and traveling salesman variables. The combination of the first and the second approaches is used in this dissertation. In the time-indexed formulation, the time is discretized and the binary decision variable x_{ij}^t equals 1 if job j starts at time t on machine i. While, in the start time formulation, the integer decision variable s_{ij} defines the start time of job j on machine i.

The time-indexed formulation of the scheduling example is given below with x_{ij}^t as the decision variable. The objective function minimizes the cost of machine allocation. Constraint (2.1) ensures that each job starts once on each machine while constraint (2.2) enforces the machine capacity limit.

[7] This approach is similar to the disjunctive formulation of Applegate and Cook (1991).

Assuming that job j is in progress on machine i at time t, $T_{ij}^t = \{t' | t - p_{ij} < t' \leq t\}$ includes the set of possible discrete time points at which job j might start processing.

$$\min \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \sum_{t=r_j}^{d_j - p_{ij}} f_{ij} x_{ij}^t$$

$$\text{s.t.} \sum_{i \in \mathcal{I}} \sum_{t=r_j}^{d_j - p_{ij}} x_{ij}^t = 1, \quad \forall j \in \mathcal{J} \tag{2.1}$$

$$\sum_{j \in \mathcal{J}} \sum_{t' \in T_{ij}^t} c_{ij} x_{ij}^{t'} \leq C_i, \quad \forall i \in \mathcal{I}, \ \forall t \tag{2.2}$$

$$x_{ij}^t \in \{0, 1\}, \quad \forall j \in \mathcal{J}, \ \forall i \in \mathcal{I}, \ \forall t$$

Figure 2.4: Time-indexed mixed integer programming model.

2.3.1.2 Constraint Programming

The success of constraint programming (CP) in solving a wide variety of scheduling problems is well established in the literature (Beck et al., 1998; Baptiste et al., 2001, 2006). The scheduling problems are usually defined as one or several instances of the constraints satisfaction problem (CSP) (Baptiste et al., 2001). An instance of CSP can be formally described as a triple of (V, D, C) where $V = \{V_1, V_2, ..., V_n\}$ is a set of n variables, $D = \{D_1, D_2, ..., D_n\}$ is a set of the variable domains, D_i corresponding to the possible values that V_i can take, and $C = \{C_1, C_2, ..., C_m\}$ is a set of m constraints, each defined over a subset of variables. A constraint $C_k = \{V_i, ..., V_j\}$ is defined on the Cartesian product of the domains of the variables in its scope $D_i \times ... \times D_j$ and is satisfied if the assignment of the variables in its scope corresponds to one of the value tuples in the constraint relation (Beck, 1999). Representing scheduling problems using CSPs results in more modeling flexibility compared to mixed integer programming models as there is no restriction on the type of decision variables and constraints.

CP solves scheduling problems by applying constraint propagation, heuristic search and backtracking within a branch-and-bound search tree. At each node of a search tree, the constraint propagation algorithms are first used to infer all the new constraints that must be true given the set of all decisions already made. Heuristic algorithms are then used to make a decision. The decision might be the assignment of a value to a variable, but it generally can be interpreted as the non-deterministic addition of a constraint to the problem. If in a node, one of the constraints is not satisfied as a result of previous decisions and inferred constraints, backtracking techniques are applied to undo some of the previous decisions, guaranteeing a complete search (Beck and Refalo, 2003; Beck, 1999).

To invoke the constraint propagation techniques where efficient inference techniques are used to reduce the solution space by adding implied constraints, the problem needs to be represented as a conjunction of global constraints. For example, in the CP formulation of the scheduling example given in Figure 2.5, the resource capacity limit is enforced using the *cumulative* global constraint. The cumu-

lative constraint has the syntax of cumulative(s, p, c, C) where $s = \{s_1, s_2, ..., s_n\}$, $p = \{p_{i1}, p_{i2}, ..., p_{in}\}$, and $c = \{c_{i1}, c_{i2}, ..., c_{in}\}$ are arrays of the start time variables, the processing time values, and the amount of required machine capacity values of n jobs on machine i, respectively and where $C = C_i$ is the total capacity of machine i. The cumulative constraint ensures that the total amount of machine capacity used at any time does not exceed C (Hooker, 2007). During search, a cumulative constraint uses it specialized inference algorithm to remove start time values and infer sequencing constraints that are implied by the current search state (Nuijten, 1994; Caseau and Laburthe, 1996; Mercier and Van Hentenryck, 2008; Schutt et al., 2011). For more details on the techniques used to propagate cumulative constraints, interested readers are referred to Chapter 3 of the book by Baptiste et al. (2001). We use constraint programming in Chapter 4.

The constraint programming model of the scheduling example is given below where x_{ij} is a binary decision variable being equal to 1 if job j is assigned to machine i and s_j is an integer decision variable representing the start time of job j. The objective function, similar to the MIP objective, minimizes the total cost of assigning jobs to machines. Constraints (2.3) and (2.4) are logically equivalent to constraints (2.1) and (2.2), and constraint (2.5) enforces the time window constraint.

$$\min \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} f_{ij} x_{ij}$$

$$\text{s.t.} \sum_{i \in \mathcal{I}} x_{ij} = 1, \quad \forall j \in \mathcal{J} \tag{2.3}$$

$$\text{cumulative}([s_j | x_{ij} = 1], [p_{ij} | x_{ij} = 1], [c_{ij} | x_{ij} = 1], C_i), \quad \forall i \in \mathcal{I} \tag{2.4}$$

$$r_j \leq s_j \leq \max_{i \in \mathcal{I}}((d_j - p_{ij}) x_{ij}), \quad \forall j \in \mathcal{J} \tag{2.5}$$

$$x_{ij} \in \{0, 1\}, \quad \forall j \in \mathcal{J}, \ \forall i \in \mathcal{I}$$

$$s_j \in \mathbb{Z}, \quad \forall j \in \mathcal{J}$$

Figure 2.5: The constraint programming model.

2.3.1.3 Hybrid Optimization Methods

In the last 15 years, the hybrid optimization techniques combining the strengths of CP and MIP have been proved compelling in solving scheduling problems (Milano and Van Hentenryck, 2010; Beck, 2010; Sadykov and Wolsey, 2006; Beck and Refalo, 2003). One of these techniques, logic-based Benders decomposition (LBBD), inspired two of the models in this dissertation in Chapters 4 and 5.

In this section, we first provide a brief background on Benders decomposition, then formally define logic-based Benders decomposition, and finally present the formulation of the scheduling example using LBBD.

Background: The classical Benders decomposition (Benders, 1962; Geoffrion and Graves, 1974) is a mathematical programming approach for solving large-scale mixed integer programming models. It partitions the problem into a mixed integer master problem (MP) which is a relaxation of the global

model and a set of linear sub-problems (SPs). Solving a problem by classical Benders involves iteratively solving the MP to optimality and using the solution to generate the sub-problems. The linear programming dual of the SPs is then solved to derive the tightest bound on the global cost function. If this bound is less than or equal to the current MP solution (assuming a minimization problem), the MP solution and the SP solutions constitute a globally optimal solution. Otherwise, a constraint, a *Benders cut*, is added to the MP to express the violated bound and another iteration is performed.

Logic-based Benders decomposition (Hooker and Yan, 1995; Hooker and Ottosson, 2003) was developed excluding the necessity that the MP should be a mixed integer model and the SPs should be linear. Therefore, the inference duals (Hooker, 2005) of the SPs are solved rather than the linear duals to find the tightest bound on the global cost function from the original constraints and the current MP solution. Although the logic-based Benders decomposition has more flexibility than the classical Benders decomposition in modeling the problems, it is problem-specific and requires creative effort.

Representing the relaxation of SPs in the MP and designing a strong Benders cut are of great importance in decreasing the computational effort to find a globally optimal solution. The former results in MP solutions which are likely to satisfy the SPs, and the latter rules out a large number of MP solutions in each iteration (Hooker, 2007).

Logic-based Benders decomposition has been shown to be effective in a wide range of problems including scheduling (Beck, 2010; Hooker, 2005, 2007), facility and vehicle allocation (Fazel-Zarandi and Beck, 2012), and queue design and control problems (Terekhov et al., 2009).

Formal Representation: Logic-based Benders decomposition applies to the problem of the form (Hooker, 2000)

$$\begin{aligned} \min \ & f(x,y) \\ \text{s.t.} \ & C_1(x,y) \\ & C_2(x) \\ & C_3(y) \\ & x \in D_x, \quad y \in D_y \end{aligned} \tag{2.6}$$

where $C_1(x,y)$, $C_2(x)$, and $C_3(y)$ are sets of constraints including both x and y variables, only x variables, and only y variables, respectively. The domains of x and y are respectively represented by D_x and D_y. Assigning the value $\bar{x}$ to x, $(x = \bar{x}, \bar{x} \in D_x)$, results in the following sub-problem:

$$\begin{aligned} \min \ & f(\bar{x},y) \\ \text{s.t.} \ & C_1(\bar{x},y) \\ & C_3(y) \\ & y \in D_y \end{aligned} \tag{2.7}$$

The *inference dual* of (2.7) is the problem of deriving the tightest possible bound on $f(\bar{x},y)$ from

$C_1(\bar{x}, y)$ and $C_3(y)$ represented as below.

$$\begin{aligned} &\min \ \nu \\ &\text{s.t. } (C_1(\bar{x}, y) \wedge C_3(y)) \Rightarrow f(\bar{x}, y) \geq \nu \\ &\nu \in \mathbb{R} \end{aligned}$$

where $\wedge$ and $\Rightarrow$ indicate logical-and and logical implication, respectively.

The solution of the dual can be viewed as a derivation of the tightest possible bound $\hat{v}$ on $f(x, y)$ when $x = \bar{x}$. The basic idea of the Benders decomposition is to derive a function, $B_{\bar{x}}(x)$ that provides a lower bound on the objective function, $f(x, y)$, for any given $x \in D_x$. Denoting the objective function of (2.6) as z, the bounding procedure results in a valid inequality $z \geq B_{\bar{x}}(x)$, which is called a "Benders cut".

In iteration H of the Benders algorithm, the following master problem is solved whose constraints are the set of constraints including only x variable and the Benders cuts generated so far. In the formulation, $x^1, x^2, ..., x^{H-1}$ are the solutions of the previous $(H-1)$ master problems. The solution of the H-th master problem, $x^H = \bar{x}$ then defines the H-th sub-problem as previously represented in (2.7).

$$\begin{aligned} &\min \ z \\ &\text{s.t. } C_2(x) \\ &\quad z \geq B_{x^h}(x), \quad h = 1, 2, ..., H-1 \\ &\quad z \in \mathbb{R}, \quad x \in D_x \end{aligned} \tag{2.8}$$

Letting $\nu_1^*, \nu_2^*, ..., \nu_{H-1}^*$ denote the optimal value of the previous $(H-1)$ sub-problems, the algorithm continues until the optimal value of the H-th master problem, z_H^* equals $\nu^* = \min\{\nu_1^*, ..., \nu_{H-1}^*\}$. At iteration H of the problem, z_H^* and ν^* provide lower and upper bounds on the optimal value of the objective function. Under fairly weak conditions, the algorithm converges finitely to an optimal solution. More details can be found in the book by Hooker (2000).

Example: A logic-based Benders decomposition (LBBD) method is formulated below where the MP assigns jobs to machines to minimize the total machine allocation cost and the sub-problems schedule the assigned jobs on each machine such that the machine capacity and the time window constraints are satisfied. The MP uses MIP for solving, while CP is used to schedule sub-problems.

As in the CP model represented in Figure 2.5, the decision variables are the binary machine allocation variable x_{ij} and the integer start time variable s_j.

The master problem incorporates a number of the constraints in the global MIP and CP models. It does not represent the start times of jobs nor does it fully represent the capacity of the machines. As is common in Benders decomposition, the master problem includes a relaxation of the sub-problems (Constraints (2.10)) and Benders cuts (Constraints (2.11)). The sub-problem relaxation ensures that the total available area on machine i, the area of the rectangle with height C_i and width from the smallest release date to the largest due dates must be greater than the sum of the areas of the jobs assigned to

$$\min \sum_{i\in\mathcal{I}} \sum_{j\in\mathcal{J}} f_{ij}x_{ij}$$

$$\text{s.t.} \sum_{i\in\mathcal{I}} x_{ij} = 1, \quad \forall j \in \mathcal{J} \tag{2.9}$$

$$\sum_{j\in\mathcal{J}} c_{ij}p_{ij}x_{ij} \leq C_i(\max_{j\in\mathcal{J}}(d_j) - \min_{j\in\mathcal{J}}(r_j)), \quad \forall i \in \mathcal{I} \tag{2.10}$$

$$\sum_{j\in\mathcal{J}_{hi}} (1 - x_{ij}) \geq 1, \quad \forall i \in \mathcal{I}, \ \ \forall h \in \{1, ..., H-1\} \tag{2.11}$$

$$x_{ij} \in \{0, 1\}, \quad \forall j \in \mathcal{J}, \ \ \forall i \in \mathcal{I},$$

Figure 2.6: The master problem formulation.

machine i. Denoting $\mathcal{J}_{hi}$ as the set of jobs that resulted in an infeasible sub-problem for machine i in iteration $h < H$, the Benders cut in iteration H enforces that the set of jobs or any superset assigned to machine i in iteration h is not reassigned to the same machine.

$$\text{cumulative}([s_j|x^h_{ij} = 1], [p_{ij}|x^h_{ij} = 1], [c_{ij}|x^h_{ij} = 1], C_i), \quad \forall i \in \mathcal{I} \tag{2.12}$$

$$r_j \leq s_j \leq \max_{i\in\mathcal{I}}(d_j - p_{ij}), \quad \forall j \ (x^h_{ij} = 1) \tag{2.13}$$

$$s_j \in \mathbb{Z}, \quad \forall j \ (x^h_{ij} = 1)$$

Figure 2.7: The sub-problem formulation.

The SP for machine i in iteration h is formulated as a constraint program. The constraints of SP model are similar to the constraints (2.4) and (2.5) in CP model of Figure 2.5 with the difference that the jobs are assigned to machines before the SP models are created.

2.3.2 Stochastic Sequencing/Scheduling

There are two independent areas in the job scheduling literature addressing the interdependency between production sequencing/scheduling and maintenance situation of no control over machine conditions. The problem concerns the optimal allocation of the available machine processing times to competing production operations where unexpected machine breakdowns limit the machine availability. In this section, we review the Stochastic Sequencing/Scheduling literature. It is worth mentioning that this literature deals with many different uncertainties that might arise from various sources including machine breakdowns, unexpected arrival of new orders, early or late arrival of raw materials, and staffing problems. Here, we narrow our review to uncertainty stemming from machine failures. Furthermore, as sequencing is a special case of scheduling, we refer to this literature as Stochastic Scheduling in the rest of this chapter.

The goal of this literature, similar to Random Yield without Maintenance literature (see Section

2.2.2), is to find an optimal policy, determining the allocation of the production operations to the machines at each decision time point. However, there is one difference: in the Stochastic Scheduling literature, the objective function involves a certain number of production jobs and for example minimizes the total expected cost summed over all jobs, whereas the Random Yield without Maintenance literature taking a long-term perspective optimizes the system performance in the steady state and for example minimizes the expected total cost per time unit.

Considering the goal of the Stochastic Scheduling literature, its most common solution technique is based on probability theory distinct from the approaches reviewed in Section 2.3.1. The majority of results are obtained for single machine problems that are more amenable to rigorous analytical analysis. Different problem variations of this literature can be distinguished as follows:

- Preempt-resume or preempt-repeat model: In a preempt-resume model, the work done on a job is not lost due to breakdown (Glazebrook, 1984, 1987; Pinedo and Rammouz, 1988) and the job is assumed to start where it left off before the breakdown. However, in a preempt-repeat model, the job which is preempted due to a machine failure must be restarted (Frostig, 1991; Cai et al., 2003, 2004).

- Static or dynamic scheduling policy: A static policy is a prespecified decision rule, called a sequence, which determines the order of executing n jobs. For example, $\lambda = (\lambda_1, \lambda_2, ..., \lambda_n)$ is a decision rule where $\lambda_k = i$ if job i is the k-th to be processed under λ. The static policy is determined at the beginning of the decision horizon and dose not change (Glazebrook, 1984, 1987; Pinedo and Rammouz, 1988; Frostig, 1991; Cai and Zhou, 1999, 2000; Cai et al., 2003, 2004). However, a dynamic policy allows the decision maker to revise the decision rule at any decision epoch within the decision horizon considering all the information accumulated up to that time (Cai et al., 2005, 2009). The class of dynamic policies contains static policies and so an optimal dynamic solution will be no worse than the best static policy.

- Expectation or stochastic optimization: There are several forms of optimization in stochastic scheduling. The weakest form is in the *expectation* sense and the stronger one is in the *stochastic* sense. For example, if the objective is to minimize the makespan, the weakest optimization finds a scheduling policy with the expected makespan less than or equal to the expected makespan of any other scheduling policy. Stochastic optimization results in a scheduling policy with the makespan stochastically less than or equal to the makespan of any other schedule. Stochastic optimization implies optimization in expectation (Pinedo, 2002). The examples of expectation optimization can be found in Pinedo and Rammouz (1988); Cai and Tu (1996); Cai and Zhou (2000) and of stochastic optimization in Frostig (1991).

- Failure time distribution with constant or increasing failure rate: Machine breakdowns occur randomly if the failure rate is constant (Glazebrook, 1987; Cai and Zhou, 2000, 2006), whereas their occurrence increases in time with increasing failure rate distributions (Cai et al., 2003, 2004).

Contrary to above papers assuming a random processing time, Adiri et al. (1989) studied the problem of minimizing the sum of the completion times of n jobs with known processing times subject to ma-

chine breakdown. Assuming known processing times adds to the combinatorial part of the problem and consequently obtaining general results is challenging. Considering a single breakdown, i.e., machine breakdown occurs only once during the processing of all n jobs and assuming the preempt-repeat model, they showed: (1) the Shortest Processing Time (SPT) dispatching rule stochastically minimizes the sum of completion times if the failure time distribution is concave.[8] SPT is the most famous dispatching rule in the scheduling literature which processes the jobs in non-decreasing order of their processing times. It is known that SPT minimizes the sum of job completion times in a single machine problem with no breakdown if all jobs are ready at time 0 (Pinedo, 2002), (2) the problem of deciding whether there is a schedule with sum of completion times less than a given value is NP-complete assuming that the time of single breakdown is known, and (3) the maximum relative error of the SPT dispatching policy for the problem with the deterministic single breakdown is 25% from the optimal scheduling policy. Adiri et al. (1991) then extended their results to the problem with the objective of stochastically minimizing the number of tardy jobs. Under certain conditions on processing times and due dates, the optimal scheduling policies are derived for both the preempt-repeat and the preempt-resume models.

Birge et al. (1990) also assumed known processing times and focused on deriving a bound on the difference between the performance of the optimal static policy and the Weighted Shortest Processing Time (WSPT)[9] dispatching rule assuming both the preempt-resume and the preempt-repeat models.

Although the majority of the Stochastic Scheduling literature is devoted to single machine problems, Allahverdi and Mittenthal (1995) studied the problem of stochastically minimizing makespan in a two-machine flowshop assuming unexpected machine breakdowns and the preempt-resume model. They first showed that the optimal scheduling policy is a permutation schedule, i.e., the sequence of the jobs is the same on both machines, as in the deterministic counterpart. Second, they characterized specific conditions on the breakdown process for which Johnson's algorithm[10] stochastically minimizes the makespan as in the deterministic problem. Allahverdi and Mittenthal (1994) studied the problem of stochastically minimizing the makespan and minimizing the expected sum of the completion times in a sub-category of the flowshop problems, i.e., a two-machine ordered[11] flowshop problem. Three other papers of Allahverdi (1995; 1996; 1997) studied different two-machine stochastic flowshop problems, focusing on deriving sufficient conditions that guarantee the optimality of specific scheduling policies.

2.3.3 Dynamic Sequencing/Scheduling

Dynamic Scheduling[12] is a methodology developed in the scheduling literature to find the allocation of operations to machines where operational uncertainties like machine breakdowns or the unexpected arrival of new orders prevent the execution of the schedule as planned (Aytug et al., 2005; O'Donovan et al., 1999).

[8] A concave failure time distribution includes distributions with decreasing and constant failure rates (Adiri et al., 1989).

[9] The Weighted Shortest Processing Time (WSPT) dispatching rule minimizes the sum of the weighted completion times in a deterministic single machine problem by processing the jobs in non-decreasing order of $\frac{p_j}{w_j}$ ratio (Pinedo, 2002).

[10] For the description of Johnson's rule, readers are referred to Chapter 6 of Pinedo's book (Pinedo, 2002).

[11] For the definition of an ordered flowshop problem, interested readers are referred to the paper by Allahverdi and Mittenthal (1994).

[12] Since scheduling includes sequencing, we refer to Dynamic Sequencing/Scheduling as Dynamic Scheduling.

At the highest level, Dynamic Scheduling is based on the combination of reactive and predictive generation of a schedule. In reactive scheduling, all decisions are usually made without anticipation and online, while in predictive scheduling, decisions are made offline. For the surveys of the literature, see the work of Bidot et al. (2009), Thomas and Szczerbicka (2007), Herroelen and Leus (2005), Aytug et al. (2005), and Davenport and Beck (2000).

In a completely reactive approach, all decision are made in the real time usually using one of the following techniques:

- Dispatching rules: Dispatching rules usually make decisions based on a priority index calculated from job and machine attributes (O'Donovan et al., 1999). Although priority dispatching rules are the best solution approach for the dynamic problems in the presence of limited computational power (Thomas and Szczerbicka, 2007) or quick and drastic changes in the problem parameters (Branke and Mattfeld, 2005), they lack the ability to optimize a global objective function (Branke and Mattfeld, 2005) or to plan into future (Thomas and Szczerbicka, 2007). Discussions on dispatching rules and their applicability for various uncertainty situations can be found in papers by Panwalkar and Iskander (1977), Haupt (1989), and Morton and Pentico (1993).

- Online stochastic optimization: The online stochastic optimization (OSCO) (Van Hentenryck and Bent, 2006) framework combines the online algorithms and stochastic programming. OSCO makes decisions one at a time by first sampling the distribution of future events and then solving deterministic problems, each representing a possible realization of the uncertain future.

In a completely predictive approach, the known statistics on uncertainty are usually used to make offline decisions. Two techniques of this category are:

- Redundancy-based techniques: The main characteristic of redundancy-based techniques is the allocation of extra time and/or resources so that the unexpected events during execution can be dealt with using some of this reserved time and resources (Davenport and Beck, 2000; Beck and Wilson, 2007). Several examples of this approach are: inserting idle time into the schedule to cope with machine disruptions where the idle time is calculated based on the failure rate and the expected repair time (Mehta and Uzsoy, 1998, 1999; O'Donovan et al., 1999); extending the duration of the critical jobs considering the expected up-time and the expected repair time (Gao, 1995); forcing any solution to respect constraints on the slack[13] of the jobs (Davenport et al., 2001); and modifying the objective function to be a linear combination of the expected makespan and expected delay (Leon et al., 1994).

- Probabilistic techniques: Probabilistic techniques, similar to approaches in Stochastic Scheduling literature (see Section 2.3.2), use representations of uncertainty to reason about the possible outcomes when the schedule is executed (Beck and Wilson, 2007). The work of Leon et al. (1994) is an example of this technique where the jobshop scheduling problem in the presence of machine

[13]The slack of a job is the time that it can be delayed without breaking any constraints and increasing the cost of the schedule.

disruption is represented as a discrete stochastic control problem and a game-theoretic approach minimizing the expected makespan and deviations from an offline schedule is developed to solve the problem.

Predictive-reactive approaches are based on the idea of rolling horizon where the predictive component generates a schedule in advance over a short time period considering the known information about uncertainty, for example, the distributions of failure and repair times. The constructed schedule is then executed and whenever disruptions occur, the reactive component modifies the schedule to either permit the execution or to improve the quality of the schedule considering the observed information. The dynamic problem is viewed as a collection of linked static sub-problems where the myriad of algorithms developed for the static scheduling problems (see Section 2.3.1) are applicable in the predictive phase. Connecting the static sub-problems using rescheduling strategies is done in the reactive phase where the previous schedule is modified. The rescheduling might be as simple as the Right Shift rule or might construct a new schedule for all the activities that have not yet executed. Some examples of rescheduling strategies are: Sadeh et al. (1993) where a set of control rules such as WSPT and Apparent Tardiness Cost (ATC) are used either to choose the next job to schedule or to identify the set of jobs which need to be fully rescheduled; El Sakkout and Wallace (2000) where a full rescheduling of the jobs is performed such that the absolute difference between the start times of the jobs in the revised schedule and the original start times is minimized; and Bean et al. (1991) and Akturk and Gorgulu (1999) where the rescheduling is performed such that the revised schedule matches the original one after a certain time. We use a predictive-reactive approach in Chapter 4 of this dissertation.

2.3.4 Sequencing/Scheduling with Availability Constraints

Addressing the problem of scheduling planned maintenance activities along with production jobs is the concern of Sequencing/Scheduling with Availability Constraints literature.[14] More specifically, there are a number of maintenance activities that must be inserted into the schedule among the regular production jobs such that a given operational performance measure is optimized. This problem has been studied from two perspectives. The first deals only with the fact that a machine undergoing maintenance is unavailable for production jobs (Schmidt, 2000; Lee, 2004; Ma et al., 2010). The second perspective models different processing times for a production job depending on whether it is scheduled before or after maintenance (Lee and Leon, 2001). Both perspectives typically focus on analyzing the computational complexity of the problems and/or deriving the properties of the optimal schedules. The derived properties are used to develop polynomial time approximation algorithms or efficient heuristics, or are modeled as constraints to reduce the computational effort.

The problem of the first category can be defined as follow. A set of jobs $\mathcal{J} = \{J_i | i = 1, ..., n\}$ and a set of machines $\mathcal{M} = \{M_j | j = 1, ..., m\}$ are given. Machine M_j is not available for processing the jobs within S_j time intervals $[B_j^s, F_j^s]$, $s = 1, ..., S_j$ where B_j^s and F_j^s denote the start time and the finish time of the s-th unavailability interval (Ma et al., 2010). The goal of the problem is to pack the jobs into the gaps

[14] We refer to this literature as Scheduling with Availability Constraints literature.

created between unavailability intervals, optimizing an operational performance measure such as finishing all the jobs as soon as possible. In different problem variations, jobs may be resumable (Lee, 1996), non-resumable (Lee, 1996), and semi-resumable (i.e., the disrupted job has to partially re-start when the machine becomes available again) (Lee, 1999). Furthermore, one or several unavailability intervals (maintenance periods) might be considered where their start and end times are either known or decision variables. A number of different combinations of the unavailability intervals and job characteristics have been studied (Lee, 1996; Liao and Chen, 2003; Akturk et al., 2004; Chen, 2006; Ji et al., 2007; Kovacs and Beck, 2007). While the majority of this literature deals with deterministic problems where limited availabilities of machines only result from planned maintenance, Cassady and Kutanoglu (2003; 2005) and Kuo and Chang (2007) studied a single machine scheduling problem assuming that the machine is not continuously available due to both planned maintenance and random machine breakdowns.

The above scheduling problems have no correlation between machine conditions and processing times, ignoring the practical relevance of maintenance on machine deterioration and restoration processes (Kellerer et al., 2012; Rustogi and Strusevich, 2012). Lee and Leon (2001) were the first to introduce such maintenance considerations into the scheduling literature, initiating the study of the second category of problems. More specifically, the authors represented maintenance as a rate-modifying activity that changes the processing times of production jobs scheduled after maintenance to $\lambda_j p_j$ where $0 < \lambda_j < 1$ and p_j represents the processing time of job j before maintenance. In the work of Lee and Leon and many subsequent models (e.g., Mosheiov and Sarig (2009); Mosheiov and Sidney (2010)) only a single rate-modifying activity is considered and the processing time of a job does not depend on its position in the schedule or its start time, only whether it comes before or after maintenance. However, recent work has studied the problem of dividing the jobs into groups where the number of groups indicates the number of maintenance activities and the processing time of each job depends both on its assigned group and its position within the group (Kuo and Yang, 2008; Yang and Yang, 2010; Lodree and D., 2010; Rustogi and Strusevich, 2012; Kellerer et al., 2012). The focus of this work is the development of polynomial-time algorithms when the problem is limited to the single machine.

Some other works incorporating practical maintenance considerations in production scheduling problems include: Kubzin and Strusevich (2006) where delaying the maintenance activity increases the time needed to perform it; and Xu et al. (2008; 2010) where the concept of ϵ-almost periodic maintenance is defined to account for non-periodic maintenance.

The review of this literature indicates there exists no work that reasons with an explicit representation of machine condition or the effect of machine deterioration and restoration on processing times. Furthermore, unlike the broader maintenance literature (Dekker et al., 1996; Wang, 2002; Nicolai and Dekker, 2008; Pintelon and Parodi-Herz, 2008), maintenance is considered as a short-term decision when reasoning about it in combination with production scheduling. That is, the problem is defined over a fixed horizon where maintenance and machine deterioration act on the same time scale as the production jobs. In practice, a machine does not deteriorate as fast as the production jobs are processed and so maintenance decisions are naturally made over longer time horizons than detailed scheduling decisions (Cassady and Kutanoglu, 2005; Budai et al., 2006; Grigoriev et al., 2006; Aghezzaf and Najid,

2008). In Chapter of 5 and 6 of this dissertation, we extend this literature using techniques reviewed in Section 2.2 to integrate maintenance planning with production and maintenance scheduling where maintenance is considered as a long-term decision and where there is an explicit model representing the deterioration processes of machines and their effects on the processing times. This perspective on the problem takes into account common conceptualizations of maintenance as they appear in that area of the research literature (McCall, 1965; Dekker et al., 1997; Wang, 2002; Pintelon and Parodi-Herz, 2008) and introduce them to the scheduling literature.

2.3.5 Summary

We reviewed the literature addressing the relationship between production problems and maintenance situations with the goal of optimizing the operational and short-term objectives of the production process. First, the common concepts and solution approaches of scheduling literature were reviewed. Then, in Sections 2.3.2 and 2.3.3, two separate areas of Stochastic Sequencing/Scheduling and Dynamic Sequencing/Scheduling that study the problem of integrated production sequencing/scheduling and the maintenance situation of no control over machine conditions were reviewed. Finally, Section 2.3.4 presented the literature studying the interdependency between production sequencing/scheduling and the maintenance situation of partial control over machine conditions.

2.4 Conclusion

In this chapter, we surveyed the literature integrating maintenance and production reasoning in two main streams with long-term and short-term decision horizons. We divided the production decisions into two problems of lot-sizing and sequencing/scheduling and the maintenance situations into two categories of no control or partial control over machine conditions. In each category, we provided a review of the literature integrating one of the two production decisions with one of the two maintenance situations.

In this dissertation, we contribute to three of these areas: the integration of production planning and maintenance assuming partial control over machine conditions; the integration of production sequencing/scheduling and maintenance with no control over machine conditions; and the integration of production sequencing/scheduling and maintenance with partial control over machine conditions. In the following chapters, we present each of our contributions in more detail.

Chapter 3

Maintenance & Production Planning with Partial Control over Machine Conditions

In many manufacturing industries, the quantity produced (yield) is uncertain, i.e., it might be less than the input production quantity due to imperfect processes, machine deteriorations and breakdowns. Adopting a reactive approach to the failures of the production process where machines, for example, are only maintained at breakdowns might result in significant production loss. Therefore, many manufacturing firms have initiatives to invest in process improvement projects such as preventive maintenance to increase the yield and minimize total cost (Gerchak and Parlar, 1990; Lin and Hou, 2005; Gupta and Cooper, 2005). To initiate improvement, a firm needs to decide the amount of resource (e.g. money) that should be allocated to a project, for example, preventive maintenance.

In this chapter, we consider a firm that manufactures one product with a single-stage process over multiple time periods to meet customer demand at the end of each period. The main causes of yield losses include machine deterioration and machine breakdowns that are internal to the production process. Therefore, investing in an improvement project such as preventive maintenance can increase the quantity produced. The problem at the beginning of each time period is to determine the production quantity and to decide whether to invest in an improvement project and if so, the amount of investment. This is an example of an integrated maintenance and production planning problem with partial control over machine conditions in a periodic review system where preventive maintenance represents the improvement project.

The ultimate goal in addressing the interdependency between maintenance and production planning where machines can be partially controlled is to determine the joint optimal maintenance and production policy. The production policy defines the production quantity (or lot-size) and the maintenance policy determines the amount of investment in maintenance to change the yield. However, as stated in Section 2.2.3.1, since characterizing the joint optimal policy is analytically hard, it is usually assumed that the production policy is fixed and the optimal maintenance policy is determined. Following the same approach, in this chapter we introduce a combined model where the production policy is predetermined, i.e., no decision is made about the production quantities. Our goal is to provide insight into the

optimal decision on the level of yield alteration by investing in maintenance. However, our modeling approach has two differences with the majority of the models reviewed in Section 2.2.3.1. First, the usual assumption of the literature is that the production time of a product is random, but, like Sloan (2004), we assume that the number of acceptable products is random. Second, unlike the models in the literature, we do not include an explicit representation of machine deterioration. In the literature, the random time to produce a product or the random number of acceptable products are dependent on machine conditions. However, we take more strategic modeling approach where the machine condition information is not available and we assume that the random number of acceptable products is dependent on the amount of money committed to perform maintenance or any other improvement projects. The decisions, therefore, in our model generally correspond to any costly improvement projects that affect yield.

The assumption that yield changes through investment in process improvement projects without explicitly modeling the machine deterioration is considered in continuous review models (Gerchak and Parlar, 1990; Lin and Hou, 2005) as explained in Section 2.2.2.2. This assumption, though, has not been studied in a periodic review system. To the best of our knowledge, only Gupta and Cooper (2005) studied the optimal direction of change in the yield distribution in a periodic review system; more details on their work are provided in Section 2.2.2.1. However, knowing the optimal direction of change is not enough to initiate a process improvement such as preventive maintenance. We need to know the optimal amount of money that should be allocated to maintenance. We address this question here.

In this chapter, we determine the structure and the properties of the optimal maintenance (investment) policy. We characterize when a threshold maintenance policy is optimal. We analyze both single period and multiple period problems. The results that apply to both cases are summarized as below:

- We prove that if the yield changes linearly in the amount of investment and the budget available for making an investment at the beginning of each period is fixed a priori, then the cost function is convex in the amount of money invested in maintenance and therefore the first-order condition provides the global optimal solution.

- Given a linear yield function and a fixed budget at the beginning of each period, we show that the threshold maintenance policy is optimal if yield does not decrease as the invested money increases. If the effect of maintenance on the process is such that the *expected* yield is non-decreasing, using Chebyshev's other inequality, we provide insight into the existence of an optimal threshold maintenance policy. To the best of our knowledge, Chebyshev's other inequality has not previously been used in the literature of production and inventory models.

- Given a linear yield function and a fixed budget at the beginning of each period, if yield is non-decreasing in the investment value, the optimal amount of investment does not increase as the amount of inventory on hand increases.

- Given a linear yield function which is non-decreasing in the investment value, the inventory threshold value does not decrease as the budget available at the beginning of the period increases.

We also provide some structural results on the relationship between different problem parameters for both single period and multiple period problems and on the structure of the optimal policy when the yield is not a linear function for the single period problem.

This chapter is organized as follows. Firstly, the problem of interest and our assumptions are defined. Section 3.2 discusses our results for the single period problem. The analysis over multiple periods is given in Section 3.3. We conclude the chapter in Section 3.4. Some of the proofs are provided in Appendix A.

3.1 Problem Definition

We consider a manufacturing firm producing a single product with a single-stage process over n discrete time periods. Let Z_i be the random demand of time period i, we assume that $Z_1, Z_2, \ldots, Z_n$ are independent and identically distributed (i.i.d) with distribution $Q(.)$ and density $q(.)$. At the beginning of time period i, the manufacturer receives a certain amount of raw material to produce u units of production. For example, one could imagine that the raw material has a long lead time and therefore was ordered previously based on a forecast of the demand. As the yield (the number of acceptable products) is random, the manufacturer decides to increase it through investing money in preventive maintenance. Assuming that the manufacturer has a budget of y_i at the beginning of time period i, the decision is to determine the amount of money invested in period i denoted as $a_i, (a_i \leq y_i)$.

As stated previously, we assume that the main causes of yield losses are internal to production process such as machine deterioration. The proportion of acceptable products is therefore dependent on the production quantity. We do not use the common stochastically proportional yield model (Yano and Lee, 1995) to describe the random yield since it is appropriate in situations where the yield losses mainly occur due to random environmental changes and variations in raw materials (Yano and Lee, 1995). We consider a more general form $Y_{a_i} = f(a_i, Y_0, u)$ for the random yield of period i which is a function of the money invested, a_i, the initial yield (the quantity produced without investment), Y_0, and the production quantity, u, with two boundary conditions: $f(0, Y_0, u) = Y_0$ and $f(y_i, Y_0, u) \leq u$. Note that Y_0 and Y_{a_i} are random variables with values over the interval $[0, u]$. Letting $\dot{Y}_{a_i} = \frac{\partial f(a_i, Y_0, u)}{\partial a_i}$ and $\ddot{Y}_{a_i} = \frac{\partial^2 f(a_i, Y_0, u)}{\partial a_i^2}$ for a given random variable Y_0 and production quantity u, we further assume that:

i) the random yield, Y_{a_i}, is increasing in the initial yield, Y_0,

ii) the random yield, Y_{a_i}, is increasing and concave in the production quantity, u: we expect the yield to be higher if there is more production quantity, but its marginal value is non-increasing, and

iii) the marginal yield, $\dot{Y}_{a_i}$, is non-increasing in the initial yield, Y_0: when the initial yield is high, investment in maintenance does not increase the number of acceptable products as much as when the initial yield is low.

To model the effect of investment in maintenance on the random yield, we introduce two different cases:

- *Positive maintenance*: The yield is non-decreasing in a_i, i.e., $\dot{Y}_{a_i} \geq 0$: the number of acceptable products does not decrease as the amount of investment increases.

- *Expected positive maintenance*: The expected yield is non-decreasing in a_i, i.e., $E[\dot{Y}_{a_i}] \geq 0$: the number of acceptable products might increase or decrease as the amount of investment increases; however, the expected number of acceptable products does not decrease.

At the beginning of period i, the initial inventory, x_i, and the total available budget, y_i, are observed. The decision on the amount of money invested in maintenance, a_i, is then made to minimize the total discounted expected cost over the remaining $(n - i + 1)$ periods from period i to n, $\Phi_{(n-i+1)}(x_i, y_i)$, given by the following recursive equation:

$$\Phi_{(n-i+1)}(x_i, y_i) = \min_{0 \leq a_i \leq y_i} \{\pi(x_i, a_i) + \rho E[\Phi_{n-i}(x_i + Y_{a_i} - Z_i, y_{i+1})]\}, \tag{3.1}$$

where $\rho \in [0, 1)$ is a discount factor, $\pi(x_i, a_i)$ is the expected total cost of period i (Equation 3.3), $x_{i+1} = x_i + Y_{a_i} - Z_i$ is the initial inventory for period $(i + 1)$ and $\Phi_0(.,.) = 0$. Note that we can either assume that the budget available at the beginning of each period is determined a priori or that there is a total budget available for investment over all time periods. In both assumptions, the investment decision in a period affects the yield and consequently the initial inventory for the next period, while in the latter assumption it also affects the total budget available for making investment in the next period, i.e., $y_{i+1} = y_i - a_i$. Since characterizing the structure of the optimal solution with the latter assumption is hard, in this chapter we only address the problem assuming that the budget for each time period is previously determined and does not carry over time periods. Therefore, Equation (3.1) is modified as:

$$\Phi_{(n-i+1)}(x_i, y_i, \ldots, y_n) = \min_{0 \leq a_i \leq y_i} \{\pi(x_i, a_i) + \rho E[\Phi_{n-i}(x_i + Y_{a_i} - Z_i, y_{i+1}, \ldots, y_n)]\}, \tag{3.2}$$

where y_i is the pre-determined available budget for period i.

Let $c, p, h \geq 0$ denote the per-item production, backlog, and holding costs. In Equation (3.3), the respective terms represent the production cost, the money invested in maintenance, expected holding cost and expected backlog cost.

$$\pi(x_i, a_i) = cu + a_i + hE[(x_i + Y_{a_i} - Z_i)^+] + pE[(Z_i - x - Y_{a_i})^+].^1 \tag{3.3}$$

In Equation (3.3), the money not used in period i, i.e., $(y_i - a_i)$, does not count negatively toward the total cost. For example, it is assumed that at each period, the firm only has access to the part of the budget which is spent on improvement projects and the firm cannot save the extra money. This assumption is realistic where each firm is a branch of a bigger company and the budget allocation decision is made centrally in the head company. If the firm can save the remaining budget, the expected total cost of

[1]The per-period expected total cost equation is different from the well-known newsvendor problem (periodic review inventory problem with random demand, perfect yield, full backlogging, linear ordering, holding, and penalty costs) because yield is random and dependent on the investment decision. An idea for representing the per-period expected total cost as a newsvendor equation is discussed in Section 7.1.2.

period i equals $cu + a_i + hE[(x_i + Y_{a_i} - Z_i)^+] + pE[(Z_i - x - Y_{a_i})^+] - (y_i - a_i)$.

We use Equation (3.3) in our analysis assuming that the firm cannot save the remaining budget of each period. However, unless otherwise indicated, our results hold true in case the firm can save the money.

Assuming the initial inventory of x_1 and the budget of $y_1, \ldots, y_n$ available for period 1 to n, the goal of the problem is to determine the investment in each period such that the total discounted expected cost from period 1 to period n, $\Phi_n(x_1, y_1, \ldots, y_n)$, is minimized.

3.2 Single Period Analysis

In this section, we analyze the problem over single time period. Since our analysis is mainly based on differentiation, we first present two propositions which provide sufficient conditions for most of the functions used in the chapter to be differentiable rather than simply assuming that the derivative exists. We then derive the optimal policy, provide some insights to the problem, and finally present a numerical study illustrating some of the results.

3.2.1 Sufficient Conditions for the Existence of Derivatives

Propositions 3.1 and 3.2, stated below, provide sufficient conditions such that derivatives exist for functions of the form that are used in this chapter.

Proposition 3.1. *If $g(t) = E[(V + Ut)^+]$ where V and U are random variables, $E[|U|] < \infty$, and $Pr(V + Ut = 0) = 0$, then $g'(t) = E[UI(V + Ut > 0)]$. Note that $x^+ = \max(0, x)$.*

Proof. See Section A.1.1. □

Proposition 3.2. *Let $g(t) = E[Q(V + Ut)]$ where V and U are random variables, $E[|U|] < \infty$, $Pr(V + Ut = 0) = 0$, and $Q(x)$ is a CDF such that $Q(x) = 0$, $\forall x < 0$. Assume also, for simplicity, that $|Q(x + h) - Q(x)| \leq C|h|$ where C is a positive constant. Then $g'(t) = E[UQ'(V + Ut)]$.*

Proof. See Section A.1.2. □

3.2.2 Single Period Optimal Policy

Let us denote the initial inventory, the budget available, and the amount of investment as x, y, and a, respectively. The random variable $W(x, a)$, given below, represents the cost over one time period where $Y = x + Y_a$.[2]

$$W(x, a) = cu + a + h(Y - Z)^+ + p(Z - Y)^+.$$

[2]In this section, we exclude the subscript that indicates the period number from most of notation since we only have one period. We however include the subscript in some notation to make it easier to compare with their counterparts in multiple period problem of Section 3.3. Furthermore, since the demands of all periods are independent and identically distributed random variables, we use Z to refer to the demand of any period in the rest of the chapter.

Knowing that $(Y-Z)^+ - (Z-Y)^+ = Y - Z$, we have

$$\begin{aligned} W(x,a) &= cu + a + h(Y-Z)^+ + p[(Y-Z)^+ - (Y-Z)] \\ &= cu + a + (h+p)(Y-Z)^+ - pY + pZ. \end{aligned}$$

The expected value of $W(x,a)$ gives the cost over one period, $\pi(x,a)$, as below:

$$\begin{aligned} \pi(x,a) = E[W(x,a)] &= cu + a + (h+p)E[(Y-Z)^+] - pE[Y] + pE[Z] \\ &= cu + a + (h+p)E[(x + Y_a - Z)^+] + p(E[Z] - x - E[Y_a]). \end{aligned}$$

The optimization problem over one time period can be written as follows where $\Phi_1(x,y)$ denotes the optimal expected cost over one time period:

$$\Phi_1(x,y) = \min_{0 \le a \le y} (\pi(x,a)).$$

We denote $g(a) = \pi(x,a)$ and $B = (h+p)$.

Lemma 3.1. *The expected cost over one time period, $g(a)$, is convex in a if the yield, Y_a, is*

(i) linear in a, or

(ii) concave in a and there is no holding cost, $h = 0$.

Proof. Using Propositions 3.1 and 3.2, $g'(a)$ and $g''(a)$ are given as below:

$$\begin{aligned} g'(a) &= 1 + BE[\dot{Y}_a I(x + Y_a - Z)^+] - pE[\dot{Y}_a] \\ &= 1 + BE[E[\dot{Y}_a I(x + Y_a - Z)^+ | Y_a]] - pE[\dot{Y}_a] \\ &= 1 + BE[\dot{Y}_a E[I(x + Y_a - Z)^+ | Y_a]] - pE[\dot{Y}_a] \\ &= 1 + BE[\dot{Y}_a Pr(x + Y_a - Z > 0)] - pE[\dot{Y}_a] \\ &= 1 + BE[\dot{Y}_a Pr(Z < x + Y_a)] - pE[\dot{Y}_a] \\ &= 1 + BE[\dot{Y}_a Q(x + Y_a)] - pE[\dot{Y}_a], \\ g''(a) &= BE[\ddot{Y}_a Q(x + Y_a) + \dot{Y}_a^2 q(x + Y_a)] - pE[\ddot{Y}_a]. \end{aligned}$$

To prove the convexity of $g(a)$, it is enough to show that $g''(a) \ge 0$. We have the following two cases:

1. If yield is linear in a, then $\ddot{Y}_a = 0$ and we have:

$$g''(a) = BE[\dot{Y}_a^2 q(x + Y_a)] \ge 0.$$

2. If yield is concave in a and $h = 0$, then $\ddot{Y}_a \leq 0$ and we have:

$$\begin{aligned} g''(a) &= pE[\ddot{Y}_a Q(x + Y_a) + \dot{Y}_a^2 q(x + Y_a)] - pE[\ddot{Y}_a] \\ &= pE[\ddot{Y}_a(Q(x + Y_a) - 1) + \dot{Y}_a^2 q(x + Y_a)] \geq 0. \end{aligned}$$

The inequality follows since $Q(.) \leq 1$.

In each of the above cases, we showed that $g''(a) \geq 0$ which completes the proof. □

Theorem 3.1. *The optimal policy, given a production quantity u, is a threshold policy if one of the following conditions holds true:*

(i) Y_a *is linear in a and maintenance is positive;*

(ii) Y_a *is linear in a,* $q(.)$ *is non-increasing, and maintenance is expected positive;*

(iii) Y_a *is concave in a,* $h = 0$*, and maintenance is positive;*

(iv) Y_a *is concave in a,* $h = 0$*,* $q(.)$ *is non-increasing, and maintenance is expected positive.*

Proof. Given Lemma 3.1, $g(a)$ is convex in all four stated conditions. Denoting the optimal amount of investment when one period is remaining as a_1^*, it will solve $g'(a_1^*) = 0$ for a given x. We need to first show that the solution exists. To have a feasible solution in the interval $[0, y]$, the following conditions need to be true:

$$\begin{aligned} L_1(0) &> 1, \\ L_1(y) &< 1, \end{aligned}$$

where $L_1(a) = pE[\dot{Y}_a(1 - Q(x + Y_a))] - hE[\dot{Y}_a Q(x + Y_a)]$ denotes the marginal yield saving where the first and the second terms are equal to the marginal backlog saving and the marginal holding cost, respectively. If the above conditions are not true, we have the following cases for the optimal amount of investment, a_1^*:

1. If $L_1(0) \leq 1$, then $a_1^* = 0$.
2. If $L_1(y) \geq 1$, then $a_1^* = y$.

Now, we assume that the solution exists. Let $\bar{x}_1(u, y)$ denote the inventory level for the production quantity u and the budget y where the optimal investment is 0 and one time period is remaining, it solves

$$1 + BE[\dot{Y}_0 Q(\bar{x}_1(u, y) + Y_0)] - pE[\dot{Y}_0] = 0.$$

Further, using Proposition 3.2, we have

$$\frac{\partial g'(a)}{\partial x} = BE[\dot{Y}_a q(x + Y_a)].$$

We discuss each of the above conditions below:

(i) In condition (i), since maintenance is positive ($\dot{Y}_a > 0$), $\frac{\partial g'(a)}{\partial x}$ is increasing in x and the optimal investment is 0 for all $x \geq \bar{x}_1(u, y)$. Therefore, the optimal policy is a threshold policy defined as

$$a_1^* = \begin{cases} 0, & x \geq \bar{x}_1(u, y) \\ > 0, & x < \bar{x}_1(u, y) \end{cases}.$$

(ii) In condition (ii), the demand density, $q(.)$, is a non-increasing function. Since Y_a is increasing in Y_0 (assumption (i) in Section 3.1), $q(x + Y_a)$ is then non-increasing in Y_0. In addition, $\dot{Y}_a$ is non-increasing in Y_0 (assumption (iii) in Section 3.1), $\dot{Y}_a$ and $q(x + Y_a)$ are therefore similarly ordered in Y_0. Using Chebyshev's other inequality (Fink and Jodeit, 1984)[3]

$$\begin{aligned} \frac{\partial g'(a)}{\partial x} &= BE[\dot{Y}_a q(x + Y_a)] \\ &\geq BE[\dot{Y}_a]E[q(x + Y_a)] \geq 0, \end{aligned}$$

implying that $\frac{\partial g'(a)}{\partial x}$ is increasing in x, and for all $x \geq \bar{x}_1(u, y)$, the optimal investment is 0. The second inequality follows because of expected positive maintenance ($E[\dot{Y}_a] \geq 0$). Therefore, the optimal policy is a threshold policy as in condition (i).

(iii) The proof for condition (iii) is similar to condition (i).

(iv) The proof for condition (iv) is similar to condition (ii).

Therefore, the optimal policy is a threshold policy in all four stated conditions. □

Given a convex cost function, Theorem 3.1 shows that positive maintenance guarantees the existence of a threshold policy over a single time period. However, if maintenance is expected positive, another condition is needed. We provide an example for each, below.

Example 1: Assume that maintenance is positive where the yield function is $Y_a = (1 - \frac{a}{K})Y_0 + \frac{a}{K}u$ and $K \geq y$. Since Y_a is linear and increasing in a, or more specifically $\dot{Y}_a = \frac{u - Y_0}{K} \geq 0$, condition (i) in Theorem 3.1 guarantees that the optimal policy is a threshold one. In this example, $E[Y_a] \geq E[Y_0]$ and $\text{Var}(Y_a) \leq \text{Var}(Y_0)$.

Example 2: Assume that the yield function, f, is given as $Y_a = (1 - \frac{a}{K})Y_0 + \alpha\frac{a}{k}u$ where $E[Y_0] \leq \alpha u$ and $K \geq y$. We have $\dot{Y}_a = \frac{\alpha u - Y_0}{K}$ which is not necessarily non-negative, however, $E[\dot{Y}_a] = \frac{\alpha u - E[Y_0]}{K}$ is non-negative because of our assumption. Therefore, maintenance is expected positive and since Y_a is linear in a, if we further assume that $q(.)$ is non-increasing, then condition (ii) in Theorem 3.1 guarantees that the optimal policy is a threshold type. In this case, $E[Y_a] \geq E[Y_0]$ and $\text{Var}(Y_a) \leq \text{Var}(Y_0)$.

Given Theorem 3.1 and Examples 1 and 2, we can conclude that investing money in maintenance to improve the production process so that the expected value of the yield increases and the variance of

[3]Chebyshev's other inequality guarantees that If U and V are similarly ordered in X (both are non-increasing or non-decreasing), then $E[U(X) \cdot V(X)] \geq E[U(X)] \cdot E[V(X)]$.

the yield decreases does not necessarily guarantee that a threshold policy is optimal. Even if we set α in Example 2 such that we have necessary conditions for Y_a to be smaller than Y_0 in the convex order, i.e., $E[Y_a] = E[Y_0]$ and $\text{Var}(Y_a) \leq \text{Var}(Y_0)$, the optimal policy is not necessarily a threshold type. Recalling from Section 2.2.2.1, Gupta and Cooper (2005) showed that if the yield after a process improvement project is smaller than the initial yield in the convex order, the expected profit increases. However, as already discussed, the same condition does not guarantee an optimal threshold type policy. To have an optimal threshold policy, we need a stronger condition on the expected value of the yield: it should increase up to a certain level so that the marginal yield ($\dot{Y}_a$) is positive, i.e., investing more money in maintenance does not decrease the number of acceptable products. However, the stated condition is not a necessary condition to have a threshold investment policy. Example 2 represents a yield function where the marginal yield is not positive, but the expected marginal yield is positive, i.e., investing more money does not decrease the number of acceptable products on average. As previously mentioned, if we assume that the demand density function is non-increasing, for example, having an exponential or a uniform distribution, then the threshold policy is optimal.

3.2.3 Insights

In this section, we provide some insights to the single period problem comparing the relationships between problem parameters.

Total Budget and Optimal Investment: If the budget allocated for investing in maintenance, y, increases, the solution space, $a \leq y$, is a superset of the previous solution space. Therefore, as stated in Remark 3.1, the optimal investment, a_1^*, does not decrease.

Remark 3.1. *For a given inventory level, the optimal investment, a_1^*, is non-decreasing in the budget, y.*

It is worth mentioning that Remark 3.1 does not hold true if the firm saves the remaining budget where it counts negatively toward the total cost. More specifically, let assume that $a_1^*(y_1)$ and $a_1^*(y_2)$ indicate the optimal amount of investment for the budget y_1 and y_2, respectively. If $y_2 \geq y_1$, it is straightforward to show that $a_1^*(y_2) - a_1^*(y_1) \geq -(y_2 - y_1)$.

Inventory Level and Optimal Investment: The optimal investment in a period depends on both the total available budget and the available inventory. If the inventory on hand increases, the firm has more acceptable finished products to satisfy demand. The need for investment in maintenance to increase the number of acceptable products therefore decreases and the optimal investment consequently does not increase. Proposition 3.3 states the conditions where this relationship between inventory level and the optimal investment exists. It is worth mentioning that in case of multiple period problem, it might be beneficial to invest more in maintenance when the need for production is reduced (see Section 3.3.2).

Proposition 3.3. *For a given budget y, if one of the following conditions holds true, then the optimal investment, a_1^*, is non-increasing in the inventory level, x.*

(i) Maintenance is positive.

(ii) Maintenance is expected positive and the demand density is non-increasing.

Proof. See Section A.1.4. □

Note that in Remark 3.1 and Proposition 3.3, the yield function can be any general function in the investment value.

Inventory Threshold Value and Total Budget: When the total budget increases, the firm has more resources to commit to maintenance to increase the number of acceptable finished products. Therefore, it would make sense to invest in maintenance even if the amount of inventory on hand is large. Proposition 3.4 states the situations when the inventory threshold value does not decrease as the total budget for investment increases.

Proposition 3.4. *For a given production quantity u, if the conditions of Theorem 3.1 hold true, then the inventory threshold value, $\bar{x}_1(u, y)$, is non-decreasing in the total budget, y.*

Proof. See Section A.1.5. □

Inventory Threshold Value and Production Quantity: We intuitively expect that as production quantity, u, increases, the number of acceptable finished products increases and the threshold value, $\bar{x}_1(u, y)$, decreases for a given y: the firm initiates preventive maintenance if the amount of inventory at the beginning of the time period is low because it needs more product at the end of the period and wants to achieve this by increasing the yield. However, Remark 3.2 shows that there is a non-monotonic relationship between the production quantity and the inventory level for starting maintenance.

In the following remark, we assume that the conditions of Lemma 3.1 hold true on the yield function.

Remark 3.2. *If maintenance is positive, $\dot{Y}_a \geq 0$, then the inventory threshold value, $\bar{x}_1(u, y)$, is non-monotonic in the production quantity, u.*

We know that $\bar{x}_1(u, y)$ solves

$$1 + BE[\dot{Y}_0 Q(\bar{x}_1(u, y) + Y_0)] - pE[\dot{Y}_0] = 0.$$

Denoting $g(\bar{x}_1(u, y), u) = BE[\dot{Y}_0 Q(\bar{x}_1(u, y) + Y_0)]$ and $J(u) = pE[\dot{Y}_0] - 1$, we then have

$$g(\bar{x}_1(u, y), u) = J(u).$$

As we assumed in Section 3.1, Y_a is increasing in u. If $\dot{Y}_a \geq 0$ (as stated in Remark 3.2), we conclude that $\dot{Y}_a$ is also increasing in u. Therefore, both $g(\bar{x}_1(u, y), u)$ and $J(u)$ are increasing functions of u. Then, as shown in Figure 3.1, if the production quantity denoted as u_0 increases to u_1 or u_2, the threshold value does not necessarily decrease. For $u_1, (u_1 > u_0)$ as shown in Figure 3.1, the threshold value decreases, i.e., $\bar{x}_1(u_1, y) < \bar{x}_1(u_0, y)$. However, for $u_2, (u_2 > u_0)$, the threshold value increases, i.e., $\bar{x}_1(u_2, y) > \bar{x}_1(u_0, y)$.

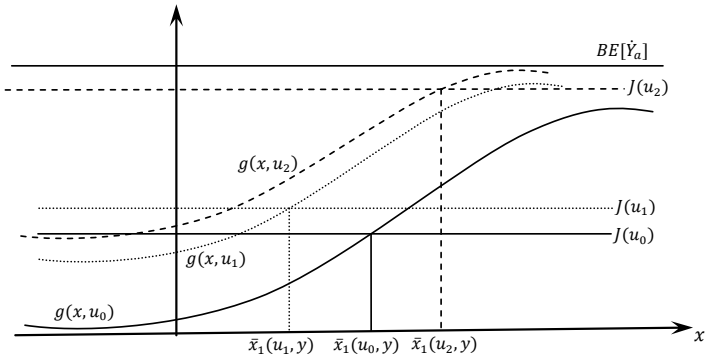


Figure 3.1: Changes in the inventory threshold value, $\bar{x}_1(u, y)$, according to the changes in the production quantity, u, when maintenance is positive.

3.2.4 Numerical Example

In this section, we provide a numerical example illustrating some of the results in the previous section.

We assume that the demand has an exponential distribution with a mean of 100, the initial yield has a uniform distribution, the holding and the backlog costs equal 1 and 6, respectively, and the total available budget is 90. Further we assume the random yield function is represented as $Y_a = (1 - \frac{a}{K})Y_0 + \frac{a}{K}u$.

Figure 3.2 illustrates the results of Proposition 3.3 that the optimal amount of investments, a_1^*, does not increase as the amount of initial inventory, x, increases for different production quantities and $K = 100$. Figure 3.3 shows that the production quantity, u, is not monotone with respect to the inventory threshold value, $\bar{x}_1(u, y) = \bar{x}_1(u, 90)$, for both values of $K = 100$ and $K = 150$ as stated in Remark 3.2. One observation in Figure 3.3 is that the inventory threshold value, $\bar{x}_1(u, 90)$, decreases for higher values of K. Increasing the number of acceptable products is more expensive when K is higher. Intuitively, when process improvement projects are costly, the firm does not make an investment unless the inventory on hand is low.

3.2.5 Summary of Single Period Analysis

The summary of the results by analyzing the single time period problem is provided below.

- If the yield is a linear function of the amount of investment or is a concave function of the investment value with a zero holding cost, we have:
  - The investment policy is a threshold policy for a given production policy if investing more money in maintenance never decreases the number of acceptable products (positive maintenance).
  - In a practical situation, it seems more reasonable that the marginal yield is not positive, but the expected marginal yield is positive (expected positive maintenance). Investing in maintenance might decrease or increase the number of acceptable products, but the expected number of acceptable products does not decrease. Our analysis shows that the investment

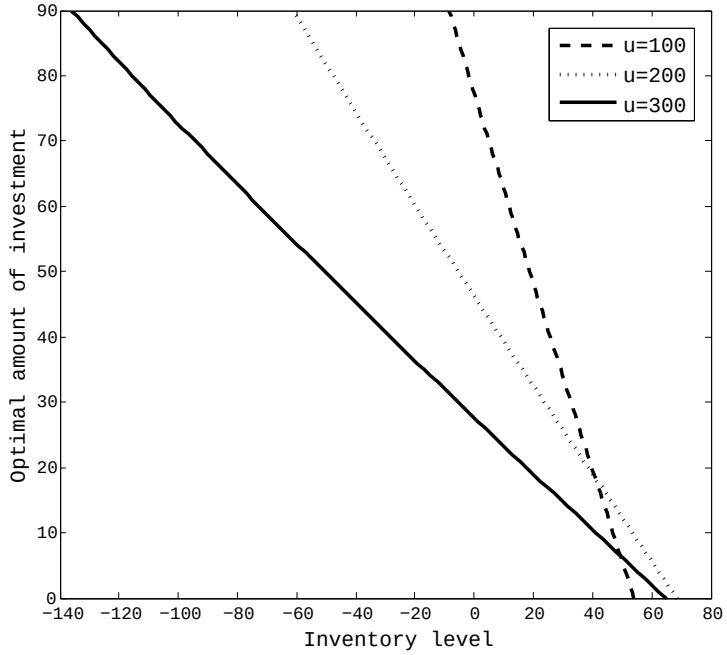


Figure 3.2: The optimal amount of investment is non-increasing in the initial inventory for different production quantities and $K = 100$.

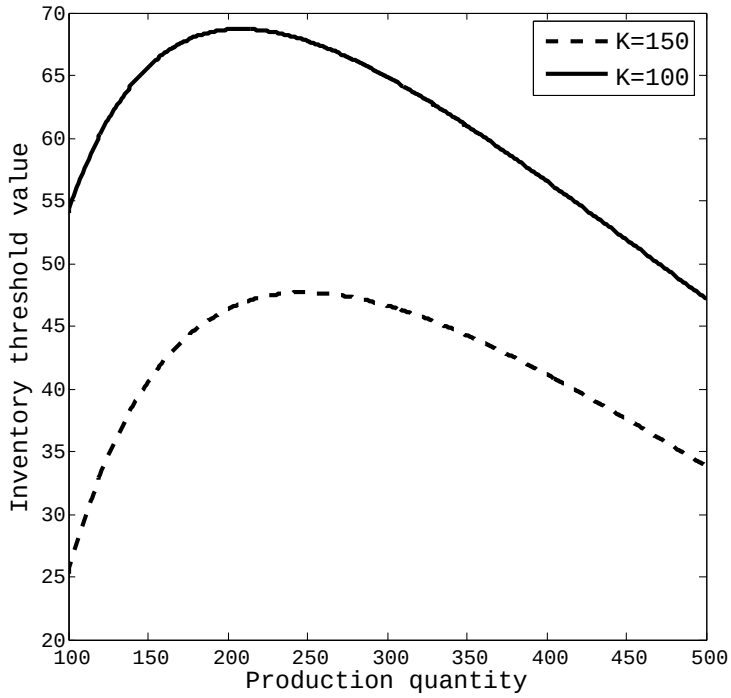


Figure 3.3: The non-monotonicity of production quantity with respect to the inventory threshold value for different K values.

policy in this situation is a threshold policy if the demand density function is non-increasing, for example, having an exponential or a uniform distribution.

  - Increasing the production quantity does not necessarily decrease the threshold on the inventory level.
  - The inventory threshold value does not decrease if the budget increases in both cases of positive maintenance and expected positive maintenance with non-increasing demand density.

- In both cases of positive maintenance and expected positive maintenance with non-increasing demand density, the optimal amount of investment does not increase if more inventory is on hand.
- The optimal amount of investment does not decrease if more budget is available.

3.3 Multiple Period Analysis

In this section, we address the multiple period problem. First, we derive the multiple period optimal policy and we then provide some insights to the problem.

3.3.1 Multiple Period Optimal Policy

Assuming x and y_i as the initial inventory and the budget for the i-th period,[4] the optimization problem over n time periods is given below. Recall that we assume the budget available for each period is previously determined and the money does not carry over the periods.

$$\Phi_n(x, y_1, \ldots, y_n) = \min_{0 \le a \le y_1} \{\pi(x, a) + \rho E[\Phi_{n-1}(x + Y_a - Z, y_2, \ldots, y_n)]\}.$$

Proposition 3.5. *The expected cost over one time period, $\pi(x, a)$, is a jointly convex function given the conditions of Lemma 3.1 hold true on the yield function.*

Proof. See Section A.2.1. □

Proposition 3.6. *$\Phi_1(x, y_n)$ is convex in x given the conditions of Lemma 3.1 hold true on the yield function.*

Proof. See Section A.2.2. □

Proposition 3.7. *$\Phi_n(x, y_1, \ldots, y_n)$ is convex in x if Y_a is linear in a.*

Proof. We use induction to show the convexity of Φ_n in x. In Proposition 3.6, we have shown that $\Phi_1(x, y_n)$ is convex in x. Assuming that $\Phi_{n-1}(x, y_2, \ldots, y_n)$ is convex in x and knowing that Y_a is linear in a, $\Phi_{n-1}(x + Y_a - Z, y_2, \ldots, y_n)$ is convex in (x, a) as $x + Y_a - Z$ is a linear function of x and a (Theorem 5.7 of Rockafellar (1970)). Since $\pi(x, a)$ is convex in (x, a) (Proposition 3.5); therefore,

[4] Since in this section we do not explicitly refer to the initial inventory level of any other period except period i, we exclude the subscript that represents the period number.

$\pi(x,a) + \rho E[\Phi_{n-1}(x + Y_a - Z, y_2, \ldots, y_n)]$ is convex in (x,a). Using the same reasoning as used in the proof of Proposition 3.6 (see Section A.2.2.) results in the convexity of $\Phi_n(x, y_1, \ldots, y_n)$ in x. □

Theorem 3.2. *The optimal policy over n time periods, given a production quantity u, is a threshold policy if the yield is linear in the investment value and maintenance is positive.*

Proof. We re-write the optimal expected discounted cost over n periods as

$$\Phi_n(x, y_1, \ldots, y_n) = \min_{0 \le a \le y_1} \{J_n(x, a, y_2, \ldots, y_n)\},$$

where,

$$J_n(x, a, y_2, \ldots, y_n) = \pi(x,a) + \rho E[\Phi_{n-1}(x + Y_a - Z, y_2, \ldots, y_n)].$$

Assuming that derivatives of $\Phi_n(x, y_1, \ldots, y_n)$ in x exist and that Y_a is linear in a, we have

$$\frac{\partial J_n}{\partial a} = 1 + BE[\dot{Y}_a Q(x + Y_a)] - pE[\dot{Y}_a] + \rho E[\dot{Y}_a \frac{\partial \Phi_{n-1}(x + Y_a - Z, y_2, \ldots, y_n)}{\partial x}],$$
$$\frac{\partial^2 J_n}{\partial a^2} = BE[\dot{Y}_a^2 q(x + Y_a)] + \rho E[\dot{Y}_a^2 \frac{\partial^2 \Phi_{n-1}(x + Y_a - Z, y_2, \ldots, y_n)}{\partial x^2}].$$

Using Proposition 3.7, $\frac{\partial^2 J_n}{\partial a^2} \ge 0$ and the optimal solution will solve

$$\frac{\partial J_n}{\partial a} = 0.$$

The reasoning on the existence of optimal solution is the same as in Theorem 3.1 with the only difference that $L_n(a)$, given below, replaces $L_1(a)$:

$$L_n(a) = pE[\dot{Y}_a(1 - Q(x + Y_a))] - hE[\dot{Y}_a Q(x + Y_a)] - \rho E[\dot{Y}_a \frac{\partial \Phi_{n-1}(x + Y_a - Z, y_2, \ldots, y_n)}{\partial x}].$$

Denote the optimal amount of investment when n periods are remaining by a_n^* and the inventory level for which the optimal investment is 0 by $\bar{x}_n(u, y_1)$. We have

$$\frac{\partial J_n}{\partial a \partial x} = BE[\dot{Y}_a q(x + Y_a)] + \rho E[\dot{Y}_a \frac{\partial^2 \Phi_{n-1}(x + Y_a - Z, y_2, \ldots, y_n)}{\partial x^2}].$$

As Proposition 3.7 and positive maintenance guarantee that $\frac{\partial J_n(x,a,y_2,\ldots,y_n)}{\partial a \partial x}$ is increasing in x, the optimal investment is 0 for all $x \ge \bar{x}_n(u, y_1)$ which completes the proof. □

Theorem 3.2 shows that condition (i) of Theorem 3.1, i.e., a linear yield in the amount of investment and positive maintenance, guarantees the existence of the threshold policy over multiple periods. If maintenance is expected positive, which is more likely in the real situation, to have a threshold opti-

mal policy over n periods, we need three conditions to guarantee $\frac{\partial J_n(x,a,y_2,\ldots,y_n)}{\partial a \partial x}$ is increasing in x using Chebyshev's other inequality. The conditions are:

(i) yield is linear in the investment,

(ii) the demand density function is non-increasing, and

(iii) the second derivative of the optimal total discounted expected cost over $(n-1)$ periods, $\Phi''_{n-1}(x, y_1, \ldots, y_n)$, is non-increasing in x.

Therefore, the conditions that make the threshold policy optimal over one time period, Conditions (i) and (ii), are not sufficient to guarantee the existence of a threshold policy over n periods in case of expected positive maintenance. Condition (iii) should also hold true to have a threshold optimal policy, however, this condition is not verifiable and useful in practice since it is stated on the cost function, not on the problem parameters.

3.3.2 Insights

Similar to Section 3.2.3, in this section, we present several insights to the multiple period problem comparing the relationships between problem parameters.

Total Budget and Optimal Investment: If the firm allocates more budget for investment in the first period, it has more resources for increasing the production quantity. In the other words, the solution space, $a \leq y_1$, becomes larger and since it includes the solution space before budget increase, the optimal investment when n periods are remaining, a_n^*, does not decrease.

Remark 3.3. *For a given inventory level, the optimal investment when n periods are remaining, a_n^*, is non-decreasing in the total budget available at the beginning of the first period, y_1.*[5]

Inventory Level and Optimal Investment: When n periods are remaining, Proposition 3.8 states that if the inventory on hand increases in case of positive maintenance, the need for increasing the yield decreases. Thus, the firm does not increase the optimal amount of investment in maintenance.

If maintenance takes the potential production capacity, resulting in periods of process unavailability, it might be optimal to invest more in maintenance when the inventory level on hand is high and there is a reduced need for production. Performing maintenance will likely increase the initial inventory level for the next periods, increasing the holding cost, but it will also decrease the backlog cost. It is worth mentioning that the expected positive maintenance situation can be considered as an example where maintenance takes the potential production capacity since investing more money does not guarantee an increase in the production quantity. Therefore, it might be optimal to increase the investment in maintenance even if the current inventory on hand is high enough in order to ensure a high inventory level in future periods. The possibility of being optimal to increase the investment for high inventory levels in the expected positive maintenance case further implies that the threshold policy is not necessary optimal without some additional assumptions as discussed in Section 3.3.1.

[5]This remark does not hold true if the firm saves the remaining budget where it counts negatively toward the total cost of one period.

Proposition 3.8. *For a given budget y_1, the optimal investment, a_n^*, is non-increasing in the inventory level, x, if maintenance is positive and the yield is linear in the investment value.*

Proof. The proof is the same as the proof of Proposition 3.3 where $J_n(x, a, y_2, \ldots, y_n)$ and $a_n^*(x, y_1)$ replace $\pi(x, a)$ and $a_1^*(x, y)$. Note that, we first need to prove that $J_n(x, a, y_2, \ldots, y_n)$ is supermodular in (x, a) which is shown in Section A.2.3. □

Inventory Threshold Value and Total Budget: The following proposition states the same result as Proposition 3.4 for n periods. If the budget at the beginning of the period increases, the firm has more resources to improve the production process and invests in maintenance even if the inventory on hand is high enough.

Proposition 3.9. *For a given production quantity u, the inventory threshold value, $\bar{x}_n(u, y_1)$, is non-decreasing in the budget y_1 if maintenance is positive and the yield is linear in the investment value.*

Proof. The proof is similar to the proof of Proposition 3.4. Note that the supermodularity of $\Phi_n(x, y_1, \ldots, y_n)$ in (x, y_1) is proved in Section A.2.3. □

It is worth mentioning that Propositions 3.8 and 3.9 correspond to Propositions 3.3 and 3.4 in Section 3.2.3. However, if one period is remaining, the properties hold true in more general cases where maintenance can be also expected positive.

3.3.3 Summary of Multiple Period Analysis

The summary of our results for multiple period problem is:

- If the yield is a linear function of the amount of investment and maintenance is positive:
  - The optimal investment policy is a threshold type policy.
  - The optimal amount of investment in maintenance does not increase if there is more inventory on hand.
  - The inventory threshold value does not decrease if the budget for investing in maintenance increases.
- If the budget increases, the optimal amount of investment does not decrease.

3.4 Conclusion

Uncertainties in production systems resulting in a random yield can be due to internal causes such as machine deterioration and machine breakdowns. Therefore, there is an interest to invest in process improvement projects such as preventive maintenance to increase the yield. A firm must jointly optimize the production quantity and invest in maintenance. In this chapter, we study this problem, addressing the integration of maintenance and production planning with partial control over machine conditions in

a periodic review system. However, because of the non-convexity of the cost function, we analyze the problem by fixing the production quantities to gain insight into the structure of optimal maintenance policy. As mentioned by Gupta and Cooper (2005), the assumption of fixed production quantities is reasonable when the firm is committed to them ahead of time.

We have mainly focused on understanding the structure of the optimal maintenance policy. If the yield is a linear function of the amount of money invested in maintenance and the marginal yield is positive, our results show that:

- the optimal policy is a single critical level type of the inventory level. At the beginning of each time period, a firm must observe the inventory level and if it is below a certain amount, it is optimal to invest in maintenance,
- the optimal amount of investment does not increase if the available inventory increases, and
- the inventory threshold value does not decrease if the firm has more budget to invest in maintenance.

However, in a real situation, the marginal yield is not always positive. It is more likely that the expected yield will not decrease as the amount of money invested in maintenance increases, i.e., the expected marginal yield is positive. Our analysis shows that if the demand probability density function is non-increasing, the threshold maintenance policy is optimal, though, only over a single time period. We have also provided some structural results when the yield is not a linear function of the investment value which are only valid for the single period problem.

Furthermore, we have discussed the technical problem of providing sufficient conditions, albeit very general ones, such that the derivatives exist for the functions commonly used in the production and inventory literature.

In this chapter, we took a strategic perspective and addressed the interdependency between production planning and maintenance where the goal is to find the optimal production quantity and investment in maintenance increasing the number of acceptable finished products. In the next chapter, we assume that the production quantity and the amount of investment are determined and therefore study the relationship between production and maintenance taking an operational view. Our goal in the next chapter is to determine the optimal allocation of resources to either production or maintenance to maximize the number of acceptable finished products by their due dates.

Chapter 4

Maintenance Planning & Production Scheduling with No Control over Machine Conditions

Production scheduling concerns the optimal allocation of machines to competing customer orders to maximize customer satisfaction. However, unexpected machine breakdowns might result in periods of unavailability where machines are under repair and no orders can be processed. Performing preventive maintenance on machines can partially control their conditions, decreasing the number of failures. In some production systems, it is, however, not justifiable to preventively maintain machines for example because of the random deterioration processes of machines[1] or higher cost of preventive maintenance than corrective maintenance. Therefore, the maintenance policy is the failure-based policy where there is no control over machine conditions and machines are maintained only at failures (see Section 2.2.1.3). In these production systems, utilizing the available information on machine breakdowns and incorporating them into the production schedule to hedge against failures is a challenging problem. In this chapter, we address this challenge in the context of an aircraft fleet management problem where the flights and the aircraft correspond to the production activities and the machines, respectively and where the aircraft are maintained only at failures because of high preventive maintenance cost.

Motivated by the work of Safaei et al. (2010; 2011), we study the problem of scheduling a military aircraft repair shop, where a number of flights are planned over a long horizon. Every flight, also called *a wave*, has requirements for a specific number of aircraft of different types. Flights might be partially carried out without their requirements. Aircraft are checked for failures before and after each flight: if an aircraft is diagnosed as failed, it enters the repair shop and is minimally repaired. Aircraft flow over a long horizon is illustrated in Figure 4.1. The goal is to determine the optimal assignment of aircraft to waves and a schedule of aircraft repairs that will maximize the flight coverage, that is, the extent to which the aircraft requirements of the flights are met. This problem is an example of an

[1] If a machine has a random deterioration process, its failure rate is constant and does not increase as the age of the machine increases.

integrated maintenance and production scheduling problem with no control over machine conditions where machine breakdowns (aircraft failures) limit machine availabilities for production (carrying out the flights) and where machines (aircraft) are minimally repaired only at failures.

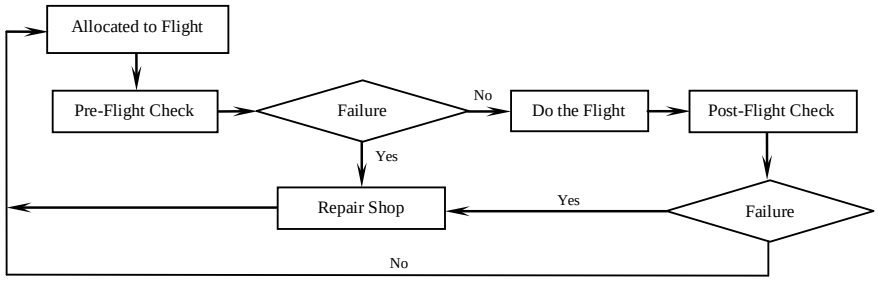


Figure 4.1: Aircraft flow among waves, checks, and the repair shop over a long horizon.

Dynamic Scheduling, reviewed in Section 2.3.3, is one of the areas dealing with the interdependency between maintenance and production scheduling assuming no control over machine conditions. Adopting dynamic scheduling approaches for solving the problem in this chapter, we show that reasoning about uncertainty in constructing the repair schedule increases the availability of aircraft (machines) for carrying out the flights (executing the production activities). More specifically, the central idea of our solution approach is to view the dynamic repair shop as successive static sub-problems over shorter time periods. A solution of the static sub-problem determines an assignment of aircraft to flights and a schedule of repair jobs maximizing the flight coverage. When a failed aircraft enters the repair shop while the previous repair schedule is still under execution, we reschedule the repair activities by solving a new static sub-problem.

In this chapter, we first provide a background, including a formal problem definition. We then prove that the static sub-problem is NP-hard and explore several techniques to solve it: mixed integer programming (MIP); constraint programming (CP); logic-based Benders decomposition (LBBD) using either MIP or CP; and a dispatching heuristic motivated by the Apparent Tardiness Cost (ATC) dispatching rule. To connect the static sub-problems, we design three different rescheduling policies based on the length of the scheduling horizon and how frequently rescheduling is done.

We perform two separate empirical studies. The first indicates that the integration of the dispatching heuristic and LBBD results in the lowest mean run-time of the techniques tested to optimally schedule the repair shop. The second experiment demonstrates that both defining the static scheduling problem over a longer horizon and rescheduling more frequently provide the flights with 10% higher coverage than either one of them alone.

The remainder of this chapter is organized as follows: We formally define the problem, and provide an overview of the relevant literature in Section 4.1. We then prove the NP-hardness of the static sub-problem in Section 4.2. Section 4.3 defines a number of solution approaches for it, presents the details of the proposed policies for rescheduling the dynamic repair shop, and describes our model of the aircraft failures. The computational results on the performance of different scheduling techniques and on how and when rescheduling should be done are described in Section 4.4. A discussion of our

solution approach and results are presented in Section 4.5. We end with conclusion in Section 4.6.

4.1 Background

In this section, the formal definition of the problem is given and the relevant literature on repair shop scheduling is reviewed.

4.1.1 Problem Definition

Figure 4.2 is a snapshot of the problem at time 0, where circles represent aircraft. A number of flights (five are shown) and their corresponding pre- and post-flight checks are already scheduled over a long horizon. It is assumed that the total number of aircraft is constant over a long horizon. A number of aircraft (three in the diagram) are ready for the pre-flight check while others are currently in the shop awaiting repair before they can proceed to a pre-flight check. Failure is only detected during a check and we assume that a check will always correctly assess the status of an aircraft at negligible cost and that the duration of a check is incorporated in the length of the corresponding wave.

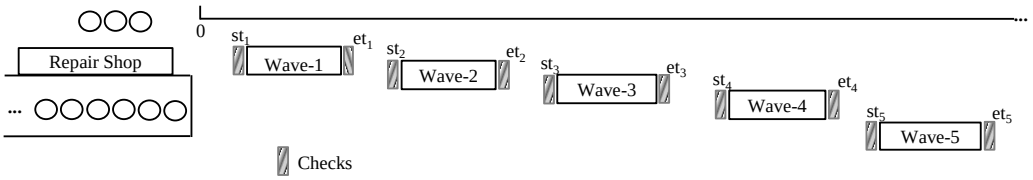


Figure 4.2: Snapshot of the problem at time 0 over a long horizon.

The goal is to assign aircraft to waves to maximize coverage while at the same time creating a feasible repair schedule. The scheduling problem is under the constraints that the repair shop has limited capacity and the aircraft are subject to breakdown. We assume that once an aircraft fails, it goes to the repair shop and waits until its repair operations are performed.

We use the following notation to represent the problem.

- N is the set of aircraft. λ_n is the failure rate of the aircraft $n \in N$ denoting the frequency of failure per time unit. For example if the failure rate is 0.2 per day, it means that the mean time to aircraft failure is 5 days.

- K is the set of aircraft types. I_k denotes the set of aircraft type $k \in K$ where η_k aircraft are ready (i.e., not in the repair shop at time 0). Let $|I_k|$ denote the number of aircraft of type k, $|I_k| - \eta_k$ aircraft of type k are then in the repair shop at time 0. $\bar{\lambda}_k$ is the mean failure rate over all aircraft of type k.

- R is the set of repair resources (called *trades*). The maximum capacity of trade $r \in R$ is C_r which is the maximum number of units of trade r that can be used at any one time point.

- W is the set of waves. Each wave, $w \in W$, has a start-time, st_w, and an end-time, et_w. Each wave requires at most a_{kw} aircraft of type k.

- J is the set of existing jobs in the repair shop. Each job is associated with a specific aircraft type. M_r is the set of jobs requiring trade r. Each job might require more than one trade to be completed. The processing time of job j on trade r is p_{jr} and c_{jr} is the capacity of trade r required by job j.

To model the deterioration of an aircraft, each time it flies a wave its failure rate, λ_n, increases by γ percent, i.e., its failure rate is $(1 + 0.01 \times \gamma)\lambda_n$ after the flight. If an aircraft fails, its failure rate after repair returns to what it was just before the failure. In the other words, as in one of the standard repair models in the maintenance literature, repair is minimal (Wang, 2002). The probability of diagnosing aircraft n as failed in pre- and post-flight checks is a function of its failure rate right before the checks denoted as $f^{pre}(\lambda_n)$ and $f^{post}(\lambda_n)$. The probability of failure detection in pre-flight checks is smaller than the post-flight checks because an aircraft is either just released from the repair shop or has already passed a previous post-flight check successfully (Safaei et al., 2011).

To find the probability of failure of an aircraft in pre- and post-flight checks for a specific wave, we need to track the complete history of the aircraft. For example, if we assume that a given aircraft is repaired and assigned to the first wave, then there are three paths: the aircraft fails the pre-flight check; the aircraft passes the pre-flight check, flies the wave, and fails the post-flight check; or the aircraft passes the pre-flight check, flies the wave, and passes the post-flight check. Therefore, the availability of the aircraft for the second wave can be represented as a random variable whose expected value depends on the probability of these three different paths and the scheduling decisions to repair the failed aircraft before the second wave. Similarly the availability of the aircraft for subsequent waves depends on its entire path through the checks, repair shop, and waves. As the number of waves and aircraft increase, the size of the state space will become prohibitive. Furthermore, the repair scheduling decisions themselves impact the aircraft histories: the probability that an aircraft is available for the third wave is different depending on if it was repaired in time for the first wave or only for the second wave. The details on approximating the failure probabilities are presented in Section 4.3.1.1.

As the complexity of the problem has not been shown, we prove that it is NP-hard in Section 4.2.

4.1.2 Literature Review

As already mentioned, the problem of this chapter is an example of integrated maintenance and production scheduling problem assuming no control over machine conditions which is reviewed in detail in Sections 2.3.2 and 2.3.3. In this section, we provide necessary background on repair shop scheduling problem, the context of our example problem.

4.1.2.1 Repair Shop Scheduling

Repair shops have been mainly studied as a machine-repairman problem (Haque and Armstrong, 2007; Stecke, 1992) which has a set of workers and a set of machines that are subject to failures and therefore

need repair. Workers and machines respectively correspond to trades and aircraft, in our problem. As the number of workers is less than the number of machines, it is necessary to allocate the repair jobs to the workers with the goal of optimizing a given performance measure (e.g., the total expected machine downtime) over the long term. Derman et al. (1980) did the early work on solving the scheduling problem of a repair shop with a single repairman. They showed that repairing the failed machines in non-decreasing order of failure rate stochastically maximizes the number of working machines. The literature on the scheduling of a repair shop was then extended by considering multiple repairmen, preemptive and non-preemptive repair, and different failure and repair distributions. A comprehensive review of the literature on the scheduling of a repair system is provided by Iravani et al. (2007).

The analytical models in the literature are mainly developed using Markov Decision Processes (dynamic programming) and guarantee the optimality of a given performance measure in the long term. These models often do not consider the combinatorics of the real scheduling problems such as different repair capacity limits, different due dates, and different resource and processing requirements. Therefore, they typically result in a static dispatching-type repair policy similar to that found by Derman et al. (1980). However, in our problem, the waves have different plane requirements and the processing times and the resource requirements of the repair activities become known when they enter the repair shop. Therefore, we believe that a better performance can be achieved by dealing directly with the combinatorics and explicitly scheduling the repair shop to meet the waves. To handle the uncertain and combinatorial structure of the scheduling problems, our solution approach is based on the ideas of dynamic scheduling algorithms reviewed in Section 2.3.3.

Other areas of literature with similarities to our static problem are the operational level maintenance scheduling problem, in general, and the flight and maintenance planning problem of military aircraft, specifically. The former literature addresses the problem of finding a schedule for given maintenance activities such that the sum of maintenance costs is minimized. The focus is on the operational level, determining the maintenance activities performed in each time period (Budai et al., 2006). Starting with the early work of Wagner et al. (1964), this literature was extended through developing mathematical models and effective solution approaches for a variety of applications (Frost and Dechter, 1998; Haghani and Shafahi, 2002; Budai et al., 2006; Grigoriev et al., 2006). The latter literature studies the problem of maintenance planning and mission assignment of military aircraft where the goal is to decide which aircraft to fly and which one to perform maintenance, maximizing their long-term availability. Similar to the maintenance scheduling literature, mathematical programming is the common approach to solve the problems of this literature. Kozanidis et al. (2012) recently proposed a mixed integer non-linear programming model to optimize the joint flight and maintenance plan of mission aircraft.

Safaei et al. (2010) modeled the static problem addressed here as an operational level maintenance scheduling problem using MIP. Their MIP includes an assignment problem and two network problems: the former assigns the aircraft to the waves and the latter calculates the expected number of available aircraft for the waves as well as the expected number of available workers for the repair jobs. They later extended their work by using slightly different MIP model where the time-indexed approach is used to enforce the workforce availability constraint and they verified the validity of their model by a number

of instances under different combinations of workforce sizes (Safaei et al., 2011).

The difference between the static problem addressed in this chapter and the previous works on operational maintenance scheduling is that our objective function (flight coverage) depends not only on the scheduling decisions but also on the outcomes of the pre- and post-flight checks. These two quite different components of the problem motivate the decomposition approach, logic-based Benders decomposition.

4.2 The Complexity of the Static Repair Shop Problem

In the static repair shop scheduling problem, we maximize the number of aircraft assigned to waves subject to the condition that the sum of the probabilities of aircraft surviving the pre-flight check is greater than or equal to a fixed threshold value (see Section 4.3.1.1). We establish the NP-hardness of the static repair shop problem by reduction from the PARTITION problem (Garey and Johnson, 1979).

Theorem 4.1. *The static problem is NP-hard.*

Proof. Consider an arbitrary instance of PARTITION problem (Garey and Johnson, 1979) as follows: Given $B \in \mathbb{Z}^+$, a set $A = \{z_1, z_2, \ldots, z_{2n}\}$, $z_k \in \mathbb{Z}^+$ and $\sum_{k=1}^{2n} = 2B$, does there exist a partition of A into two disjoint subsets A_1 and A_2 such that $\sum_{z_j \in A_1} z_j = \sum_{z_j \in A_2} z_j = B$?

Given an instance of PARTITION problem, a specific instance of the decision version of the static problem can be constructed such that there is one wave, there are $2n$ failed aircraft in the repair shop ($|N| = 2n$), there are $2n$ aircraft types ($|K| = 2n$), and there is one repair resource with capacity $C = 1$. The start-time of the wave is $st_1 = B$, requiring all $2n$ aircraft. Each failed aircraft, j, has a different type, and corresponds to one repair job in the repair shop with the processing time $p_{j1} = z_j$ and resource requirement $c_{j1} = 1$ for the single resource. The probability of failure in the pre-check of the wave for aircraft j is $(1 - \frac{z_j}{\max_j(z_j)})$. The repaired aircraft j contributes to the flight coverage if it survives the pre-check with probability $\frac{z_j}{\max_j(z_j)}$. For this specific instance of the static problem, consider the following decision problem.

Decision Problem: Does there exist a schedule σ such that the sum of the probabilities of aircraft surviving the pre-check is at least $\frac{B}{\max_j(z_j)}$ and the flight coverage of the σ, Γ_σ, satisfies $\Gamma_\sigma \geq 0$? The decision problem is clearly in class NP. Also, it is easy to verify that the construction of the decision problem can be done in polynomial time. It is proven below that there is a schedule σ such that $\Gamma_\sigma \geq 0$ and the sum of the probabilities of the aircraft surviving the pre-check is at least $\frac{B}{\max_j(z_j)}$ if and only if there exists a solution to the PARTITION problem.

If part ($\Rightarrow$): If there exists a partition, then there is a schedule σ where all aircraft in subset A_1 are repaired for the first wave. Therefore the sum of the probabilities of the aircraft surviving the pre-check equals $\sum_{j \in A_1} \frac{z_j}{\max_j(z_j)} = \frac{B}{\max_j(z_j)}$ and the flight coverage is obviously greater than or equal to 0.

Only if part ($\Leftarrow$): Suppose that there is a schedule σ with the sum of the probabilities of the aircraft surviving the pre-check greater than or equal to $\frac{B}{\max_j(z_j)}$ and the flight coverage of greater than or equal to 0. Further assume that Q is the set of all aircraft repaired before the first wave in schedule σ. Since

the start time of the first wave is B, then $\sum_{j \in Q} z_j \leq B$. Furthermore, the sum of the probabilities of the aircraft surviving the pre-check equals $\sum_{j \in Q} \frac{z_j}{\max_j(z_j)}$ which is greater than or equal to $\frac{B}{\max_j(z_j)}$. Therefore, we can conclude that $\sum_{j \in Q} z_j = B$ meaning that there is a solution to the PARTITION problem. □

4.3 Solution Approach

The main idea of our solution approach is to view the dynamic problem as linked successive static sub-problems which is a common approach in dynamic scheduling. This view results in a rescheduling strategy based on scheduling static sub-problems over shorter time periods. Therefore, we have two sub-goals: how to solve and how to connect the static sub-problems. In this section, we first present different solution techniques for solving the static sub-problems and then define three rescheduling strategies designed to connect them. Finally, we describe our approach for modeling the dynamic events, i.e., aircraft failures.

4.3.1 Scheduling Techniques

We investigate a number of approaches to solve the repair shop scheduling problem including mixed integer programming, constraint programming, logic-based Benders decomposition, a dispatch rule, and a simple hybrid approach. Each of the approaches is described in detail in this section.

4.3.1.1 Mixed Integer Programming

We propose a novel mixed integer programming model where the uncertainty in the outcome of the checks is modeled as expectation. This model is different from and, as we show below in Section 4.4.1.2, significantly faster than those of Safaei et al. (2010; 2011). Table 4.1 summarizes the notation defined in Section 4.1.1 and defines the decision variables of the MIP model.

In this section, without loss of generality, we interpret W as the set of waves in the current static sub-problem and consider the start-times of the waves as due dates to finish the repair of the aircraft. Therefore, we define $D = \{d_i | i = 1, 2, ..., |W|, |W| + 1\}$ to be an ordered set of due dates consisting of the wave start-times plus a big value, B, sorted in ascending order. More specifically, d_i equals to the start-time of the i-th wave, st_i. Because of the limited repair capacity, it is possible that some of the failed aircraft cannot be repaired in time for any of the waves. In such a case, the due date of the repair job is assigned to $d_{|W|+1} = B$. In our model, B equals the sum of the start-time of the last wave and the maximum processing times of all the jobs over all the trades, i.e., $d_{|W|} + \max_{j,r}(p_{jr})$ and we do not enforce the repair resource capacity after $d_{|W|}$.

As explained in Section 4.1.1, the exact calculation of the aircraft failure probability and consequently the expected number of available aircraft is intractable since it depends on the complete aircraft histories. Therefore, we distinguish aircraft based on their type and use a recursive equation (Equation 4.4) to approximate the expected number of available aircraft. The details of Equation (4.4) are provided later in this section. For each aircraft type of k, the average failure rate, $\bar{\lambda}_k$, is used to calculate the prob-

ability of failure during pre- and post-flight checks, respectively: $\xi_k^{pre} = f^{pre}(\bar{\lambda}_k)$ and $\xi_k^{post} = f^{post}(\bar{\lambda}_k)$. Furthermore, the failure rate of each aircraft is assumed to remain constant in the scheduling horizon of the static problem and to not increase after flying a wave. Therefore, our approximation is likely to underestimate the number of actual aircraft failures.

Notation	
$N = \{1, 2, ..., n, ..., \|N\|\}$	The set of aircraft
$K = \{1, 2, ..., k, ..., \|K\|\}$	The set of aircraft types
$R = \{1, 2, ..., r, ..., \|R\|\}$	The set of trades
$W = \{1, 2, ..., w, ..., \|W\|\}$	The set of waves
$J = \{1, 2, ..., j, ..., \|J\|\}$	The set of repair jobs (failed aircraft) in the repair shop
λ_n	The failure rate of aircraft n
I_k	The set of the aircraft of type k
η_k	The number of aircraft of type k at the repair shop at time 0
$\bar{\lambda}_k$	The average failure rate over all aircraft of type k being equal to $\frac{\sum_{n \in I_k} \lambda_n}{\|I_k\|}$
ξ_k^{pre}	The probability that aircraft type k fails in pre-flight check
ξ_k^{post}	The probability that aircraft type k fails in post-flight check
st_w	The start-time of wave w
et_w	The end-time of wave w
a_{kw}	The maximum number of aircraft of type k required by wave w
M_r	The set of the repair jobs requiring trade r
C_r	The maximum capacity of trade r
p_{jr}	The processing time of job j on trade r
c_{jr}	The capacity of trade r required to process job j
$D = \{d_1, ..., d_i, ..., d_{\|W\|+1}\}$	The set of due dates where $d_i = st_i, \forall i \leq \|W\|$ and $d_{\|W\|+1} = B$
B	The big value equal to $d_{\|W\|} + \max_{j,r}(p_{jr})$
Decision Variables	
Z_{kw}	The number of aircraft of type k assigned to fly in wave w
x_{ij}	$x_{ij} = 1$ if the ith due date is assigned to job j, and $x_{ij} = 0$ otherwise
st_{jr}	The start-time of job j on trade r
Inferred Variables	
U_{kw}	The number of aircraft of type k whose repair due date is st_w
E_{kw}	The expected number of available aircraft of type k for wave w
et_{jr}	The end-time of job j on trade r

Table 4.1: Summary of notation; the decision variables and inferred variables for the MIP model.

The MIP model is shown in Figure 4.3 where Z_{kw}, the number of aircraft of type k that is assigned to fly in wave w, is a true decision variable: we can choose to send fewer aircraft on a wave than are currently (in expectation) available. In contrast, E_{kw} is the expected number of aircraft of type k available for wave w and is based on the probabilistic outcomes of previous waves and the number of newly repaired aircraft (U_{kw}). We refer to this model as *MIP* and rely on the default branch-and-bound search in the IBM ILOG CPLEX 12.3 solver, a state-of-the-art commercial MIP solver to solve it.

$$\text{Maximize} \sum_{w=1}^{|W|} \sum_{k=1}^{|K|} Z_{kw} \tag{4.1}$$

Subject to:

$$U_{kw} = \sum_{j \in I_k,\ i=w} x_{ij}, \qquad \forall k,\ \forall w \tag{4.2}$$

$$E_{k1} = (\eta_k + U_{k1})(1 - \xi_k^{pre}), \qquad \forall k \tag{4.3}$$

$$E_{kw} = (E_{k(w-1)} - Z_{k(w-1)} + U_{kw})(1 - \xi_k^{pre}) + \sum_{v \in \mathcal{V}_w} Z_{kv}(1 - \xi_k^{post})(1 - \xi_k^{pre}), \qquad \forall w (w \neq 1),\ \forall k \tag{4.4}$$

$$Z_{kw} \leq a_{kw}, \qquad \forall k,\ \forall w \tag{4.5}$$

$$Z_{kw} \leq E_{kw}, \qquad \forall k,\ \forall w \tag{4.6}$$

$$\sum_{i=1}^{|W|+1} x_{ij} = 1, \qquad \forall j \tag{4.7}$$

$$st_{jr} + p_{jr} = et_{jr}, \qquad \forall j,\ \forall r \tag{4.8}$$

$$et_{jr} \leq \sum_{i=1}^{|W|+1} x_{ij} d_i, \qquad \forall j,\ \forall r \tag{4.9}$$

$$\sum_{j \in M_r} c_{jr}((t \geq st_{jr}) \wedge (t < et_{jr})) \leq C_r, \qquad \forall t (t \leq st_{|W|}),\ \forall r \tag{4.10}$$

$$x_{ij} \in \{0, 1\}, \qquad \forall i,\ \forall j \tag{4.11}$$

$$0 \leq E_{kw} \leq |N|, \qquad \forall k,\ \forall w \tag{4.12}$$

$$st_{jr}, et_{jr} \in \mathbb{Z}^+ \cup \{0\}, \qquad \forall j,\ \forall r \tag{4.13}$$

$$Z_{kw} \in \mathbb{Z}^+ \cup \{0\}, Z_{kw} \leq |N|, \qquad \forall k,\ \forall w \tag{4.14}$$

Figure 4.3: The global MIP model for the static repair shop scheduling problem.

The details of MIP model are summarized as follows:

- The objective function (4.1) maximizes the number of aircraft assigned to waves. Although we have modeled the uncertain outcome of the flight checks as expectation, the objective function is not the expected wave coverage because (i) each wave has specific upper bounds on plane requirements and (ii) the maximum wave coverage for each wave is 1. If the expected number of available aircraft, E_{kw}, is more than the requirement, a_{kw}, for a given wave, the extra aircraft do not fly the wave and so do not contribute to the coverage. By not flying "extra" planes, we do not decrease the probability that they will be available for the next wave.

- Equation (4.2) calculates the number of aircraft of type k whose repair due date is st_w. In the other words, summing the decision variables x_{ij} where job j is an aircraft of type k and where the i-th due date corresponds to the start-time of wave w gives the number of aircraft type k leaving the repair shop right before the pre-flight check of wave w.

- Equation (4.3) calculates the expected number of available aircraft of type k for the first wave.
- Equation (4.4) calculates the expected number of available aircraft of type k for the other waves. The first term includes those aircraft available but not used for the previous wave, i.e., $(E_{k(w-1)} - Z_{k(w-1)})$, and those newly arrived from the repair shop, i.e., U_{kw}. The second term sums over all aircraft that become available because they have completed waves since the previous wave started where $\mathcal{V}_w = \{v | v \in W, st_{w-1} < et_v \leq st_w\}$.
- Constraints (4.5) and (4.6) ensure that the number of aircraft that is assigned to fly in each wave is less than or equal to the number of aircraft required and the expected number available.
- Constraint (4.7) assigns exactly one due date to each job.
- Equation (4.8) calculates the end-time of the jobs.
- Constraint (4.9) guarantees that the end-time of each job is less than or equal to its assigned due date.
- Constraint (4.10) is a logical-and constraint enforcing the capacity limit of trade r by summing over the capacity required by the set of jobs under repair at time t. Since the jobs after the start-time of the last wave do not contribute to the coverage, the capacity constraint is enforced only until the start-time of the last wave, i.e., $st_{|W|}$. The logical "$\wedge$" constraint evaluates to 1 if and only if its two component constraints both evaluate to 1 and to 0 otherwise. A logical inequality evaluates to 1 if and only if it is true and 0 otherwise. For example, if job j is under repair at time t, both logical inequalities, $(st_{jr} \leq t)$ and $(t < et_{jr})$ evaluate to 1 and the logical-and constraint, therefore, evaluates to 1. To linearize this constraint, we rely on the default approaches in IBM ILOG CPLEX for handling logical constraints. These approaches translate the logical constraints into their equivalent linear counterparts by creation of new variables and constraints (CPLEX, 2011).
- Constraints (4.11) to (4.14) define the domains of the decision variables.

4.3.1.2 Constraint Programming

To formulate the problem using CP, we use the same decision variables as in Table 4.1. However, instead of x_{ij}, we define $\mathcal{D}_j$ corresponding to the assigned due date for job j. The CP model differs from MIP in several constraints defined below.

The global cardinality constraint (gcc) has the syntax of gcc($card, value, base$) where $card$, $value$, and $base$ are arrays of variables, values, and variables, respectively. The gcc constraint is satisfied if $value[i]$ is taken by $card[i]$ elements of $base$. In our CP model, for each aircraft type k, Constraint (4.15) enforces that U_{kw} counts the number of times that the start-time of wave w is assigned as a due date to the jobs associated with a failed aircraft of type k.

The cumulative constraint ensures that the total amount of resource capacity used at any time on machine r does not exceed its total capacity, C_r.

Maximize Objective (4.1)
Subject to:
Constraints (4.3) to (4.6), (4.12), (4.14)

$$\text{gcc}([U_{k1}, U_{k2}, ..., U_{k|W|}], [st_1, st_2, ..., st_{|W|}], [\mathcal{D}_{j\in I_k}]), \qquad \forall k \qquad (4.15)$$

$$\text{cumulative}([st_{jr}|j \in M_r], [p_{jr}|j \in M_r], [c_{jr}|j \in M_r], C_r), \qquad \forall r \qquad (4.16)$$

$$0 \leq st_{jr} \leq \mathcal{D}_j - p_{jr}, \qquad \forall j, \forall r \qquad (4.17)$$

$$\mathcal{D}_j \in \{st_1, st_2, ..., st_{|W|}, B\}, \qquad \forall j \qquad (4.18)$$

Figure 4.4: The CP model for the static repair shop scheduling problem.

Constraint (4.17) enforces the time windows: job j on trade r cannot be started later than $(\mathcal{D}_j - p_{jr})$. Constraint (4.18) defines the domain of the decision variables $\mathcal{D}_j$.

We implement this model using IBM ILOG CP Optimizer 12.3 where the default search is used. The start-time variables, st_{jr}, are defined by IloIntervalVar objects. To implement the global constraints, we use IloDistribute class for the gcc constraint and IloPulse and IloAlwaysIn functions for the cumulative constraint. Note that, the cumulative constraint is implemented for any time point t until $st_{|W|}$.

4.3.1.3 Logic-based Benders Decomposition

As the static problem requires making two different decisions, assigning aircraft to the waves and scheduling repair jobs for failed aircraft, a decomposition approach may be well suited. A logic-based Benders decomposition (LBBD) method can be formulated where the master problem assigns aircraft to waves to maximize wave coverage and the sub-problems create the repair schedules given the due dates derived from the master problem solution. We propose four variations: *Benders-MIP* and *Benders-MIP-T*, where the master problems are solved using MIP, the latter with a tighter sub-problem relaxation ("T" stands for tighter); and *Benders-CP* and *Benders-CP-T* with a constraint programming-based master problem. All models use CP for the scheduling sub-problems.

The Due Date Assignment Master Problem (DAMP): MIP Model To formulate the master problem as a MIP model, we use a binary variable x_{ij} for job j and the i-th due date with the same meaning as in the global MIP model. A MIP formulation of DAMP is as follows:

Maximize Objective (4.1)
Subject to: (4.19)
Constraints (4.2) to (4.7), (4.11), (4.12), (4.14)

$$\sum_{j\in M_r} c_{jr}p_{jr} \leq C_r \max_{j\in M_r}\Big(\sum_{i=1}^{|W|+1} x_{ij}d_i\Big), \qquad \forall r \qquad (4.20)$$

MIP cuts (4.21)

The master problem incorporates a number of the constraints in the global MIP model. It does not represent the start-times of jobs nor does it fully represent the capacity of the trades. As is common in Benders decomposition, the master problem includes a relaxation of the sub-problems (Constraints 4.20) and Benders cuts (Constraints 4.21).

The Sub-problem Relaxation Defining the area of job j as the area of a rectangle with height c_{jr} and width p_{jr}, Constraint (4.20) is the relaxation of the capacity of a trade, expressing a limit on the area of jobs that can be executed. The limit is defined using the area bounded by the capacity of the trade and the time interval $[0, M]$ where M is the maximum due date assigned to the jobs on the trade. This relaxation is due to Hooker (2005; 2007).

We tighten the relaxation of sub-problems in the Benders-MIP-T approach by enforcing an analogous limit on multiple intervals: $[0, st_w]$ for each wave w. For each interval, the sum of the areas of the jobs whose assigned due date is less than or equal to the end-time of the interval must be less than or equal to the available area. This relaxation is a special case of the interval relaxation due to Hooker (2005; 2007). Formally, the tighter relaxation replaces Constraint (4.20) with:

$$\sum_{j \in M_r} c_{jr} p_{jr} ((\sum_{i=1}^{|W|+1} x_{ij} d_i) \leq st_w) \leq st_w C_r, \quad \forall r, \ \forall w \tag{4.22}$$

where $((\sum_{i=1}^{|W|+1} x_{ij} d_i) \leq st_w)$ is a logical inequality evaluating to 1 if and only if the assigned due date to job j is less than or equal to st_w.

The Benders Cuts Before defining the cut formally, we demonstrate the intuition with an example. Consider a due date set, $D = \{14, 17, 20, 35\}$, and, for a given trade with five jobs, the current master solution: $x_{21} = 1, x_{12} = 1, x_{43} = 1, x_{14} = 1,$ and $x_{15} = 1$. Job 1 is assigned to the second due date, 17, job 2 has the first due date, 14, and so on. If the current solution is infeasible due to the resource capacity of the trade, then we know that at least one of the jobs must have a later due date than it has in the current master solution. We can, therefore, constrain the sum of the consecutive x_{ij} up to and including the ones currently assigned to 1 to be one less than the number of jobs. In our example, the cut would be:

$$(x_{11} + x_{21}) + (x_{12}) +$$
$$(x_{13} + x_{23} + x_{33} + x_{43}) + (x_{14}) + (x_{15}) \leq 5 - 1$$

These variables represent the possible due dates less than or equal to those currently assigned for all jobs. By constraining these variables to be at most one less than the number of jobs, at least one job must be assigned a later due date.

Formally, assume that in iteration h, the solution of the DAMP assigns a set, Q, of due dates to the jobs on trade r. Assume further that there is no feasible solution on trade r with the assignments in Q. The cut after iteration h is:

$$\sum_{j \in M_r} \sum_{i \in I_{jr}^h} x_{ij} \leq |M_r| - 1, \quad \forall r \tag{4.23}$$

where for each job j on trade r, $I^h_{jr} = \{i' | i' \leq i, \text{ and } x^h_{ij} = 1\}$ is the set of due date indices less than or equal to the due date index assigned to job j in iteration h and $|M_r|$ is the number of jobs on trade r. The validity of this cut is proved in Section 4.3.1.6.

The Due Date Assignment Master Problem: CP Model We also formulate the DAMP using CP. Let $\mathcal{D}_j$ be the variable corresponding to the due date for job j similar to the global CP model.

Maximize Objective (4.1)

Subject to:

Constraints (4.3) to (4.6), (4.12), (4.14), (4.15), (4.18)

$$\sum_{j \in M_r} c_{jr} p_{jr} \leq C_r \max_{j \in M_r}(\mathcal{D}_j), \qquad \forall r \tag{4.24}$$

CP cuts (4.25)

The master problem modeled using CP includes several constraints of the global CP model. Constraint (4.24) represents the relaxation of repair capacity limit of the trades guaranteeing that the sum of processing areas for the set of jobs on the same trade does not exceed the maximum available area.

A tighter relaxation in a CP-based DAMP replaces (4.24) with the following inequality defining the Benders-CP-T approach where the logical inequality ($\mathcal{D}_j \leq st_w$) evaluates to 1 if and only if the due date of job j, $\mathcal{D}_j$, is less than or equal to the start-time of wave w, st_w.

$$\sum_{j \in M_r} c_{jr} p_{jr} (\mathcal{D}_j \leq st_w) \leq st_w C_r, \quad \forall r, \ \forall w$$

The CP cut is based on the same reasoning as the MIP cuts. If the assigned set of due dates to the jobs on trade r is not a feasible solution for the SP, the cut will guarantee that in the next iteration at least one of the assigned due dates will have a greater value. Formally, the cut is:

$$\bigvee \mathcal{D}_j > \mathcal{D}^h_j, \quad \forall j \in M_r \tag{4.26}$$

where $\mathcal{D}^h_j$ is the due date assigned to job j in iteration h, $\bigvee$ represents the logical-or constraints and M_r is the set of jobs on trade r.

Repair Scheduling Sub-problem Given a set of due dates assigned to the jobs on a trade, the goal of the repair scheduling sub-problem (RSSP) is to assign start-times to the jobs to satisfy the due dates and the trade capacity. We use a CP formulation where the RSSP for each trade is modeled by the cumulative constraint.

$$\text{cumulative}([st_{jr} | j \in M_r], [p_{jr} | j \in M_r], [c_{jr} | j \in M_r], C_r), \qquad \forall r$$

$$0 \leq st_{jr} \leq \mathcal{D}^h_j - p_{jr}, \qquad \forall j, \ \forall r \tag{4.27}$$

Recall that $[st_{jr} | j \in M_r]$ is the tuple of the start-time variables of the jobs on trade r, $\mathcal{D}^h_j$ is the value assigned to the due date for job j in master problem in iteration h. The parameters p_{jr}, c_{jr}, C_r are as

defined in Table 4.1. Constraint (4.27) enforces the time windows similar to constraint (4.17). It is worth mentioning that in the RSSP, the due date of job j, $\mathcal{D}_j^h$, is a value; however, it is a decision variable, $\mathcal{D}_j$, in the global CP model.

Since CP approaches are shown to be significantly more efficient than MIP for simple scheduling problems with resource capacity constraints (Hooker and Ottosson, 2003; Hooker, 2005, 2007), we do not experiment with MIP formulations of the sub-problems.

To implement the master problems in Benders-MIP and Benders-MIP-T, we use IBM ILOG CPLEX 12.3 solver; while Benders-CP and Benders-CP-T master problems and the RSSP are implemented in IBM ILOG CP Optimizer 12.3. The details on the implementation of the global constraints are similar to Section 4.3.1.2.

4.3.1.4 A Dispatching Heuristic

Since the static problem is NP-hard, solving it to optimality may be prohibitively expensive. We therefore investigate a heuristic approach, inspired by the Apparent Tardiness Cost (ATC) heuristic, a composite dispatching rule that is typically applied to single machine scheduling problem with the sum of weighted tardiness objective (Pinedo, 2005). The heuristic computes a ranking index for each job and sorts the jobs in ascending order of the index. The heuristic then iterates through the jobs, scheduling each job at its earliest available time. The ranking index we use is as follows:

$$I_j = ST(k_j) \exp(-\frac{FN_j}{FC_j}), \quad \forall j$$

If we let k_j denote the type of aircraft j, then $ST(k_j)$ is the start-time of the first wave that requires an aircraft of type k_j. FN_j is the fraction of the total number of aircraft of type k_j required by the first wave that requires k_j, and FC_j is the maximum proportion of the capacity needed by job j over all its required trades, as follows.

$$FC_j = \max_r(\frac{p_{jr}c_{jr}}{ST(k_j)C_r})$$

Intuitively, the earlier the start-time of the first relevant wave, the higher proportion of aircraft required by that wave, and the lower the proportion of capacity required before the wave, then the sooner the job will be scheduled. The exponential function is used to place more weight on the start-time.

In preliminary experiments, three other dispatching heuristics were investigated, with the chosen heuristic performing best. The first two heuristics rank the jobs with slightly different ranking indices equal to $I_j = ST(k_j) \times \frac{\max_r(p_{jr})}{FN_j}$ and $I_j = ST(k_j) \times \frac{\max_r(p_{jr}c_{jr})}{FN_j}$, respectively. The third heuristic is a two-stage approach based on a decomposition. The first stage finds the number of each aircraft type assigned to each wave and the second stage schedules the jobs in increasing order of $\max_{j,r}(p_{jr}c_{jr})$ considering the values determined in the first stage as upper bounds on the number of jobs required before each wave. Our preliminary experiments demonstrated that our chosen dispatching rule results in on average of 6% higher wave coverage compared to the first two heuristics and in the same coverage as the third heuristic while having the advantage of being easy to understand and implement.

4.3.1.5 Hybrid Heuristic-Complete Approaches

A hybrid heuristic-complete approach in which the heuristic solution provides a lower bound for the maximization objective (Equation 4.1) may improve the performance of the complete approaches. Therefore, a simple hybrid first runs the dispatching heuristic and then uses the objective value as a starting lower bound for the complete approaches. Assume that the heuristic finds a solution, S, with $f(S)$ as the number of aircraft assigned to waves. Any of the complete approaches can now be modified by adding the following constraint:

$$\sum_{w=1}^{|W|} \sum_{k=1}^{|K|} Z_{kw} \geq f(S)$$

For LBBD variations, the above constraint is added to the master problem.

4.3.1.6 Theoretical Results

To guarantee the finite convergence of a LBBD model to a globally optimal solution, the Benders cuts must be valid and the master decision variables must have finite domains. A Benders cut is valid in a given iteration, h, if and only if (1) it excludes the current globally infeasible assignment in the master problem without (2) removing any globally optimal assignments (Chu and Xia, 2004). The former guarantees the finite convergence and the latter guarantees the optimality. As the decision variables in DAMP have a finite domain, it is sufficient to prove the satisfaction of the two conditions.

Theorem 4.2. *Cut* (4.23) *is valid.*

Proof. For condition (1), for the sub-problem in iteration h on trade r, by definition:

$$\sum_{j \in M_r} \sum_{i \in I_{jr}^h} x_{ij} = |M_r|$$

Consequently, cut (4.23) excludes the current assignment of master problem.

For condition (2), consider a global optimal solution S that does not satisfy cut (4.23) as generated in iteration h. As the cut states that at least one job must have a greater due date than it had in h, to violate the cut, all jobs in S must have equal or lesser due dates than they had in iteration h. However, because the sub-problem was infeasible in iteration h, any sub-problem with only equal or lesser due dates must also be infeasible as the available capacity on the trade is the same or less. Therefore, S must be infeasible and we contradict the assumption that S is globally optimal.

Therefore, the cut is valid. □

An analogous argument holds for cut (4.26).

4.3.2 Rescheduling Strategies

The dynamic repair shop problem over the long horizon can be viewed as static scheduling sub-problems over successive time periods. Let's assume that we start repairing the failed aircraft and assigning them

to the waves based on the computed schedule at time 0. A wave might start while a repair is under way in the repair shop. If some aircraft fails the pre-flight check, it goes to the repair shop. Each failed aircraft requires a set of independent repair activities with known processing times and resource requirements. At the repair shop, some of the previously failed aircraft might be already repaired, some might be under repair, and others might be awaiting repair. Once the failed aircraft enters the repair shop, we have a new static repair scheduling sub-problem where the set of existing jobs (J), the number of aircraft not in the repair shop for each aircraft type (η_k), and the failure rates of the aircraft (λ_n) are updated. The set of existing jobs includes the recently failed aircraft and the previously failed aircraft whose repairs are still under way or are not yet started. The new static sub-problem has an added constraint, namely that the repairs currently under way cannot be disrupted.

We connect the static sub-problems using three different policies denoted as P_{ij} where i and j define the length of scheduling horizon and the frequency of rescheduling in number of waves, respectively. In all three policies, we schedule the repair activities, observe the aircraft failures, and respond to failures by rescheduling the repair activities.

The three policies discussed here are as follows:

- P_{11}: In Figure 4.5, we show that P_{11} schedules one wave at a time ($i = 1$) and reschedules after each wave ($j = 1$). P_{11} is a myopic policy aiming at providing the next first wave with the highest possible coverage.

- P_{31}: In contrast to P_{11}, for P_{31} (Figure 4.5), the scheduling horizon is three waves but rescheduling is still done after each wave. P_{31} with a longer scheduling horizon than P_{11} trades-off the coverage among the next three waves. It is worth mentioning that we have chosen three as the length of the scheduling horizon because three waves are usually scheduled daily based on the real data (Safaei et al., 2011).

- P_{33}: This policy has a scheduling horizon with a length of three waves and reschedules after every third wave (Figure 4.5). P_{33} might trade-off the lower coverage of the next first wave for higher coverages of the second and the third waves; however, it has a lower frequency of rescheduling.

4.3.3 Modeling the Aircraft Failures

To model the dynamic events, we simulate the aircraft failures in pre- and post-flight checks. Every aircraft either passes or fails each check. If the aircraft fails, a new set of repair activities with known processing times and resource requirements is added to the repair shop. If the aircraft passes, it flies the wave if required. As mentioned in Section 4.1.1, after repair the failure rate of the aircraft returns to what it was before the failure and it increases by γ percent each time it flies a wave due to deterioration. If λ_n is the initial failure rate of the aircraft $n \in N$, its failure rate after flying w waves without failure will be $\lambda_n(1 + 0.01 \times \gamma)^w$.

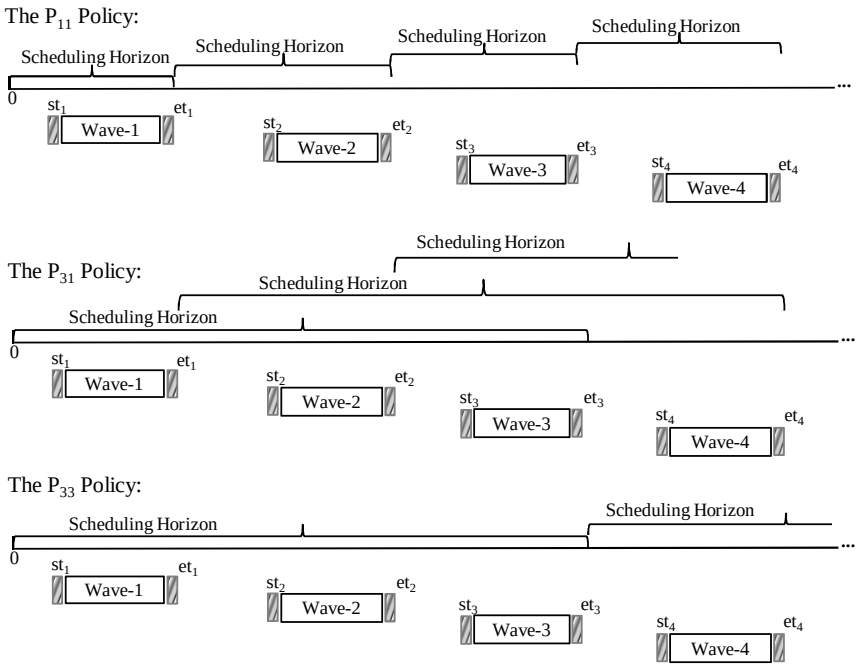


Figure 4.5: The rescheduling policies.

4.4 Computational Experiments

In this section, we present two separate empirical studies. The first study compares the scheduling techniques experimentally and presents insights into each algorithm's performance through a deeper analysis of the results. The second study investigates the impact of using different scheduling techniques and rescheduling policies on the observed wave coverage.

4.4.1 Experimental Results on Scheduling Techniques

This sub-section describes the experiment comparing different solution techniques for scheduling the static repair shop.

4.4.1.1 Experimental Setup

The problem instances have 10 to 30 aircraft (in steps of 1), 3 or 4 trades, and 3 or 4 waves. Five instances for each combination of parameters are generated, resulting in 420 instances (21 total aircraft counts by 2 trades counts by 2 waves counts by 5 instances).

Aircraft: The number of aircraft types is equal to $\frac{|N|}{5}$, where $|N|$ is the number of aircraft. The aircraft are randomly assigned to different types with uniform probability. The number of aircraft of type k is $|I_k|$. The failure rate for each aircraft is randomly chosen from the uniform distribution $[0, 0.5]$. The failure rate for aircraft of type k, $\bar{\lambda}_k$, is the mean failure rate over all aircraft of type k. The functions used to represent aircraft n probability of failures in pre- and post-flight checks, respectively, are $f^{pre}(\lambda_n) = (1 - e^{-\lambda_n})$ and $f^{post}(\lambda_n) = (1 - e^{-3\lambda_n})$. It is worth mentioning that the conditions of a reliability function

in the extreme values of failure rates hold true for the functions used. If the failure rate goes to 0, the probability of failure equals 0, and if the failure rate goes to ∞, the probability of failure equals 1.

Waves: The plane requirement of each aircraft type for each wave is randomly generated from the integer uniform distribution $[1, |I_k|]$. The length of each wave is drawn with uniform probability from $[3, 5]$. To make an instance loose enough to permit feasible solutions yet tight enough to be challenging, a lower bound on the length of the scheduling horizon (H) is needed. The sum of the processing areas of the jobs in each trade, r, divided by the trade capacity is denoted by S_r. $LB = \max_r(S_r)$ is a lower bound on the time required to schedule all jobs and we use $H = 1.2 \times LB$. The end-time for each wave, et_w, is generated as $et_{|W|} = H - rand[0, 3]$ for the final wave, $|W|$, and $et_w = st_{w+1} - rand[0, 3]$ for $w < |W|$.

Trades: The capacity limit for each trade is set at $C_r = 10$.

Repair Jobs: Eighty percent of the aircraft are in the repair shop at the beginning, resulting in $|J| = 0.8|N|$ repair jobs. The jobs are randomly assigned to the trades with replacement such that the number of jobs per trade is equal to $|J|/2$. Each job requires at least one trade and some require more than one trade. The capacity of trade r used by job j, c_{jr}, is drawn from $[1, 10]$ while the processing time, p_{jr}, is drawn from $[r, 10r]$: jobs on trades with lower indices have shorter processing times than those on trades with higher indices.

Though our problem instances are generated randomly, the setting of our experiment includes three numerical examples of Safaei et al. (2011) which are based on the real data. Furthermore, our setting consists of more instances and results in problem instances which are one and a half times bigger than the examples used in the literature (Safaei et al., 2011) where the number of aircraft is 10, 15, or 20; the number of waves is 3 or 4; and the number of trades and aircraft types are equal to 3 and 2.

All experiments were run with a 7200-second time limit on an AMD 270 CPU with 1 MB cache per core, 4 GB of main memory, running Red Hat Enterprise Linux 4.

4.4.1.2 Experimental Results

Figure 4.6 shows scatter-plots of run-times of the six complete approaches. Both axes are log-scale, and the points below the line $y = x$ indicate a lower run-time for the algorithm on the y-axis. The numbers in the boxes indicate the number of points below or above the line. Run-times are counted as equal if they differ less than 10%.

The graphs indicate a benefit for MIP over CP, for Benders-CP over MIP, for Benders-MIP over Benders-CP and MIP, for Benders-MIP-T over Benders-MIP, and for Benders-CP-T over Benders-CP. Table 4.2 presents further data, sorted in descending percentage of problems solved to optimality, for all algorithms.

The mean run-time of the MIP model given in Safaei et al. (2011) over eight scenarios with 10 aircraft and 3 waves is 294.75 seconds. However, our proposed MIP model has the run-time of 2.64 seconds on average over ten of the instances with the same number of aircraft and waves, indicating that it is significantly faster than Safaei et al.'s model.

MIP vs. CP The MIP approach has a clear superiority over the CP, achieving a lower run-time on

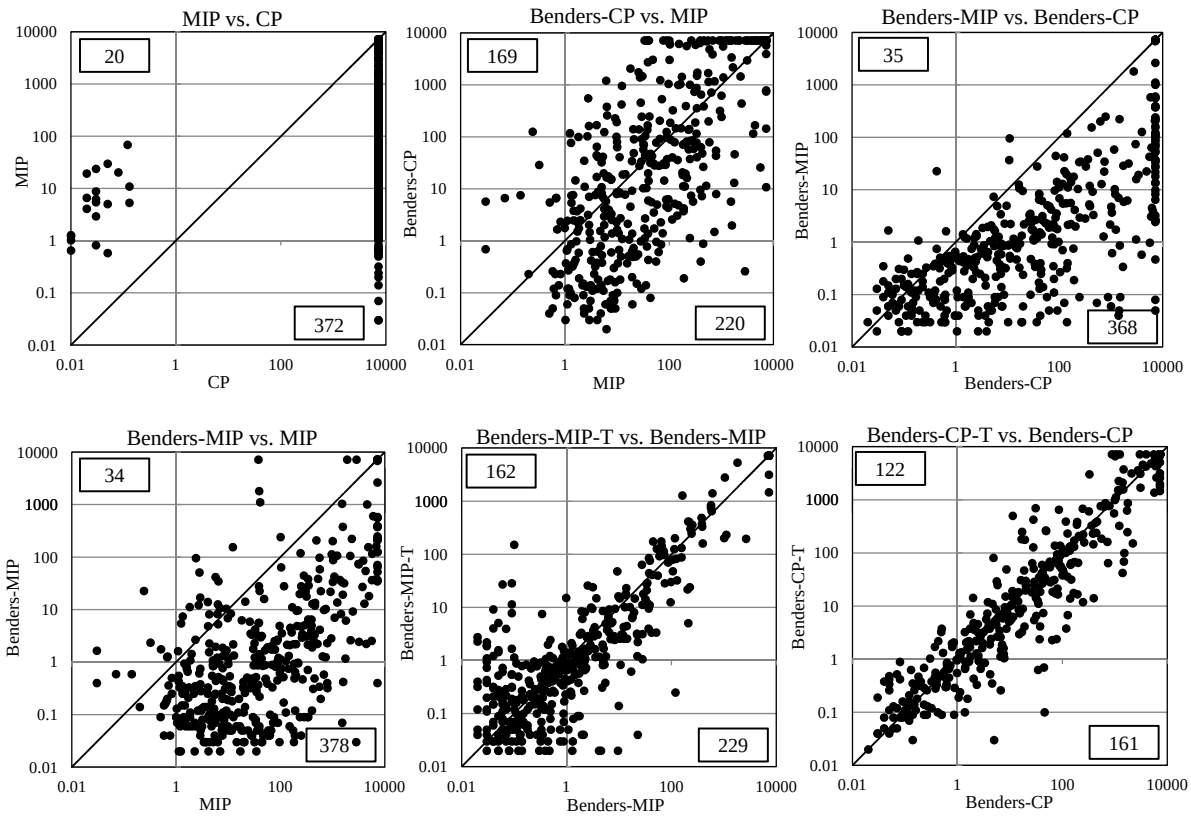


Figure 4.6: Run-times (seconds) of the six complete models.

89% of the problem instances. The CP model outperforms the MIP only on 5% of the instances where it can solve to optimality within the time limit. A further investigation of the results shows that the mean quality of CP solution is 0.27% from the best found solution across all the algorithms. Therefore, the poor performance of CP is due to its weakness in proving the optimality.

Benders-CP vs. MIP The Benders-CP approach does better than MIP in terms of run-time on 52% of the instances while performing worse on 40%. However, Table 4.2 favors MIP in terms of the overall performance, smaller mean run-time, mainly because more instances are solved to optimality within the time limit.

Benders-MIP vs. Benders-CP The Benders-MIP approach achieves a better run-time than Benders-CP on 88% of the test problems, performing worse on about 8%. The branching heuristics for Benders-CP often lead to an initial feasible master solution with tighter due dates than the initial master solution in Benders-MIP. The tighter, globally infeasible initial solution means that the CP-based master problem model requires more iterations to find a globally feasible solution.

Benders-MIP vs. MIP The Benders-MIP approach achieves a better run-time than MIP on 90% of the test problems and a worse run-time on 8%, achieving a lower mean run-time and solving a higher proportion of the problem instances. When the time horizon is short, the MIP approach is faster, however, with longer horizons and more jobs, the number of constraints and variables grows, substantially

Method	Mean Time (s)	Iter.	% MP	% SP	% Solved to optimality
Benders-MIP-T-Hybrid	211.98	66.63 (8.0)	51.39 (55.05)	48.61 (44.95)	98.10
Benders-MIP-T	213.12	66.44 (8.0)	51.84 (53.98)	48.16 (46.02)	97.86
Benders-MIP	227.94	64.66 (8.0)	61.75 (67.44)	38.25 (32.56)	97.62
MIP	837.04	-	-	-	93.57
MIP-Hybrid	924.30	-	-	-	91.19
Benders-CP	1373.16	75.72 (15.5)	84.30 (96.96)	15.70 (3.04)	85.24
Benders-CP-T	1356.70	66.42 (10.0)	85.36 (97.36)	14.64 (2.64)	85.00
Dispatching Rule	≈ 0	-	-	-	9.76
CP	6857.14	-	-	-	4.76

Table 4.2: The mean run-time, the mean (median) number of master problem iterations, the mean (median) percentage of run-time spent solving the master problem and the sub-problems, and the percentage of problems solved to optimality for all approaches.

reducing its performance.

Benders-MIP-T vs. Benders-MIP The tighter relaxation in Benders-MIP-T slightly speeds up LBBD: Benders-MIP-T has a better run-time than Benders-MIP on 55% of problems instances and worse on 39%. The mean run-time decreases by 6% and the tighter relaxation solves one more instance to optimality. Tightening the relaxation of sub-problems increases the mean number of iterations in spite of our expectation. A closer look to the results shows that the mean number of iterations of the instances solved to optimality by both approaches (97.38% of the instances) decreases: 39.15 and 41.24 for Benders-MIP-T and Benders-MIP, respectively. However, the mean number of iterations of the instances timed out by both approaches (1.9% of the instances) increases from 1051.38 for Benders-MIP to 1259.88 for Benders-MIP-T. Therefore, the increase in the number of iterations results from the timed-out instances which does not support that the tighter relaxation requires more iterations to optimality.

Furthermore, the percentage of time solving the master problem decreases compared to Benders-MIP, while the sub-problem percentage run-time increases. This latter observation is because the sub-problems for Benders-MIP that can be quickly proved insoluble by the initial propagation of CP sub-problem model, violate the tighter relaxation in the Benders-MIP-T master problem. Therefore, in the tighter model, the sub-problem solver is not called on these "easy" sub-problems, increasing the percentage of run-time spent on sub-problems.

Benders-CP-T vs. Benders-CP The tighter relaxation in CP-based master problem results in a slightly lower mean run-time; however, as shown in Figure 4.6, their performance comparison is more even.

Incomplete and Hybrid Approaches The dispatching heuristic is fast, finding a feasible solution to all problems. However, it finds (but, of course, does not prove) an optimal solution in only 9.76% of the instances and Benders-MIP-T finds and proves optimality for these instances in 0.99 seconds on average. It seems that the heuristic can find the optimal solution only when the problem instance is relatively easy. The mean quality of the heuristic solution is 16% from optimal. In industries with expensive assets, such a reduction in solution quality can translate to costly under-use of a valuable resource (e.g., a fighter aircraft costs in the vicinity of 100 million dollars). However, finding a feasible

solution almost instantaneously for large problem instances makes the heuristic approach compelling in situations where the long run-time might delay carrying out the waves. For example, if a wave starts within a very short time, flying the wave with lower coverage (achieved by the dispatching heuristic) is better than not carrying it out because of the long solving time of the complete approaches.

To evaluate the effect of combining the dispatching heuristic with the complete approaches, we examine using the hybrid heuristic-complete approach. A smaller feasible set is the direct consequence of defining a bound on the cost function. As the MIP model searches the feasible set, while LBBD methods explore the infeasible space, one intuition is that the MIP model should benefit more from using the heuristic solution. However, solving the master problem in LBBD requires searching in a relaxed feasible space and therefore the heuristic starting solution may also speed solving.

Table 4.2 shows a very marginal benefit for bounding the Benders-MIP-T approaches with the dispatching heuristic solution. Bootstrap paired-t tests (Cohen, 1995) also indicate that there is no significant difference in mean run-time at $p \leq 0.01$ for either hybrid.

Scalability Figure 4.7 shows our results as the number of aircraft per wave increases. We aggregate results by truncating $\frac{|N|}{|W|}$ and using the instances with three waves and both three and four trades. Note that each point represents 30 problem instances except $x = 3$ which has only 20 problems instances. We omitted $x = 10$ as we only have 10 problem instances for that point. The y-axis is log-scale.

The results show that the LBBD variations outperform the other techniques across all ratios.

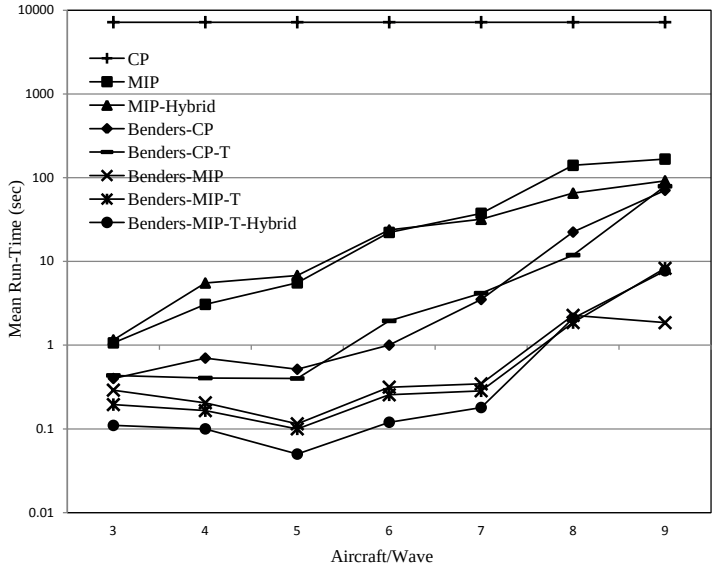


Figure 4.7: Mean run-time vs. number of aircraft per wave ($|W| = 3$).

Summary The following observations on the performance of scheduling techniques are supported by this empirical study.

- The LBBD approach combining mixed integer programming and constraint programming outperforms the mixed integer programming model. The mean run-time of Benders-MIP is almost 4 times lower than MIP. Furthermore, defining the time ratio for a given instance as the MIP run-

time divided by Benders-MIP run-time, the Benders-MIP is almost 32 times faster than MIP, on average, with a geometric mean time ratio of 31.56. The time ratios range is $[0.01, 94132.67]$ with a median of 38.32.

- A tighter relaxation slightly speeds up LBBD. Benders-MIP-T and Benders-CP-T, both, have a run-time about 1.2 times faster than Benders-MIP and Benders-CP, respectively.
- A dispatching heuristic can provide the optimal solution for the easy problem instances. However, the mean percent relative error of heuristic is almost 16% overall, indicating that the dispatching rule by itself is not effective enough for industries with high equipment cost.
- The simple hybridization of the complete approaches with the dispatching heuristic does not result in a statistically significant difference in run-time.

4.4.2 Experimental Results on Rescheduling Strategies

This sub-section describes experiment investigating the impact of applying different scheduling techniques and rescheduling policies in a dynamic repair shop.

4.4.2.1 Experimental Setup

For our problem instances, the number of aircraft, the number of trades, and the total number of waves are set to $\{10, 15, 20, 25, 30\}$, $\{4\}$, and $\{30\}$ respectively. Each combination has 5 instances for a total of 25 instances. Each instance is simulated 20 times. The parameters of the problem instances are generated as in Section 4.4.1 with the following modifications:

Aircraft: The failure rate of an aircraft is increased by $\gamma = 5$ percent each time it is used.

Repair Jobs: Repair jobs that entered repair shop after time 0 are randomly assigned to the trades. The probability of assigning a job to each trade is considered as 0.5.

Waves: The start-time of each wave is generated as $st_1 = rand[\frac{H}{3}, \frac{H}{2}]$ for the first wave, and $st_w = et_{w-1} + rand(0, 40)$ for $1 < w \leq 30$. As mentioned earlier the total number of waves is 30. The value of H is calculated as in Section 4.4.1.

Dynamic events: To simulate an aircraft failure, we generate a random value from the uniform distribution $[0, 1]$ for each aircraft at each check. If the random value is less than the aircraft's probability of failure, the aircraft fails; otherwise, it passes. The aircraft's probability of failure in pre- and post-flight checks are calculated using $(1 - e^{-\lambda_n})$ and $(1 - e^{-3\lambda_n})$, respectively. Recall that, λ_n is the failure rate of aircraft, $n \in N$, which increases by $\gamma = 5$ percent each time the aircraft flies a wave. Note that passing the pre-flight check of a wave does not necessarily mean that the aircraft flies the wave. If the number of available aircraft is more than the requirements, the aircraft that fly are randomly selected from those that passed the pre-flight check to meet the requirements. Our latter assumption implies that all aircraft ready at the beginning of a wave are checked regardless of the wave requirements. Since

we have assumed that the pre- and post-flight checks have negligible cost, our assumption is reasonable to discover the potential aircraft failures sooner which is likely to increase their availability for the subsequent waves.

We experiment with three techniques including MIP, Benders-MIP-T, and the dispatching rule discussed in Section 4.3.1. The time-limit to schedule the repair activities at each decision time point is 600 seconds. We execute the best feasible schedule found before the time-limit if an algorithm times out. In the case that Benders-MIP-T times out, the schedule created by the dispatching heuristic is executed as Benders-MIP-T does not create a feasible schedule when it times-out.

As in the static problem, the scheduling uses IBM ILOG CPLEX 12.3 and IBM ILOG CP Optimizer 12.3. The simulation is implemented in C++.

4.4.2.2 Experimental Results

In this section, we discuss our results to compare the performance of different scheduling and rescheduling techniques on the availability of the aircraft in the long run. We further investigate the effect of modeling the aircraft failures using the expected coverage.

Figures 4.8, 4.9, and 4.10 illustrate the mean observed coverage up to flight $w \in \{1, 2, ..., 28\}$ for different scheduling and rescheduling techniques. Denoting ν_{wpl} as the coverage of flight w in the l-th simulation of instance p for a given policy, $O_{wpl} = \frac{\sum_{i=1}^{w} \nu_{ipl}}{w}$ represents the mean observed coverage up to flight w in instance p and in simulation l. The mean observed coverage up to flight w, shown in the figures, is calculated as $O_w = \frac{\sum_{p=1}^{P} \sum_{l=1}^{L} O_{wpl}}{PL}$, where P and L are the number of instances and simulations, respectively. Table 4.3 shows the mean observed coverage up to flight 28, i.e., O_{28} and the variance of the observed coverage up to flight 28 for all scheduling techniques and rescheduling policies. As illustrated, Benders-MIP-T using P_{31} achieves at least a 10% higher mean coverage than any other combinations of the scheduling and rescheduling techniques and has the lowest variance.

Method	O_{28}[var(.)]			ρ(.)		
	P_{11}	P_{31}	P_{33}	P_{11}	P_{31}	P_{33}
Benders-MIP-T	0.67 [0.03]	0.77 [0.01]	0.70 [0.01]	45	56	48
MIP	0.52 [0.03]	0.64 [0.03]	0.60 [0.02]	34	46	38
Dispatching heuristic	0.61 [0.04]	0.61 [0.04]	0.63 [0.03]	42	44	47

Table 4.3: The mean (variance) of observed coverage up to flight 28 (O_{28}[var(.)]) and the mean percentage of available aircraft for the first flight (ρ).

The impact of the scheduling algorithms A complete technique is anticipated to achieve higher flight coverage because it takes the expected probabilistic information into account when creating a repair schedule, while the dispatching heuristic does not have this property. As shown in Table 4.3, Benders-MIP-T as a complete technique results in higher mean observed coverage over all policies when compared to the dispatching heuristic. However, MIP, incorporating the mean of known information on uncertainty into scheduling the repair activities, results in flights with lower coverage than the dispatching heuristic over two of the rescheduling policies, P_{11} and P_{33}. To understand the MIP performance, we make two conjectures.

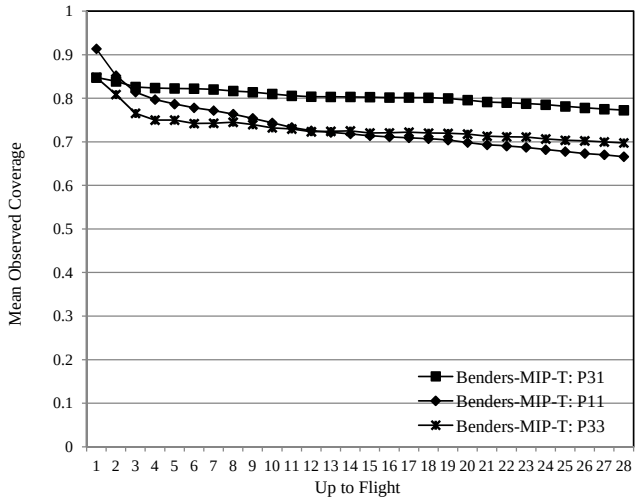


Figure 4.8: Mean observed coverage for three different policies using Benders-MIP-T.

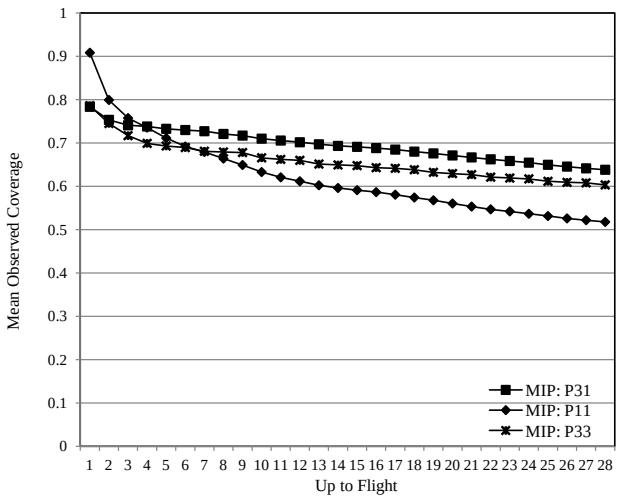


Figure 4.9: Mean observed coverage for three different policies using MIP.

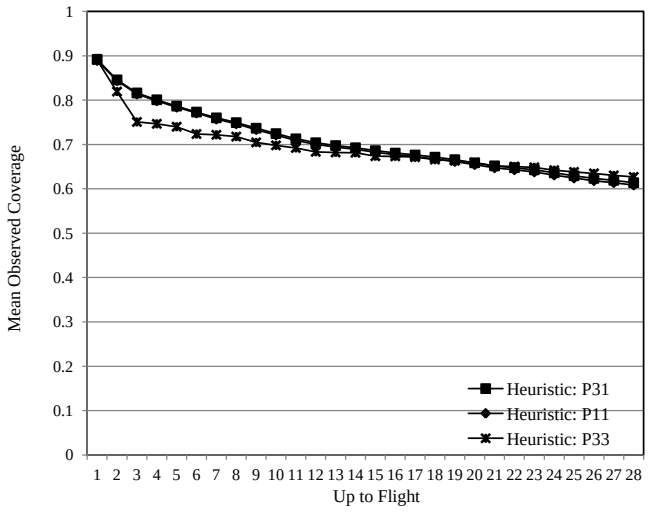


Figure 4.10: Mean observed coverage for three different policies using the dispatching heuristic.

Our first conjecture is that the poor performance of MIP algorithm is because it frequently times out on most of the static sub-problems and the best found feasible solution or the dispatching heuristic is then used to create the repair schedule. However, our results do not support the conjecture. MIP algorithm only times out on 13% of the scheduling sub-problems and a feasible solution is found in each, implying that the dispatching heuristic is never used to find a repair schedule.

Our second conjecture is that the low coverage achieved by MIP can be attributed to its different way of scheduling the repair activities compared to the other two scheduling techniques. A deeper look into the schedules of the static sub-problems shows that the dispatching heuristic and Benders-MIP-T schedule the repair activities at the earliest possible time; however, MIP usually does not. Repairing the aircraft earlier makes more aircraft available which intuitively increases the coverage in the long run, even though the number of pre-flight checks that aircraft go through increases in expectation. The quick adjustment of the schedule makes some of the failed planes again available before the start-time of the next flight. To investigate the impact of making the aircraft available earlier using a given scheduling technique, we define the mean percentage of available aircraft for the first flight as $\rho(P_{ij}) = \frac{\sum_{p=1}^{P}\sum_{l=1}^{L}\sum_{k=1}^{S}\rho_{kpl}}{PLS}$ where ρ_{kpl} and S denote the percentage of aircraft available at the beginning of the first flight of the k-th static sub-problem in the l-th simulation of instance p and the number of static sub-problems in P_{ij} policy, respectively. For example, in P_{31} policy for a given p and l, the first static sub-problem includes flights 1, 2, and 3. Then, ρ_{1pl} is the number of aircraft available for flight 1 divided by the total number of aircraft. The second static sub-problem schedules for flights 2, 3, and 4. Therefore, ρ_{2pl} is equal to the number of aircraft available for flight 2 divided by the total number of aircraft. We follow the same procedure to find ρ_{kpl} for all 28 static sub-problems in P_{31} policy. We can find $\rho(P_{11})$ and $\rho(P_{33})$ using the same argument considering that the number of static sub-problems is 30 and 10, respectively.

Comparing any pair of the scheduling and the rescheduling techniques in Table 4.3, there is a positive relationship between making the aircraft available earlier and the wave coverage in the long term which supports our conjecture: if the mean percentage of available aircraft for the first flight (ρ) increases, the mean observed coverage in the long rum (O_{28}) also increases.

The impact of rescheduling policies As illustrated in Figures 4.8 and 4.9, the P_{11} policy with either Benders-MIP-T or MIP in the short term (i.e., for the first three flights) outperforms the other two policies. However, the P_{31} policy then leads to consistently higher coverage because it schedules over a longer horizon and adjusts the schedule as soon as aircraft failures occur. Although P_{31} with the dispatching heuristic also responds quickly to the aircraft failures, it does not incorporate the length of the scheduling horizon into the ranking index for repair activities and always repairs the aircraft for the earliest possible time, resulting in flights with the same coverage as P_{11}.

Figure 4.11 displays the cumulative percentage of the flights with a coverage less than or equal to ω for Benders-MIP-T, the dispatching heuristic and MIP where ω denotes the values on the x-axis. The best performing approach will have fewer flights with a low coverage and more flights with a high coverage. Therefore, its curve will be closer to the lower right-hand corner. As illustrated, Benders-MIP-T using P_{31} performs better than any other combination. The P_{31} rescheduling policy is computationally

more expensive than the other two policies, its run-time per one static sub-problem, however, is small compared to the length of scheduling horizon being usually one day in the real applications (Safaei et al., 2011). The P_{31} policy using Benders-MIP-T has a run-time of on average 67 seconds per one static sub-problem and of less than 249 seconds on 90% of static sub-problems.

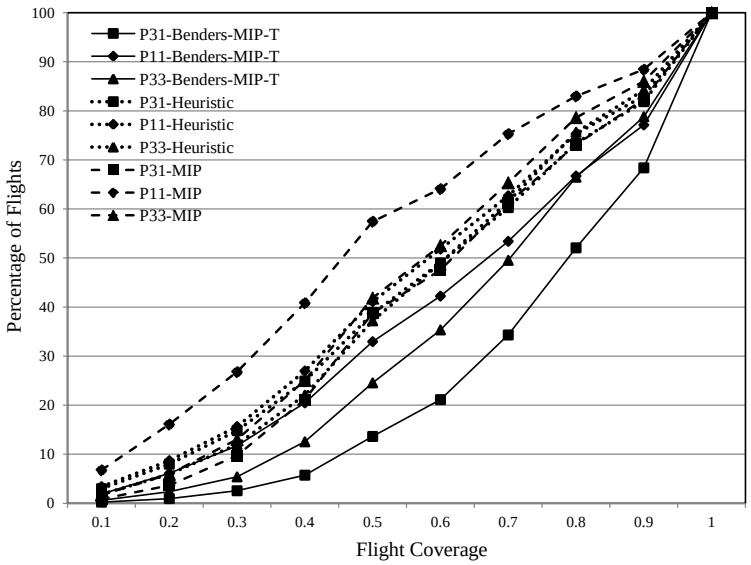


Figure 4.11: The percentage of flights with a coverage less than or equal to ω, where ω denotes the values on the x-axis.

In summary, our analysis of the results identifies the Benders-MIP-T with P_{31} (Benders-MIP-T:P_{31}) as the best combination of the scheduling and the rescheduling techniques providing the flights with a higher mean coverage over the long term. Furthermore, it has the lowest variance for the observed coverage compared to the other scheduling and rescheduling techniques.

The impact of modeling the uncertainty in expectation Because of the random aircraft failures, the coverage achieved by any scheduling algorithm is a random variable. The ultimate goal is to construct a repair schedule which is optimal for the specific realization of the uncertainty that actually occurs. However, since the complete information on the aircraft failures is not known and the future uncertainty is dependent on the previous repair decisions, it is impossible to find a repair schedule which is ideal under any realization of uncertainty. As discussed earlier, we have modeled the aircraft failures using the expected value to find the optimal repair schedule. Since treating the uncertainty in the expectation form may be far from optimal for the actual realization of uncertainty, we perform a sensitivity analysis on the failure rates of the aircraft to investigate how the optimal repair schedule by Benders-MIP-T:P_{31} is hedged against various uncertain situations.

Using the same problem instances as in Section 4.4.2.1, two other experiments are set up where the failure rate of each aircraft (λ_n) is increased to $\lambda_n + 0.05$ and $\lambda_n + 0.1$. Our results show that while the mean observed coverage up to flight 28 decreases to 0.69 and 0.62, the variance of observed coverage does not change, indicating that modeling the uncertainty using the expected probabilistic information is a reasonable approach.

To find a possible upper bound (tighter than 1) on the mean observed coverage up to flight w under any scheduling algorithm, we define a policy called "Relaxed" which relaxes the repair capacity limit and repairs any failed aircraft after its maximum processing times over all trades. Although the Relaxed policy makes the aircraft available for the waves earlier than any other repair scheduling policy, we cannot guarantee that it results in the upper bound on the observed coverage unless all waves have the same requirements. The optimal decision might be to trade off the immediate low coverage for future higher coverage when the plane requirements for the waves are different. Applying the Relaxed scheduling policy on the same instances as in Section 4.4.2.1, the mean observed coverage up to flight 28, i.e. O_w, is 24% higher than the best identified algorithm, Benders-MIP-T:P_{31}. More specifically, the Relaxed policy results in a mean coverage of 0.95 with the variance of 0.002.

The impact of longer scheduling horizon vs. more frequent rescheduling The P_{31} policy changes the repair schedule after each flight and trades-off the coverage among three consecutive flights by scheduling over a longer horizon. In contrast, the P_{11} policy schedules for one flight and reacts after each flight while the P_{33} policy reasons over a longer term without a quick response to the dynamic events. As already shown in Figures 4.8 and 4.9, the P_{31} policy with any of the complete techniques results in a higher mean coverage. The P_{11} policy outperforms the P_{33} for early waves, but the P_{33} provides the later waves with higher coverage.

The superiority of policy P_{31} indicates that both features of quick response to the dynamic events and long-term reasoning contribute to the overall performance. The contribution of each feature is significantly dependent on several parameters such as the aircraft failure rates, the plane requirements, and the repair capacity. When the failure rate is high, the probability of aircraft being diagnosed as failed in pre- and post- flight checks is higher. Therefore, the arrival rate of the aircraft to the repair shop is higher and the previously constructed schedule is more likely not to be executed as is. In such a system, frequent rescheduling is more likely to increase the coverage. When the plane requirements of the waves are widely varying and the repair capacity limit is tight, trading-off the coverage among the flights through scheduling over a longer horizon significantly contributes to the availability of the aircraft in the long term.

Summary The following observations on how and when the scheduling and rescheduling should be done are supported by the second empirical study:

- Solving the dynamic repair shop problem using the Benders-MIP-T scheduling technique and the P_{31} rescheduling policy results into an observed coverage with higher mean and lower variance than any other combination tested.

- There is a positive relationship between making the failed aircraft available as early as possible and achieving a higher coverage in the long term.

- Since the variance of the P_{31} policy with the Benders-MIP-T is not sensitive to the small changes in the aircraft failures, modeling the uncertainty with respect to the mean is a reasonable approach to balance against different uncertain scenarios.

4.5 Discussion

The experimental results demonstrate that incorporating both probabilistic and execution time reasoning into the schedule of repair activities results in a better system performance. We showed that the decomposition technique, LBBD, and the rescheduling policy P_{31} result in a 10% higher mean observed coverage in the long term, increasing the utilization of the valuable resources. The decomposition technique considers the mean of known probabilistic information about uncertainty over a longer scheduling horizon and repairs the failed aircraft at the earliest time. The P_{31} rescheduling policy takes advantage of up-to-date information more frequently. It is also shown that the variance of the coverage does not increase as the aircraft failures increase, supporting the core idea of our solution approach: the dynamic repair shop problem is viewed as a collection of static sub-problems where the uncertainty on aircraft failures is treated as expectation.

Optimizing with respect to the mean and considering a specific class of scheduling problems are limitations of our solution approach. We address each in detail below and discuss ideas to deal with them.

Modeling the uncertainty Optimizing with respect to the expected coverage can have unfavorable consequences: the constructed repair schedule may have a remarkably poor performance for particular realizations of uncertainty that might happen in actuality (Birge and Louveaux, 1997). There are a number of other possible approaches for solving the dynamic problem. We briefly discuss each method below.

Leaving some availability slack on repair resources to make the schedule more robust and flexible (Branke and Mattfeld, 2002, 2005; Davenport et al., 2001) is the first approach. For example, Branke and Mattfeld (2002; 2005) propose an anticipatory scheduling algorithm to predict the future job arrivals in a dynamic scheduling problem. A secondary objective, called flexibility, is included within each static sub-problem to penalize the early idleness of the machines. They experimentally show that this approach improves the system performance. Their conclusion is consistent with our observation in Section 4.4.2.2 on the positive relationship between repairing the aircraft earlier and achieving a higher coverage. It would be therefore interesting to adjust the MIP model such that a flexibility term is added in the objective function to quantify the value of making the repair resources available as early as possible. However, it appears that none of the existing work on such slack-based techniques uses analytical reasoning to decide the amount of slack or level of penalization of early slack that should be used for different levels of stochasticity.

Modeling each static sub-problem as a two-stage stochastic programming is the second approach (Birge and Louveaux, 1997). The first stage decision corresponds to constructing the repair schedule and occurs before aircraft failures in pre- and post-flight checks. The second stage decision, which includes the allocation of aircraft to flights, occurs after the pre-flight checks. One approach is to define Z^s_{kw} as the number of aircraft of type k assigned to fly in wave w under scenario s. Each scenario s represents a possible realization of aircraft failures in the horizon of the static sub-problem with probability $p(s)$. Therefore, the objective function (Equation 4.1) can be written as $\sum_s p(s) \sum_k \sum_w Z^s_{kw}$. The

main modeling challenge is then to calculate the probability of each scenario. As already explained in Section 4.1.1, the uncertainty in our problem is not exogenous information and is dependent on the first-stage decisions which is hard to represent it in a closed and tractable form. Computationally, two-stage stochastic programming models are substantially more challenging than most discrete optimization problems (Dyer and Stougie, 2003) and therefore the ability to solve such models of our problem to optimality is doubtful.

The third approach is to use multi-stage dynamic programming for solving the dynamic repair shop problem (Iravani et al., 2007). The goal is to construct a repair schedule at each decision epoch, marked by the arrival of newly failed aircraft to the repair shop, such that the coverage is maximized over the long term. Using the dynamic programming framework, the state of the repair shop at each decision time point is a tuple of the aircraft failure rates, the aircraft processing times, and the aircraft resource requirements. The decision or action is to assign start-times to the failed aircraft in the repair shop. There are several challenges in modeling the problem as a classical dynamic program. First, the expected wave coverage as a result of the current state and the action taken cannot be represented in a closed form expression because of the combinatorics involved in the scheduling problem. Second, the probabilities with which the repair shop transitions to a new state at the next decision epoch as a result of the current state, the action taken, and the revealed uncertainty on the aircraft failures are not known and are hard to calculate mainly, again, due to the combinatorics of the scheduling decisions and the fact that the processing times and the resource requirements of the repair operations become known upon aircraft failure. The challenges indicate that the analytical tools of the classical dynamic programming methodology cannot be used in modeling our problem. However, AI techniques which have a broader scope of applicability such as machine learning (Sutton and Barto, 1998), online stochastic combinatorial optimization (Van Hentenryck and Bent, 2006), and hindsight optimization (Burns et al., 2012) can be investigated as potential approaches in future work.

Extending the scheduling problem Although our results are demonstrated for a specific class of scheduling problems where the only constraint is the repair capacity limit, our solution approach can be adapted for more complex scheduling problems. More specifically, the proposed MIP and CP algorithms for the static sub-problem can be easily extended to handle other types of scheduling constraints such as precedence constraints. However, modeling the problem via the decomposition approach would require additional effort. The existence of the precedence constraints among the repair activities of a failed aircraft makes the scheduling of different repair resources dependent. Therefore, a separate RSSP for each repair resource cannot be defined. One possible idea is to represent the scheduling problem as a single sub-problem where appropriate relaxation and Benders cut can be developed.

4.6 Conclusion

In this chapter, we addressed the problem of integrated maintenance and production scheduling where there is no control over machine conditions. In the context of scheduling a dynamic aircraft repair shop, the goal is to maximize the flight coverage (production) in the long term considering the aircraft failures

(machine breakdowns). Minimal repair on failed aircraft should be scheduled to ensure that the flights, each requiring a certain number and type of aircraft, are carried out with their full aircraft requirement.

Our proposed solution approach solves the dynamic problem as successive static scheduling problems over shorter time periods. Several scheduling algorithms and different rescheduling policies are proposed to schedule the repair activities online with dynamic reaction to the aircraft failures. The length of the scheduling horizon and the frequency of rescheduling are the features defining our three policies.

The computational results show that an optimization approach using logic-based Benders decomposition, scheduling over a longer horizon, incorporating the mean of known information on aircraft failures, and adjusting the repair schedule as soon as new jobs enter the repair shop yields higher mean coverage and is a reasonable approach to balance against different uncertain scenarios.

To address the relationship between maintenance and production scheduling in this chapter, we assumed that machines are maintained only at failures. In the next chapter, we address the same relationship in the context of manufacturing industries, where there is partial control over machine conditions. Machines are maintained not only at failures but they are also preventively maintained before failures to decrease the number of breakdowns.

Chapter 5

Maintenance Planning & Production Scheduling with Partial Control over Machine Conditions: Deterministic Deterioration

In the previous chapter, we addressed the interdependency between maintenance and production scheduling where machines were only correctively maintained at failures in a repair shop scheduling problem. In this chapter, we study the same relationship assuming that machines can also be preventively maintained before failures.

In many industries, when machines are used for production, their condition changes as parts wear. Such wear can lead to a decrease in the speed at which operations (e.g., drilling with a dull drill-bit) can be done, thereby decreasing the available production capacity. However, preventive maintenance improves machine conditions, restoring the production capacity, while using potential production time that could be otherwise allocated to processing the customer orders. Therefore, scheduling maintenance to minimize the disruption of the production process is a challenging problem. In this chapter, we explore how information about machine conditions can be utilized to simultaneously schedule maintenance and production activities, maximizing customer satisfaction.

The problem of integrated maintenance and production scheduling with partial control over machine conditions was reviewed in Section 2.3.4. As already mentioned, the following three maintenance concepts are not considered in the scheduling literature:

- the effect of maintenance on machines, i.e., improving machine conditions (Ma et al., 2010),
- the explicit connection between processing times and machine conditions, and
- the decision regarding planning maintenance since the time windows for maintenance are typically given (e.g., Kuo and Yang (2008); Kellerer et al. (2012); Mosheiov and Sidney (2010)).

In this chapter, we address a maintenance planning/production scheduling problem over multiple time periods in a multi-machine system, modeling maintenance concepts as defined in the maintenance research literature (McCall, 1965; Cho and Parlar, 1991; Dekker et al., 1997; Wang, 2002; Nicolai and Dekker, 2008). We explicitly model the effect of machine deteriorations and restorations on processing times and consider maintenance as a long-term decision. We assume a deterministic deterioration for each machine in this chapter where a machine's speed is a deterministic function of the previous time of performing preventive maintenance.[1]

Since production capacity is dependent on both machine conditions and scheduling constraints,[2] in this chapter we design an integrated two-stage approach, representing the production capacity by incorporating the combinatorics of the production scheduling problem into the model for maintenance planning. The first stage finds the optimal maintenance plan, abstracting the production scheduling problem. It has a long-term view over the time periods where information about the customer orders is available and seeks to minimize the sum of maintenance and a lower bound on the lost production costs. The maintenance plan determines the assignment of maintenance activities to machines and time periods. The second stage has a short-term view over the current period, finding the optimal schedule of maintenance and production activities given the specified maintenance plan. The real lost production cost is then communicated via a constraint to the first stage so that the maintenance plan can be revised if it is no longer optimal given more detailed lost cost information. The decision stages iterate until the relaxation of lost production cost in the first stage solution is equal to the actual lost production cost.

We experimentally compare the performance of this integrated algorithm with three other approaches: hierarchical decision making where there is no feedback between decision stages, a short-term model where maintenance planning and scheduling are done together for each period, and a heuristic model. Our empirical results demonstrate that the integrated and long-term decision making results in higher solution quality. It is further shown that the benefit of integrated decision making increases as the ratio of maintenance cost to lost production cost decreases while planning maintenance for multiple periods is beneficial when the ratio increases.

In the following sections, we formally define the problem, describe the proposed solution approaches, present the experiments and discuss the results. Finally, we end with conclusion.

5.1 Problem Definition

We consider a multi-machine flowshop production environment, producing multiple products over a finite planning horizon. There are K discrete time periods, each T time units long. Machines deteriorate as they are used for production. To model each machine deterioration process, we assume that the speed of a machine decreases as the number of time periods since preventive maintenance increases. Machine $m \in \{1, 2, ..., M\}$ is in state $s_m \in \{0, 1, ..., \mathcal{S}_m\}$ if its most recent preventive maintenance was s_m time periods ago. In state s_m, machine m operates at speed $v^m_{s_m}$. Without loss of generality, it is assumed

[1] In the next chapter, we relax this assumption, modeling machine deteriorations as stochastic processes.

[2] In a flowshop scheduling problem, the precedence constraints, for example, limit the production capacity since a downstream machine is idle until the process of the first job is finished on the upstream machines.

that the speed of machine m in state $s_m = 0$ is $v_0^m = 1$ and $v_0^m > v_1^m > ... > v_{S_m}^m = 0$. Performing a preventive maintenance job, p, at any point on machine m takes t_p^m units of time, costs τ_p^m and changes the machine's speed to v_0^m. In other words, preventive maintenance makes the machine as good as new, such that it operates at the highest speed. The initial state of machine m at the beginning of the planning horizon is known and denoted as α_m.

At the beginning of each time period, the set of production jobs is known for the next L periods where $L < K$. The set of production jobs at time period $k \in \{1, 2, ..., K\}$ is denoted as $\mathcal{J}_k$. The production jobs are not carried over time periods: job j in time period k, $j \in \mathcal{J}_k$, can only be processed during time period k. Furthermore, job j has to be processed on all machines in sequence, requires processing time p_{jm} on machine m, and has the due date d_j. The processing time of job j on machine m is $\frac{n_{jm}}{v_{s_m}^m}$ where n_{jm} is the processing time of job j at $s_m = 0$, the best state of the machine. The due date d_j corresponds to the latest possible completion time of job j and is a time point within the k-th period. If a job is not finished by its due date, it is lost at cost h_k.

The goal of the problem is to allocate preventive maintenance to machines and time periods over the planning horizon and to assign start-times to both production jobs and preventive maintenance activities, if any, within each time period such that the total cost of lost jobs and performing maintenance is minimized.

Let the binary maintenance decision variable y_{mk} take a value of 1 if machine m at time period $k \in \{1, 2, ..., K\}$ is maintained and let the binary tardy variable u_j take value of 1 if job $j \in \mathcal{J}_k$ is lost in time period k. Thus, objective function (5.1) minimizes the sum of lost production and maintenance cost over the planning horizon.

$$\min \sum_{k=1}^{K} \sum_{j \in \mathcal{J}_k} h_k u_j + \sum_{k=1}^{K} \sum_{m=1}^{M} \tau_p^m y_{mk} \tag{5.1}$$

The problem is subject to maintenance planning and maintenance/production scheduling constraints which are defined below.

Maintenance planning constraints: Since in any time period, there is a limit on the number of machines that can be maintained denoted as C, Constraints (5.2) enforce the maintenance capacity limit in each time period.

$$\sum_{m=1}^{M} y_{mk} \leq C, \qquad \forall k \tag{5.2}$$

Maintenance/production scheduling constraints: To find the assignment of start-times to production and maintenance jobs in period k, we define several extra decision variables as shown in Table 5.1.

The detailed descriptions of the maintenance/production scheduling constraints in period k, shown in Figure 5.1, are provided below:

- In Constraints (5.3), $N_m(k)$ defines the state of machine m at time period k before performing maintenance. Defining the dummy variable $y_{m0} = 1$ and the indicator function $I(x)$ being equal to

$N_m(k)$	The state of machine m in period k before performing maintenance.
st_{jm}	The start-time of job j on machine m.
p_{jm}	The processing time of job j on machine m.
st_{pm}	The start-time of preventive maintenance job p on machine m.
x_{jim}	$x_{jim} = 1$ if job j is processed before job i on machine m.
b_{jm}	$b_{jm} = 1$ if job j is processed before preventive maintenance on machine m.

Table 5.1: Extra decision variables for maintenance/production scheduling in period k.

$$N_m(k) = (k - 1 + \alpha_m)I(\max\{l|y_{ml} = 1, 0 \le l < k\} = 0)$$
$$+ (k - \max\{l|y_{ml} = 1, 0 \le l < k\})I(\max\{l|y_{ml} = 1, 0 \le l < k\} > 0), \qquad \forall m \qquad (5.3)$$
$$p_{jm} = \frac{n_{jm}}{\nu^m_{N_m(k)}} b_{jm} + \frac{n_{jm}}{\nu^m_0}(1 - b_{jm}), \qquad \forall j \in \mathcal{J}_k,\ \forall m \qquad (5.4)$$
$$st_{jm} + p_{jm} \le st_{j(m+1)}, \qquad \forall j \in \mathcal{J}_k,\ \forall m\ (m \ne M) \qquad (5.5)$$
$$st_{pm} + t^m_p + \mathcal{B}(y_{mk} - 1) \le T, \qquad \forall m \qquad (5.6)$$
$$st_{jm} + p_{jm} \le st_{pm} + \mathcal{B}(1 - b_{jm}), \qquad \forall j \in \mathcal{J}_k,\ \forall m \qquad (5.7)$$
$$st_{pm} + t^m_p \le st_{jm} + \mathcal{B}b_{jm}, \qquad \forall j \in \mathcal{J}_k,\ \forall m \qquad (5.8)$$
$$1 - b_{jm} \le y_{mk} \qquad \forall j \in \mathcal{J}_k,\ \forall m \qquad (5.9)$$
$$st_{jM} + p_{jM} \le d_j + \mathcal{B}u_j, \qquad \forall j \in \mathcal{J}_k \qquad (5.10)$$
$$st_{jm} + p_{jm} \le st_{im} + \mathcal{B}(1 - x_{jim}), \qquad \forall j, i \in \mathcal{J}_k\ (j > i),\ \forall m \qquad (5.11)$$
$$st_{im} + p_{im} \le st_{jm} + \mathcal{B}x_{jim}, \qquad \forall j, i \in \mathcal{J}_k\ (j > i),\ \forall m \qquad (5.12)$$

Figure 5.1: Maintenance/production scheduling constraints in period k.

1 if x is true and to 0 otherwise, we have: (i) if machine m is not maintained in any of the previous periods, $I(\max\{l|y_{ml} = 1, 0 \le l < k\} = 0)$ equals 1 and machine m's state is $(k - 1 + \alpha_m)$, or (ii) if the most recent maintenance on machine m is in period $l > 0$, $I(\max\{l|y_{ml} = 1, 0 \le l < k\} > 0)$ is equal to 1 and machine m is in state $(k - l)$.

- Constraints (5.4) denote the processing times of jobs in time period k. If job j is scheduled before maintenance on machine m, $b_{jm} = 1$, the state of the machine is $N_m(k)$ and if scheduled after maintenance, the machine is in state 0.

- Constraints (5.5) enforce the precedence constraints: the job should be finished on an upstream machine before its processing starts on downstream machines.

- Constraints (5.6) ensure that maintenance activities on machines requiring maintenance at time period k, $y_{mk} = 1$, are scheduled within the length of the time period where $\mathcal{B}$ is a big value.

- Constraints (5.7), (5.8), and (5.9) define the relationships between the binary decision variables b_{jm} and the maintenance decisions. Respectively, the constraints guarantee that: if a job is processed before maintenance ($b_{jm} = 1$), then its processing is finished before maintenance is started;

if a job is processed after maintenance ($b_{jm} = 0$), then maintenance is performed before processing the job is started; if a machine does not require maintenance, $y_{mk} = 0$, all jobs are processed before maintenance, $b_{jm} = 1$.

- Since M is the last machine, Constraints (5.10) define whether job j in time period k is lost or not. If a job is not finished before or at its due date, it is then lost.

- Constraints (5.11) and (5.12) are disjunctive constraints ensuring that all jobs on a machine form a total ordering, meaning that no two jobs execute at the same time.

Since the number of production jobs is only known for the next L periods, we use a rolling horizon approach to make the decisions at the beginning of each period. Without loss of generality, the current period is considered as the first period and the future periods where the number of production jobs are known are numbered from 2 to L. Defining maintenance assignment decisions as $Y = \{y_{mk} | \forall m,\ \forall k\}$ and the scheduling decisions as $S = \{st_{jm} | j \in \mathcal{J}_k,\ \forall m,\ \forall k\}$, the optimization problem for making the current time period decisions is shown in Figure 5.2. The schedule is executed for the current time period, the decision horizon is then extended, and the same procedure repeats until the end of the planning horizon.

$$\min_{Y,S} \sum_{k=1}^{L} \sum_{j \in \mathcal{J}_k} h_k u_j + \sum_{k=1}^{L} \sum_{m=1}^{M} \tau_p^m y_{mk}$$

s.t. Constraints (5.2) to (5.12)

$$y_{mk}, u_j, x_{jim}, b_{jm} \in \{0, 1\}, \qquad \forall j, i \in \mathcal{J}_k,\ \forall m,\ \forall k \in \{1, \ldots, L\}$$

$$st_{jm}, p_{jm}, st_{pm} \in \mathbb{Z}^+ \cup \{0\}, \qquad \forall j \in \mathcal{J}_k,\ \forall m,\ \forall k \in \{1, \ldots, L\}$$

Figure 5.2: The non-linear mixed integer programming model.

The optimization problem in Figure 5.2 is a non-linear mixed integer programming model since Constraints (5.3), defining the state of machines at each period, and Constraints (5.4), denoting the processing times of the jobs, are non-linear.

5.2 Solution Approaches

To solve the optimization problem (Figure 5.2) at the beginning of each period, we design a two-stage decomposed but coupled approach, *Integrated*, where each stage is modeled as a mixed integer linear program (MILP).

In this section, the Integrated approach and three alternative approaches, *Non-integrated*, *Short-term*, and *Heuristic* are presented.

5.2.1 The Integrated Approach

There are two different decisions in the problem: (i) assigning maintenance to machines and time periods and (ii) scheduling the production jobs and maintenance activities, if any, in each period. Therefore, similar to a classical logic-based Benders decomposition (LBBD), the global problem (Figure 5.2) can be decomposed into a maintenance planning problem (MPP) and L production scheduling problems (PSP). The MPP is the master problem assigning maintenance to machines and time periods and each PSP defines one sub-problem, finding the schedule of a period. However, solving the problem using the classical logic-based Benders decomposition framework is computationally expensive, though both MPP and PSPs are mixed integer linear models (see Section 5.4.1). Therefore, as illustrated in Figures 5.3 and 5.4, we modify the LBBD such that only one PSP problem is solved at each iteration.

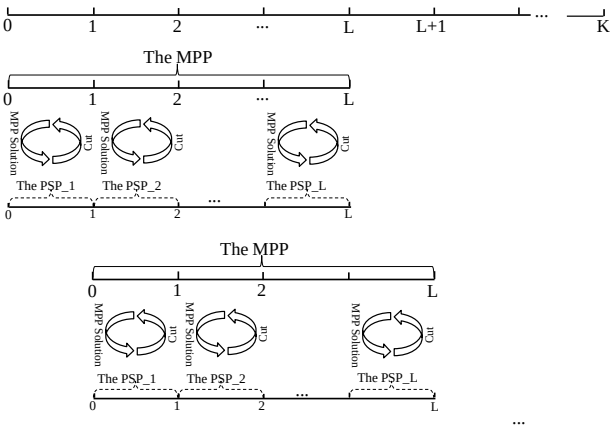


Figure 5.3: The schematic representation of the logic-based Benders decomposition approach.

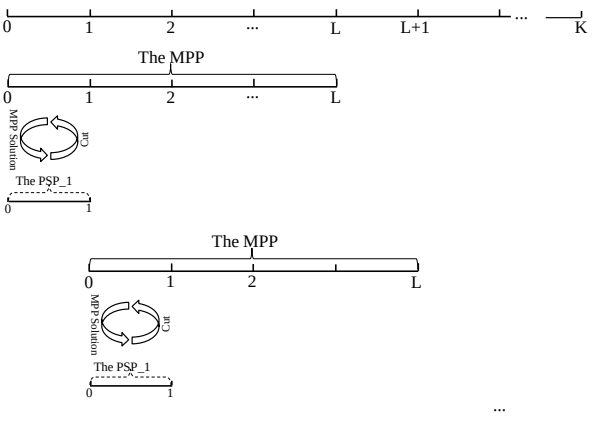


Figure 5.4: The schematic representation of the Integrated approach.

In the Integrated algorithm, the MPP is solved in the first stage to determine the assignment of maintenance to machines and time periods, minimizing the sum of maintenance and lost production costs over the L time periods where the production jobs are known. In the MPP, the PSPs and the production capacity are relaxed, the lost production cost in the first stage is therefore a lower bound on the actual lost production cost.

The PSP in the second stage creates a production and maintenance schedule for the first period, minimizing the actual lost production cost of the first period given the maintenance plan specified by the MPP. If the achieved lost production cost is equal to the lower bound computed on the lost cost of the first period in the MPP, the computed schedule is executed. Otherwise, a constraint expressing a new bound on the lost production cost of the first period, called a *cut*, is added to the MPP and the MPP is re-solved. Each cut corresponds to a new constraint improving the bound on the lost production cost which indirectly incorporates the combinatorics of the scheduling problem into the MPP. The iteration between MPP and PSP continues until the lower bound on the lost production cost of the first period in the MPP is equal to the cost calculated in the PSP. The finite convergence of the Integrated approach is demonstrated below in Section 5.2.1.3.

The decision horizon then rolls over one time period, the initial state of each machine (α_m) is updated, the customer orders become known for time period ($L+1$) and the solution procedure repeats.

In the balance of this section, we present our optimization models for both MPP and PSP, the cut, and the relaxation of the PSPs in the MPP. We have also proved a number of structural properties about the PSP. However, since our early experimentation showed that none of the properties had significant impact on the performance of the solver, we do not use them in this chapter. The properties and the details of our preliminary experiment are given in Appendix B.

5.2.1.1 The Maintenance Planning Problem (MPP)

To model the MPP as a MILP, we change the maintenance binary decision variable from y_{mk} to y^m_{lk} that equals 1 if machine m at time period k is most recently maintained in time period l where $l \leq k$. We further define the new variable Λ_k as the lost cost variable of time period k. To abstract the production scheduling problems in the MPP and to find a lower bound on the lost cost variables, we assume that maintenance is performed at the beginning of the period with negligible time and define the following notation where 0 is a dummy period. Let N^m_{lk} denote the state of machine m in period k after performing the most recent maintenance in period l.

$$N^m_{lk} = \begin{cases} 0 & k = l \\ k - 1 + \alpha_m & k > l,\ l = 0 \\ k - l & k > l,\ l > 0 \end{cases}$$

To explain the notation defined above, we distinguish three cases:

1. $k = l$: Machine m is maintained at period k, i.e., $y^m_{kk} = 1$. Maintenance makes machine m as good as new, setting its state to the best value, 0.

2. $k > l,\ l = 0$: Machine m at time period k has not been maintained in any of the previous periods, i.e., $y^m_{0k} = 1$. Machine m's state is equal to $(k - 1 + \alpha_m)$.

3. $k > l,\ l > 0$: Machine m at time period k is previously maintained at time period $l,\ l > 0$, i.e., $y^m_{lk} = 1$. Machine m is then at state $(k - l)$.

The MILP model for MPP in the first time period is shown in Figure 5.5.

The MPP objective function (5.13) minimizes the total cost composed of the lower bound on the lost cost of L periods and the maintenance cost. Constraints (5.14) and (5.15) ensure the feasibility of the maintenance plan where the former defines the previous maintenance period on machine m at time period k and the latter guarantees that if time period $l,\ (l < k)$, is the previous maintenance period on machine m before the k-th period, then l is also the previous maintenance period before period $(k - 1)$. Constraints (5.16) enforce the maintenance capacity limit in each time period. Constraints (5.17) are the relaxations of PSPs, calculating the lower bound on the lost cost at period k where $|\mathcal{J}_k|$ is the number of production jobs at time period k. In a flowshop system, the upper bound on total number of products produced is equal to the minimum number of products produced over all machines. The upper bound

$$\min \sum_{k=1}^{L} \Lambda_k + \sum_{k=1}^{L} \sum_{m=1}^{M} \tau_p^m y_{kk}^m \tag{5.13}$$

$$\text{s.t.} \sum_{l=0}^{k} y_{lk}^m = 1, \qquad \forall m,\ \forall k \in \{1,\dots,L\} \tag{5.14}$$

$$y_{lk}^m - y_{l(k-1)}^m \leq 0, \qquad \forall m,\ \forall k \in \{1,\dots,L\},\ \forall l \in \{1,\dots,k-1\} \tag{5.15}$$

$$\sum_{m=1}^{M} y_{kk}^m \leq C, \qquad \forall k \in \{1,\dots,L\} \tag{5.16}$$

$$\Lambda_k \geq h_k(|\mathcal{J}_k| - \min_m(\sum_{l=0}^{k} \frac{\nu_{N_{lk}^m}^m \times T}{\min_{j\in\mathcal{J}_k}(n_{jm})} y_{lk}^m)), \qquad \forall k \in \{1,\dots,L\} \tag{5.17}$$

Cuts

$$y_{lk}^m \in \{0,1\}, \Lambda_k \geq 0 \qquad \forall m,\ \forall k \in \{1,\dots,L\},\ \forall l \in \{1,\dots,k\}$$

Figure 5.5: The MPP model.

on the number of finished jobs on machine m given that it was last maintained in period l, i.e., $y_{lk}^m = 1$, equals

$$\frac{\nu_{N_{lk}^m}^m \times T}{\min_{j\in\mathcal{J}_k}(n_{jm})}$$

where the numerator is the upper bound on the total available processing time and the denominator is the minimum processing time required by a job on machine m in period k. The cuts are explained in Section 5.2.1.3.

To linearize the non-linear Constraints (5.17), they are replaced by the following two constraints where δ_k is a dummy decision variable.

$$\Lambda_k \geq h_k(|\mathcal{J}_k| - \delta_k), \qquad \forall k \in \{1,\dots,L\}$$

$$\delta_k \leq \sum_{l=0}^{k} \frac{\nu_{N_{lk}^m}^m \times T}{\min_{j\in\mathcal{J}_k}(n_{jm})} y_{lk}^m, \qquad \forall m,\ \forall k \in \{1,\dots,L\}$$

5.2.1.2 The Production Scheduling Problem (PSP)

After the maintenance assignment decisions denoted as y_{lk}^{mh} are found by the MPP in iteration h, the states of machines are known. The PSP problem for finding the optimal maintenance and production schedule in the first time period for a given maintenance plan by the MPP is shown in Figure 5.6 where in Constraints (5.4) to (5.12): (i) k equals 1; (ii) y_{m1} changes to y_{11}^{mh}, and (iii) $N_m(1)$ equals α_m denoting the state of machine m before performing maintenance at the first period.

If we relax the PSP by assuming there is no deterioration and that $|M| = 2$, then the PSP problem

$$\min h_1 \sum_{j=1}^{|\mathcal{J}_1|} u_j$$

s.t. Constraints (5.4) to (5.12)

$$u_j, x_{jim}, b_{jm} \in \{0,1\}, \qquad \forall j, i \in \mathcal{J}_1,\ \forall m$$

$$st_{jm}, p_{jm}, st_{pm} \in \mathbb{Z}^+ \cup \{0\}, \qquad \forall j \in \mathcal{J}_1,\ \forall m$$

Figure 5.6: The PSP model.

corresponds to a two machine flowshop with the objective of minimizing the number of tardy jobs, an NP-complete problem (Lenstra et al., 1977). Therefore the PSP problem which generalizes the two machine flowshop is also NP-complete.

5.2.1.3 The MPP Cuts

As noted above, the MPP and PSP are iteratively solved, with each optimal MPP solution defining a PSP and each PSP returning cuts if the lost production cost of the first period from the MPP cannot be achieved. Assume that in iteration h, the first period lost production cost in the MPP is less than the optimal lost production cost in the PSP, represented as Λ_1^h. The cut after iteration h is:

$$\Lambda_1 \geq \Lambda_1^h(1 - \sum_{m \in Q^h}(1 - y_{11}^m) - \sum_{m \notin Q^h} y_{11}^m) \tag{5.18}$$

where $Q^h = \{m | y_{11}^{mh} = 1\}$ denotes the set of machines requiring maintenance in iteration h found in the MPP.

The cut is a *no-good* cut guaranteeing that if the same set of machines are maintained ($m \in Q^h$) and the same set of machines are not maintained ($m \notin Q^h$) in the current first period, the lost production cost of the first period in the MPP (Λ_1) should be greater than or equal to Λ_1^h. As the MPP and the PSP find, respectively, a lower bound and an upper bound on the lost production cost of the first period in each iteration, iterating between stages terminates when the bounds are equal. Furthermore, the finite number of possible maintenance plans guarantees the finite convergence of the Integrated approach.

Changing the cut to $\Lambda_1 \geq \Lambda_1^h(1 - \sum_{m \notin Q^h} y_{11}^m)$ would make it stronger, but is unsound due to the non-monotonic behavior of Q^h: depending on the problem, maintaining a subset of Q^h can decrease *or* increase the lost production cost making the stronger cut invalid (see Example 1).

The stronger cut is not valid unless we make further assumptions. For example, if we assume that the maintenance duration of all machines is less than the increase in the processing times of all jobs, then maintaining fewer machines never decreases the lost production cost,[3] making the stronger cut valid. However, we do not make such an assumption in this chapter.

Example 1: A facility with 3 machines (M1, M2, M3) and 2 production jobs (J1, J2) is considered where

[3]For more details, see Property 2 in Appendix B.

the length of the time period is 40, the due dates of production jobs are 24 and 35, the processing time of each production job on each of three machines is 10 and decreases to 5 if scheduled after maintenance. The durations of maintenance activities on machines (P1, P2, P3) are 30, 5, and 15, respectively.

Assuming that the MPP at iteration h decides to maintain machines 1, 2, and 3 ($Q^h = \{1, 2, 3\}$), the optimal schedule is shown in Figure 5.7 where the number of on-time jobs is one. If the subset $\{1, 2\}$ is maintained in the next iteration, none of the jobs is then on-time, increasing the lost production cost. However, maintaining the subset $\{2, 3\}$ makes both jobs on-time decreasing the lost production cost.

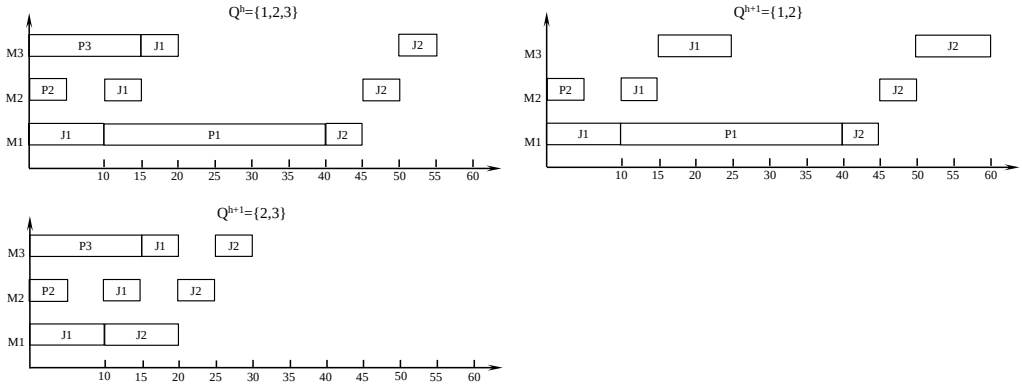


Figure 5.7: The optimal schedules for Example 1.

5.2.1.4 Relaxation of the PSP in the MPP

As noted, Constraints (5.17) are the relaxation of the PSPs in the MPP, expressing a lower bound on the lost production cost. We tighten the lower bound for the first time period by applying Moore's algorithm on the last machine. Moore's algorithm finds the optimal number of tardy jobs in a single machine problem when all jobs are ready at time 0 with the computational complexity of $O(n \log n)$ (Pinedo, 2002).

The last machine is considered as a single machine where the due dates of the production jobs are changed to $d'_j = d_j - \Delta$ since all are not available at time 0. Δ corresponds to the sum of the minimum processing times of the jobs on the upstream machines denoted as $\sum_{m=1}^{M-1} \min_{j \in \mathcal{J}_1}(n_{jm})$. Since Δ is calculated assuming that all previous machines are processing at their best states, that the processing times of all upstream jobs on a given machine are equal to the minimum processing time over all jobs, and that there are no resource constraints so that all upstream jobs on the same machine are processed at the same time, then the following constraint, added to the MPP, is a lower bound on the lost production cost of the first time period.

$$\Lambda_1 \geq h_1 U^1 y_{11}^M + h_1 U^0 y_{01}^M \tag{5.19}$$

U^1 and U^0 represent the value of Moore's algorithm when the last machine is maintained and is not, respectively. Similarly, the processing times of the jobs on the last machine are n_{jM} or $\frac{n_{jM}}{v_{\alpha_M}^M}$ in Moore's

algorithm. Note that, Moore's algorithm to find U^1 and U^0 is just applied before starting to iterate. We use both relaxations, i.e., Constraints (5.17) and (5.19), in our model.

5.2.2 The Non-integrated Approach

The Non-integrated approach (Figure 5.8) is the standard hierarchical decision making procedure where there is no iteration between the MPP and PSP. The MPP (Figure 5.5) solves the maintenance planning problem over L periods minimizing the sum of maintenance and a lower bound on the lost production costs. The PSP (Figure 5.6) then finds the optimal lost production cost for the current time period given the maintenance activities specified by the MPP. The schedule is executed, the decision horizon then rolls over one time period updating the machine states (α_m), and the same procedure repeats.

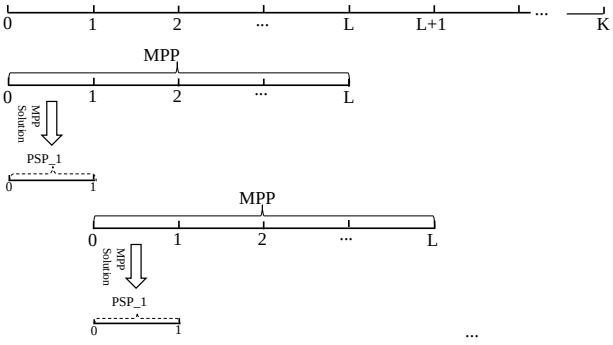


Figure 5.8: The schematic representation of the Non-integrated approach.

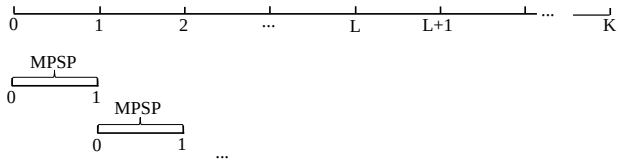


Figure 5.9: The schematic representation of the Short-term approach.

5.2.3 The Short-term Approach

The Short-term approach has a reasoning horizon of one time period (Figure 5.9) considering maintenance as a short-term decision. The maintenance and production scheduling problem (MPSP) determines which machines are maintained and finds the optimal schedule, minimizing the sum of maintenance and lost production costs simultaneously. The computed schedule is then executed, the machine states (α_m) are updated, and the same procedure repeats for the next time period.

The MPSP model for the first period is shown in Figure 5.10, where $k = 1$ and $N_m(1) = \alpha_m$ in Constraints (5.2) and Constraints (5.4) to (5.12).

5.2.4 Heuristic Approaches

We investigate two heuristic approaches for the PSP and the MPSP models inspired by Moore's algorithm.

$$\min h_1 \sum_{j=1}^{|\mathcal{J}_1|} u_j + \sum_{m=1}^{M} \tau_p^m y_{m1}$$

s.t. Constraints (5.2), Constraints (5.4) to (5.12)

$$y_{m1}, u_j, x_{jim}, b_{jm} \in \{0,1\}, \qquad \forall j, i \in \mathcal{J}_1,\ \forall m$$

$$st_{jm}, p_{jm}, st_{pm} \in \mathbb{Z}^+ \cup \{0\}, \qquad \forall j \in \mathcal{J}_1,\ \forall m$$

Figure 5.10: The MPSP model.

5.2.4.1 A Heuristic for the PSP

In the heuristic algorithm, the maintenance activities are performed first on machines that have to be maintained, i.e., $\forall m \in Q^1$. Q^1 is the set of machines determined for maintenance in the first iteration of the MPP. Moore's algorithm is then applied on the last machine, M, as explained in Section 5.2.1.4 where

$$\Delta = \sum_{\substack{m \in Q^1 \\ m \neq M}} (t_p^m + \min_{j \in \mathcal{J}_1}(n_{jm})) + \sum_{\substack{m \notin Q^1 \\ m \neq M}} \min_{j \in \mathcal{J}_1}(\frac{n_{jm}}{\nu_{\alpha_m}^m})$$

$$d'_j = \begin{cases} d_j - (\Delta + t_p^M) & \text{if } M \in Q^1 \\ d_j - \Delta & \text{if } M \notin Q^1 \end{cases}$$

The sequence found by Moore's algorithm is used to schedule the jobs on all machines.

5.2.4.2 A Heuristic for the MPSP

The heuristic is the same as one for the PSP with the only difference that the decision on which machines require maintenance is also incorporated. Machines are ordered in increasing order of their indices and the first C machines in an initial state greater than or equal to $\frac{S_m}{2}$ are maintained. Recall that $\mathcal{S}_m$ is the worst state of machine m. The maintained machines then form set Q^1 and the *Heuristic for the PSP* is applied to find a feasible schedule.

5.3 Empirical Study

The next sub-section describes the problem instances and the experimental details. We then compare the performance of the solution approaches experimentally and present insights into each algorithm's performance through a deeper analysis of the results.

5.3.1 Experimental Setup

The problem instances have $M \in \{3,4,5,6\}$ machines and $|\mathcal{J}| \in \{5,10,15\}$ jobs in each time period. Note that in our experimental study, the number of jobs at each time period is equal, i.e., $|\mathcal{J}_k| = |\mathcal{J}|$ in

a given instance. Twenty instances for each combination of parameters are generated, resulting in 240 instances.

Machines Each machine has five states and is randomly assigned to one of the deterioration processes shown in Table 5.2. The deterioration process is classified into three categories of slow, medium, or fast, defining the speed of the machine in different states. The initial state of each machine, α_m, is drawn from the discrete uniform distribution $[0, 3]$ assuming that no machine is in the worst state at the beginning of the planning horizon. The maintenance cost for each machine, τ_p^m, is generated from the discrete uniform distribution $[50, 100]$.

Deterioration process	States				
	0	1	2	3	4
Slow	1	0.9	0.6	0.3	0
Medium	1	0.75	0.5	0.25	0
Fast	1	0.6	0.3	0.15	0

Table 5.2: The speed of a machine at each state in different deterioration processes.

Time periods The length of time period, T, is set at 79, 152, and 224 in instances with 5, 10, and 15 jobs, respectively (see next paragraph for more details). As with the maintenance cost, the lost production cost per each job at time period k, h_k, is generated from the discrete uniform distribution $[50, 100]$. The maintenance capacity at each time period, C, is equal to $\lfloor \frac{M}{2} \rfloor$.

Production jobs To generate the processing times of the jobs at the best state of machines, i.e., n_{jm}, we assume that they are uniformly distributed with mean μ and variance σ^2. Further we assume that ν_a denotes the average speed of a machine. The average processing time of a job on a machine regardless of its state is then uniformly distributed with mean $\frac{\mu}{\nu_a}$ and variance $\frac{\sigma^2}{\nu_a^2}$. The sum of the average processing times of all jobs has an approximately normal distribution with mean $|J| \times \frac{\mu}{\nu_a}$ and variance $|J| \times \frac{\sigma^2}{\nu_a^2}$. Setting $\nu_a = 0.5$, μ and σ^2 are found such that the probability that the sum of the average processing times of all jobs is less than eighty percent of the length of the time period equals 0.75. In our experiment, μ and σ^2 equal 5.5 and 6.75 in all instances and the length of the time periods are set based on the number of jobs, as described above. n_{jm} is then drawn from the discrete uniform distribution $[1, 10]$. The due date of job j is generated from the discrete uniform distribution $[f^d \times \sum_{m=1}^{M} n_{jm}, \max(T, f^d \times \sum_{m=1}^{M} n_{jm})]$, where $f^d = 1.5$ and T is the length of each time period.

Maintenance Activities The maintenance duration on machine m, t_p^m, is drawn from the discrete uniform distribution $[0.05 \times T, 0.15 \times T]$.

There are $K = 24$ time periods in the planning horizon where the number of production jobs are always known for the next $L = 4$ periods. The CPU time limit to find the maintenance and production schedule at each time period is 900 seconds. As noted above, the length of the time periods varies between 79, 152, and 224 time units. Since it is not uncommon in practice to have one time unit correspond to 10 or 15 minutes, the CPU time limit being less than 2% of the length of the time period is compatible with the online execution requirement. We execute the best feasible maintenance and production schedule found by the time-limit if an algorithm times out. In the case that no feasible

solution is found before the time limit, the schedule found by a heuristic is executed: *Heuristic for the PSP* is executed when the PSP times out and *Heuristic for the MPSP* is executed when the MPSP times out.

All experiments were run on an AMD 270 CPU with 1 MB cache per core, 4 GB of main memory, running Red Hat Enterprise Linux 4. The MILP solver is CPLEX 12.3.

5.3.2 Computational Results

In this section, we discuss our results to compare the performance of different algorithms on the total cost of maintenance and lost production. The total cost is calculated over the first 21 time periods to reduce end-of-horizon effects. The algorithms are Integrated, Non-integrated, Short-term, and Heuristic. The Heuristic algorithm refers to the Heuristic for the MPSP defined in Section 5.2.4.2.

Figure 5.11 shows the mean and the standard deviation of the normalized total cost for different algorithms and different number of jobs. The number of jobs differs between 5, 10, and 15, each representing a different problem set with 80 instances. The total cost of each instance for each algorithm is normalized by dividing by the total cost achieved using the Heuristic algorithm. The graph shows a lower mean and standard deviation for the Integrated approach for all problem sets, indicating its superiority over the other three approaches. Table 5.3 presents further data for each algorithm and each problem set: the mean and the standard deviation of the normalized total cost, the number of instances for which the best known solution is found, and the number of timed-out instances. An instance is counted as a timed-out if it reaches the time limit without finding the optimal solution in at least one time period.

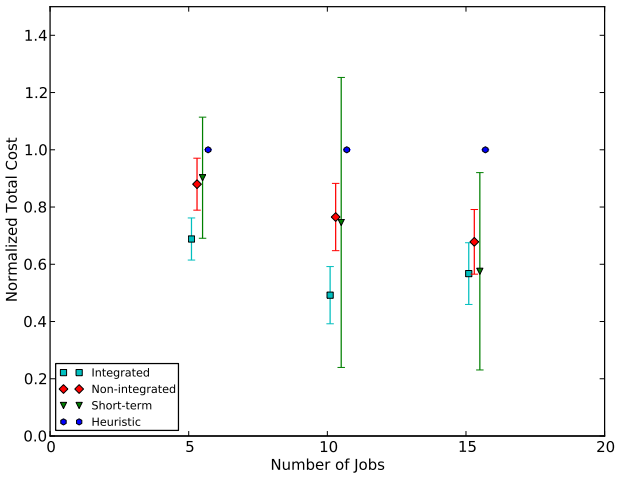


Figure 5.11: The mean and the standard deviation of the normalized total cost for different algorithms and different number of jobs.

Table 5.4 shows the mean and the standard deviation of the total run-time of each period problem for different number of jobs and the Integrated, Non-integrated, and Short-term approaches, and the mean

	Integrated				Non-integrated				Short-term				Heuristic			
$\mathcal{J}$	mean	std	best	timed-out	mean	std	best	timed-out	mean	std	best	timed-out	mean	std	best	timed-out
5	0.69	0.07	73	0	0.88	0.09	1	0	0.9	0.21	6	0	1	0	0	0
10	0.49	0.10	79	22	0.77	0.12	0	2	0.75	0.51	2	60	1	0	0	0
15	0.57	0.11	22	80	0.68	0.11	1	79	0.58	0.34	57	80	1	0	0	0
{5, 10, 15}	0.58	0.09	174	102	0.78	0.11	2	81	0.74	0.35	65	140	1	0	0	0

Table 5.3: The mean and the standard deviation (std) of the normalized total cost, the number of instances for which the best known solution is found (best), and the number of timed-out instances.

percentage of run-time spent solving the MPPs and the PSPs in each period for different number of jobs and the Integrated and Non-integrated approaches.

	Integrated				Non-integrated				Short-term	
$\mathcal{J}$	mean	std	% MPPs	% PSPs	mean	std	% MPPs	% PSPs	mean	std
5	0.97	1.37	27.80	72.20	0.052	0.03	18.26	81.74	0.1	0.18
10	87.68	201.63	1.14	98.86	4.58	36.71	1.16	98.84	214.81	359.36
15	619.96	378.09	0.05	99.95	359.61	388.16	0.05	99.95	658.59	377.89

Table 5.4: The mean and the standard deviation (std) of the total run-time of each period problem, the mean percentage of run-time spent solving MPPs and PSPs in each period.

Integrated vs. Non-integrated The Integrated approach outperforms the Non-integrated, achieving a lower normalized total cost and finding the best known solutions on 99% of the instances.

Integrated vs. Short-term The Integrated algorithm results in a lower normalized total cost on 73% of the problem instances and a higher value on 27%. A closer look to the results shows that for 89% of the instances where Short-term outperforms Integrated, both algorithms time out. If the Integrated approach times out, it executes the best feasible schedule found for that time period. Therefore, the comparison between the performance of the algorithms reduces to comparison between different heuristics.

Integrated vs. Heuristic Although the Heuristic approach is fast, the Integrated algorithm has a significant superiority over it, decreasing the mean normalized cost by 42% and resulting in a lower normalized total cost for all problem instances.

5.4 Discussion

A more detailed data analysis of the results suggest that the superiority of the Integrated over the Non-integrated and the Short-term decreases as the maintenance becomes more expensive and more inexpensive, respectively.

In both Integrated and Non-integrated algorithms, the maintenance decision is made primarily based on long-term reasoning and both decide to do the same amount of maintenance over the MPP horizon. However, having the same number of maintenance jobs does not mean that the two approaches find the same schedule. In particular, recall that the iterations of the Integrated approach result in the total lost production cost over the MPP horizon being composed of the actual lost production cost in the first period plus a lower bound from the later periods. This asymmetry results in the Integrated approach preferring to schedule its maintenance in the first period because that leads to reduced lost production

cost. In other words, because the exact lost cost instead of a lower bound is used in the first time period, lost cost appears more expensive in the first time period, and therefore Integrated prefers to perform maintenance to decrease it. The outcome then is that Integrated adopts a schedule which is less expensive than Non-integrated but which tends to schedule more maintenance in the first period. When maintenance cost is high, the bias to perform maintenance earlier in each MPP horizon tends to result in *more frequent* maintenance over the planning horizon. Therefore, a higher maintenance cost over the 21 time periods results in a higher total cost since the savings on the lost production costs is insignificant compared to the maintenance cost. Adjusting the Integrated approach to have a symmetric view over all periods such that the total lost production cost consists of the actual lost costs of all periods in the MPP horizon is likely to remove the bias of the Integrated approach. As we will see in Section 5.4.1, however, such an adjustment results in other algorithmic challenges.

Turning to the comparison of Integrated and Short-term, the primary difference is the long-term maintenance reasoning done by the former. A limitation of the Integrated compared to the Short-term is likely to arise when maintenance is inexpensive. If maintenance costs less than failing to satisfy a customer order, then it is almost always best to do more maintenance. Furthermore, the Short-term approach will be able to find such solutions because maximizing maintenance is worthwhile both in the long and short runs.

To verify our interpretations, we define $\rho = \frac{\tau_p^m}{h_k}$ as the ratio of maintenance cost to lost production cost and use the 240 problem instances as defined in Section 5.3.1. We run two further experiments with the modification that the maintenance cost of each machine is multiplied by 0.5 and 1.5, respectively: $0.5 \leq \rho \leq 2$ in the first experiment is changed to $0.25 \leq \rho \leq 1$ and $0.75 \leq \rho \leq 3$. Figure 5.12 illustrates the mean and the standard deviation of normalized total cost for different algorithms and different ρ values over all 240 problem instances.

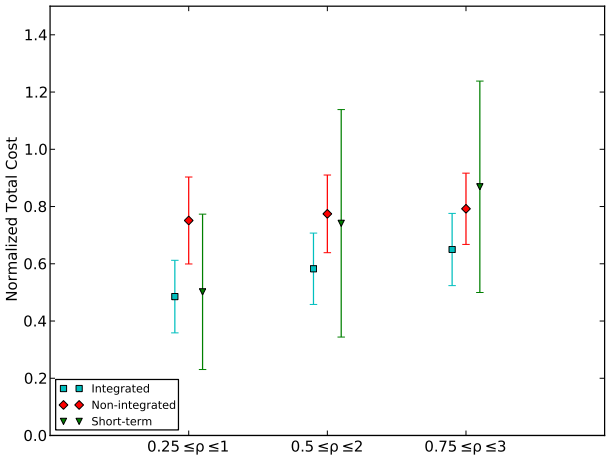


Figure 5.12: The mean and the standard deviation of the normalized total cost for different algorithms and different ρ values.

Table 5.5 shows the difference between the means of normalized total costs for different algorithms. As the ρ values increase, i.e., performing maintenance becomes more expensive, the difference between the Non-integrated and the Integrated approaches decreases while the difference between the Short-term and the Integrated increases, supporting our interpretations.

ρ	Non-integrated:Integrated	Short-term:Integrated
$0.25 \leq \rho \leq 1$	0.27	0.02
$0.5 \leq \rho \leq 2$	0.19	0.16
$0.75 \leq \rho \leq 3$	0.14	0.22

Table 5.5: The difference between the means of normalized total costs for different algorithms and different ρ values.

5.4.1 The Extended Integrated Approach

As already discussed, the Integrated approach has an asymmetrical view over the PSPs in the MPP horizon: because the MPP lost cost value in the current period converges to the actual lost cost but the same value is represented only by a lower bound in later periods, the Integrated approach has a bias to perform immediate maintenance. The lost cost is essentially more expensive in the current period than in subsequent periods. Adjusting the Integrated approach to represent the actual lost production cost from all periods will remove this bias while also allowing the MPP to reason with more accurate lost cost information.

We can therefore use the logic-based Benders decomposition representation of the problem shown in Figure 5.3, called the Extended Integrated approach. The extension is that for each MPP solution, a PSP for each period within the known horizon is solved to find the actual lost costs for each of the L time periods. While this increases the number of PSPs, given a maintenance plan, each PSP is independent and they can be solved in parallel with multiple processors.

While the Extended Integrated approach is actually a standard logic-based Benders decomposition, the approach has two critical weaknesses in our context.

1. Observe that the lost production cost of time period k is dependent on both the set of maintained machines in period k and the machine speeds, and therefore the machine conditions, at the beginning of the period. While the L PSPs can be solved independently, a cut for a time period $k > 1$ cannot simply refer to the maintenance decisions in period k. In a subsequent iteration, a change in maintenance decisions in an earlier period would change the machine conditions at the beginning of period k and, therefore, would change the lost cost impact of the maintenance decisions in period k. A cut that only includes the maintenance decisions for time period k is therefore invalid. In fact, a valid cut for period k in the Extended Integrated approach must refer to the maintenance decisions for the first k periods and provide a bound on the *sum* of the lost costs over the first k periods.

Formally, the cuts after iteration h are:

$$\sum_{i=1}^{k} \Lambda_i \geq (\sum_{i=1}^{k} \Lambda_i^h)(1 - \sum_{i=1}^{k} \sum_{m \in Q_i^h} (1 - y_{ii}^m) - \sum_{i=1}^{k} \sum_{m \notin Q_i^h} y_{ii}^m), \forall k \in \{1, \ldots, L\} \tag{5.20}$$

Q_k^h indicates the set of machines maintained in period k in iteration h. The iterations between the MPP and the PSPs continue until the total lost cost over L time periods is equal to the one computed in the MPP.

2. At each iteration of the MPP, the PSPs return cuts until the convergence criterion is achieved. The maximum number of iterations therefore equals the maximum number of times that the PSPs might return cuts to the MPP. Since the cuts in the Integrated approach (Equations (5.18)) involve only the lost production cost variable for the first period, the maximum number of iterations is $\sum_{i=0}^{C} \binom{M}{i}$ enumerating all possible ways of assigning maintenance to i machines and the first period considering the maintenance capacity limit of C. However, the cuts in the Extended Integrated approach (Equations (5.20)) involve the lost production cost variables for all L periods. The maximum number of iterations consequently increases to $(\sum_{i=0}^{C} \binom{M}{i})^L$. The Extended Integrated approach will then be expected to have an extremely high computational expense not because of the linear increase in the number of PSPs in each MPP iteration (i.e., solving $(L-1)$ more PSPs), but because of the exponential increase in the number of MPP iterations.

These weaknesses make the Extended Integrated model unlikely to be successful. To confirm this analysis, we ran it on the 240 problem instances of Section 5.3 where $0.5 \leq \rho \leq 2$ and where the CPU time limit is 900 seconds for each period. As expected, it times out on 198 problem instances and the mean of the normalized total cost over all instances marginally increases to 0.59 compared to 0.58 for the Integrated approach in Table 5.3.

5.5 Conclusion

In this chapter, we studied an integrated maintenance planning and production scheduling problem in a multi-machine and multi-period production system where machine conditions are partially controlled. At the beginning of each time period, two decisions are made: which machines are to be maintained, if any, and when each production and each maintenance activity should be executed in order to minimize the total maintenance and lost production costs over the planning horizon.

To precisely model the production capacity as a function of both machine states and scheduling combinatorics, we propose an integrated two-stage algorithm. In the first stage of the algorithm, maintenance planning is done over time periods where the customer orders are known. The production scheduling problem and production capacity are abstracted in the first stage and the objective is to find a maintenance plan for each machine, minimizing the sum of maintenance cost and a lower bound on lost production cost. The second stage then schedules maintenance and production activities in the current

period, minimizing the actual lost production cost assuming the given maintenance plan. The iteration between two stages continues, with feedback, until the lower bound and the actual lost production cost of the current period converge.

The computational results demonstrate that the Integrated approach yields lower total cost than three other approaches tested: a Non-integrated approach, a Short-term, integrated approach, and a Heuristic approach. The benefit for Integrated decision making over Non-integrated, furthermore, increases for lower maintenance cost relative to lost production cost. Finally, the benefit of long-term decision making in the Integrated approach over a myopic, Short-term approach increases with higher relative maintenance cost. These observations suggest that at extreme low or high relative maintenance cost, Short-term and Non-integrated approaches should be adopted. However, for a broad range of intermediate relative costs, Integrated provides superior quality solutions.

To model each machine deterioration in this chapter, we assumed that the speed of a machine is a deterministic function of the number of time periods since preventive maintenance. This assumption might be a plausible approximation for modeling machine conditions where the production process is smooth and machines do not go under dramatic load fluctuations. In the next chapter, we generalize this assumption modeling each machine deterioration as a stochastic process covering a wider range of manufacturing industries.

Chapter 6

Maintenance Planning & Production Scheduling with Partial Control over Machine Conditions: Markovian Deterioration

In this chapter we continue studying the interdependency between maintenance planning and production scheduling where machines are maintained both correctively and preventively.

Machine deterioration is one of the main causes of production capacity reduction in many manufacturing industries (Kaufman and Lewis, 2007; Sloan, 2008). Maintenance operations improve machine conditions, but also occupy potential production time, possibly delaying the customer orders. Therefore, the challenge is to determine the maintenance and the production schedule to maximize customer satisfaction. In the previous chapter, we motivated this challenge from the perspective of the scheduling literature and introduced common maintenance conceptualizations as they appear in maintenance research area to scheduling literature. More specifically, we modeled machine deterioration as a deterministic function of the number of time periods since previous maintenance and considered a finite, though long, decision horizon. In this chapter, we study the same challenge but from the perspective of the research on maintenance.

In the maintenance literature, the problem of integrated maintenance and production scheduling utilizing machine condition information has mainly been addressed on the tactical level, adopting a long-term decision horizon. It has been developed in two directions. The first stream assumes that all customer orders are similar and addresses the problem of production planning: how much to produce (see Section 2.2.3 for review of this literature). The second stream assumes different customer orders, but a single machine and addresses the problem of product dispatching: which product to produce next (see Section 2.2.4 for review of this literature). In this chapter, we address the problem of integrated maintenance and production scheduling on the operational level assuming a multi-machine production environment where different customer orders have to be scheduled on machines in sequence and are due

at different times. The goal is to determine when each machine is maintained and when each order on each machine is started, maximizing the number of orders satisfied by their due dates.

Characterizing machine conditions by a discrete set of states and assuming that the customer orders become known at the beginning of each period, we design two different modules to solve the problem. The first module uses a Markov decision process (MDP) model to determine the maintenance plan, abstracting the combinatorics of the production scheduling problem, over a long-term infinite horizon. The maintenance plan is a decision rule identifying machines for maintenance based on their states and the number of customer orders. We also derive sufficient conditions to guarantee that the optimal maintenance plan has a switching curve structure which is monotone in both machine state and the number of customer orders. The set of customer orders and the maintenance activities, if any, then constitute the set of operations in the second module. We solve a mixed integer programming (MIP) model to assign a start-time to each operation within the time period. The planned maintenance and production schedule is then executed, the real cost of the period is realized, the new states of machines and the number of customer orders are observed, and the procedure repeats to find the schedule for the next time period.

To gain insight into situations where exploiting online condition monitoring information is beneficial, we compare the designed algorithm with a heuristic approach where both maintenance planning and production scheduling are done using dispatching type policies. Our computational results demonstrate that incorporating accurate information about machine deterioration decreases the total discounted cost of maintenance and lost production on average 21%. It is further shown that the benefit of using online condition monitoring information increases for medium failure industries and high discount factors where the maintenance planning decision has a larger effect on the short-term production scheduling decision and where the long-term impact of short-term decisions has a more significant weight on the total discounted cost.

This chapter is organized as follows. The problem of interest is first defined in Section 6.1. Our solution approaches are then explained and the sufficient conditions for the switching curve optimal maintenance policy are derived in Sections 6.2 and 6.3, respectively. The details on the execution of the planned schedules given by different algorithms can be found in Section 6.4. Computational results are then presented, followed by a discussion. We complete this chapter with a conclusion in Section 6.7. The exact method for deriving the average production rate used in production scheduling problem and the details of the experimental setup are provided in Appendices C and D, respectively.

6.1 Problem Definition

Figure 6.1 is a snapshot of the problem at time 0, where rectangles represent machines. Machines deteriorate as they are used for production; the filled colors in Figure 6.1 illustrate machine conditions where darker colors indicate higher levels of deterioration. Maintenance improves machine conditions but it takes production time and delays the delivery of the customer orders. Each order has a specific processing requirement and a due date and should be processed on all machines in sequence. The goal is

to simultaneously schedule maintenance and customer orders to minimize the total cost of maintenance and lost production in the long term.

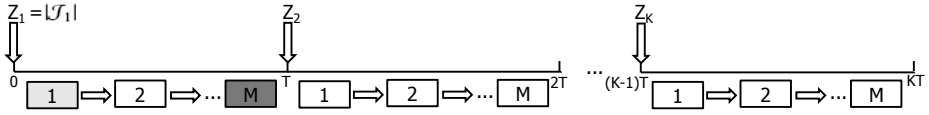


Figure 6.1: Snapshot of the problem at time 0.

Formally, we consider a series (flowshop) manufacturing facility with M machines producing products to meet demand due at different time points over K discrete periods, each with length T. All machines deteriorate as they are used for production. The deteriorating machine $m \in \{1, \ldots, M\}$ can be in one of N_m operational states $\{0, \ldots, N_m - 1\}$ or in a failure state N_m. The state process $(X_t^m : t \in \mathbb{R}^+)$, the state of machine m at time t, follows a continuous time homogeneous Markov chain with state space $\mathbb{S}_m = \{0, \ldots, N_m\}$. The state transition rate matrix of machine m is defined as $Q^m = [q_{ik}^m]_{(N_m+1)\times(N_m+1)}$ where $-q_{ii}^m$ is the rate at which the machine changes its state when in state i and q_{ij}^m is the rate of transition to state j leaving state i.

$$q_{ik}^m = \lim_{h\to 0} \frac{\Pr(X_h^m = k | X_0^m = i)}{h}, \quad i, k \in \mathbb{S}_m,\ i \neq k,$$
$$q_{ii}^m = -\sum_{k\neq i} q_{ik}^m.$$

As a machine deteriorates, its production rate decreases. The production rate of machine m depends on its state and is denoted as $r^m(i), i \in \mathbb{S}_m$. In each state of machine m at time t, two actions can be performed. Therefore, the action space of machine m is $a_t^m \in \mathbb{A}_m = \{0, 1\}$ where a_t^m is the action taken on machine m at time t being equal to 1 if maintained and 0 otherwise. Performing maintenance on machine m at state i takes t_p^m units of time, costs $\tau^m(i)$, and transitions it to state k with probability R_{ik}^m. The deterioration process and maintenance operations behaviors of machine m are summarized below where "B" stands for behavior.

- B1: Each state represents a level of machine deterioration. Higher states indicate higher levels of deterioration or worse machine conditions, that is, state i is worse than state k if $i > k$.

- B2: Machine conditions deteriorate as a result of production without performing maintenance. Machine states do not therefore improve, i.e., $q_{ik}^m = 0, \forall k < i$.

- B3: Maintenance improves machine conditions, thus, machine states do not worsen, i.e., $R_{ik}^m = \Pr(X_{t+t_p^m}^m = k | X_t^m = i, a_t^m = 1) = 0,\ \forall k > i$. It is worth mentioning that R_{ik}^m only depends on machine states, not on time t or maintenance duration, t_p^m: $R_{ik}^m = \Pr(X_{t'}^m = k | X_t^m = i, a_t^m = 1),\ \forall t' > t$.

- B4: Production rate does not increase as the machine deteriorates, it is non-increasing in the machine state, i.e., $r^m(i) \geq r^m(i+1)$. Furthermore, the production rate at failure state equals 0, $r^m(N_m) = 0$.

- B5: Maintenance cost does not decrease as the machine deteriorates, it is non-decreasing in the machine state, i.e., $\tau^m(i) \leq \tau^m(i+1)$.

- B6: As the machine deteriorates, the rate of transition to worse states increases. In other words, the transition rate matrix is monotone (Keilson and Kester, 1977), more specifically, $\sum_{k \geq l} q^m_{ik} < \sum_{k \geq l} q^m_{(i+1)k}, \forall l \in \mathbb{S}_m,\ l \geq (i+2)$.

- B7: As the machine deteriorates, the probability of going to a better state after maintenance does not increase, i.e., $\Pr(X^m_{t+t^m_p} \leq l | X^m_t = i, a^m_t = 1) \geq \Pr(X^m_{t+t^m_p} \leq l | X^m_t = i+1, a^m_t = 1)$ (or $\sum_{k \leq l} R^m_{ik} \geq \sum_{k \leq l} R^m_{(i+1)k}, \forall l \in \mathbb{S}_m$).

Let Z_k be the random demand (the number of customer orders) of period k. We assume that $Z_1, \ldots, Z_K$ are independent and identically distributed (i.i.d) with probability mass function $g(z)$. At the beginning of each time period, the demand corresponding to a set of production jobs becomes known. The set of production jobs in time period k denoted as $\mathcal{J}_k$ is the realization of the random variable Z_k, i.e., $Z_k = |\mathcal{J}_k|$. In Figure 6.1, $|\mathcal{J}_1|$ represents the known demand of the first time period and Z_k denotes the random demand for the future period k. Each production job j in time period k, $j \in \mathcal{J}_k$, can only be processed in time period k, has to be processed on all machines in sequence and has a due date of d_j. The due date of each job is a time point within period k. The processing time of a job on machine m at state i is a random variable denoted as $Y^m(i)$ having the expected value $\frac{1}{r^m(i)}$. We assume that $Y^m(i) = Y^m(0) + (\frac{1}{r^m(i)} - \frac{1}{r^m(0)})$ where $Y^m(0)$ is the random variable representing the processing time of a job on machine m at its best state (or the nominal processing time) and $(\frac{1}{r^m(i)} - \frac{1}{r^m(0)})$ is the increase in the processing time as the machine deteriorates to state i. Since the demand becomes known at the beginning of each time period, the nominal processing times of the jobs, $n_{jm},\ j \in \mathcal{J}_k$, which are the realizations of the random variable $Y^m(0)$, also become known. Therefore, the processing time of job j on machine m at state i, $\mathcal{P}_{jm}(i)$, the realization of the random variable $Y^m(i)$, is then known being equal to $\mathcal{P}_{jm}(i) = n_{jm} + (\frac{1}{r^m(i)} - \frac{1}{r^m(0)})$. If the processing of production job j on the last machine in sequence, M, is not completed before its due date, the production job is lost at cost of h.

We denote the state of the system at the beginning of time period k as $\mathcal{X}_k = (i^1_k, \ldots, i^M_k, |\mathcal{J}_k|)$ which consists of machine states, $(i^1_k, \ldots, i^M_k)$, and the number of customer orders, $|\mathcal{J}_k|$. We further define $\mathcal{Y}_k = (y^1_k, \ldots, y^M_k, st_{jm}, st_{pm})$ to represent the maintenance and production scheduling decisions in period k where y^m_k determines if machine m is maintained at period k or not, st_{jm} is the start-time of job $j \in \mathcal{J}_k$ on machine $m \in \{1, \ldots, M\}$, and st_{pm} is the start-time of maintenance operation on machine m, if maintained in period k, i.e., $y^m_k = 1$.[1] Given the initial system state, $\mathcal{X}_1$, the goal of the problem is to find the maintenance and production scheduling decisions in each period, $\mathcal{Y}_k,\ \forall k \in K$, such that the total expected discounted cost is minimized over an infinite horizon, i.e., when $K \to \infty$. Thus, the objective

[1] If machine m is maintained at time period k ($y^m_k = 1$) and st_{pm} is the start-time of maintenance operation, it means that $a^m_{st_{pm}} = 1$ and $a^m_t = 0,\ \forall t\ (0 \leq t \leq T, t \neq st_{pm})$. In case $y^m_k = 0$, then $a^m_t = 0,\ \forall t\ (0 \leq t \leq T)$.

function is:

$$\min_{\mathcal{Y}_1,\mathcal{Y}_2,\ldots} E_{\mathbb{X}}[\sum_{k=1}^{\infty} C(\mathcal{X}_k,\mathcal{Y}_k)\rho^{k-1}|\mathcal{X}_1], \tag{6.1}$$

where ρ is the discount factor and $C(\mathcal{X}_k,\mathcal{Y}_k)$ is the total maintenance and lost production cost of period k given state $\mathcal{X}_k$ and the decision $\mathcal{Y}_k$ taken. Note that the expected value is calculated over the system state space $\mathbb{X} = \{0,\ldots,N_1\}\times\ldots\times\{0,\ldots,N_M\}\times\mathbb{Z}^+$ where $\mathbb{Z}^+$ is the set of non-negative integers.

To define the total maintenance and lost production cost of period k, we define the following extra decision variables:

u_j	$u_j = 1$ iff job j is lost.
x_{jim}	$x_{jim} = 1$ iff job j is processed before job i on machine m.
b_{jm}	$b_{jm} = 1$ iff job j is processed before preventive maintenance on machine m.

Table 6.1: Extra decision variables for maintenance/production scheduling in period k.

Therefore, the total maintenance and lost production cost in period k equals:

$$C(\mathcal{X}_k,\mathcal{Y}_k) = \sum_{m=1}^{M} E_{X^m_{st_{pm}}}[\tau^m(X^m_{st_{pm}})]y^m_k + h\sum_{j\in\mathcal{J}_k} u_j,$$

where the first term is the expected maintenance cost and the second term is the lost production cost of period k. Since the state of machine m at time of performing maintenance, i.e., st_{pm}, is random, the expected value is calculated over $X^m_{st_{pm}}$.

The problem at period k is subject to maintenance planning and maintenance/production scheduling constraints, each is defined below.

Maintenance planning constraint: Constraint (6.2) enforces the maintenance capacity limit, C, denoting the maximum number of machines that can be maintained in period k.

$$\sum_{m=1}^{M} y^m_k \le C. \tag{6.2}$$

Maintenance/production scheduling constraints: As already mentioned, the processing time of job j on machine m is dependent on machine m's state. Since the state of machine m at time st_{jm} is random and several transitions might also happen within the processing time of the job, the processing time of job j on machine m is random. Therefore the expected processing time of job j on machine m is defined as $\mathcal{P}^e_{jm}(st_{jm}, n_{jm})$ which is a function of its start-time and its nominal processing time. The details of calculating the expected processing times are provided in Section 6.3.2.1.

The detailed descriptions of the maintenance/production scheduling constraints in period k for each realization of the demand, $\mathcal{J}_k$, shown in Figure 6.2, are provided below.

- Constraints (6.3) enforce the precedence constraints: the job should be finished on an upstream machine before its processing starts on downstream machines.

$$st_{jm} + \mathcal{P}^e_{jm}(st_{jm}, n_{jm}) \leq st_{j(m+1)}, \qquad \forall j \in \mathcal{J}_k,\ \forall m\ (m \neq M) \tag{6.3}$$
$$st_{pm} + t^m_p + \mathcal{B}(y^m_k - 1) \leq T, \qquad \forall m \tag{6.4}$$
$$st_{jm} + \mathcal{P}^e_{jm}(st_{jm}, n_{jm}) \leq st_{pm} + \mathcal{B}(1 - b_{jm}), \qquad \forall j \in \mathcal{J}_k,\ \forall m \tag{6.5}$$
$$st_{pm} + t^m_p \leq st_{jm} + \mathcal{B}b_{jm}, \qquad \forall j \in \mathcal{J}_k,\ \forall m \tag{6.6}$$
$$1 - b_{jm} \leq y^m_k \qquad \forall j \in \mathcal{J}_k,\ \forall m \tag{6.7}$$
$$st_{jM} + \mathcal{P}^e_{jM}(st_{jM}, n_{jM}) \leq d_j + \mathcal{B}u_j, \qquad \forall j \in \mathcal{J}_k \tag{6.8}$$
$$st_{jm} + \mathcal{P}^e_{jm}(st_{jm}, n_{jm}) \leq st_{im} + \mathcal{B}(1 - x_{jim}), \qquad \forall j, i \in \mathcal{J}_k\ (j > i),\ \forall m \tag{6.9}$$
$$st_{im} + \mathcal{P}^e_{jm}(st_{jm}, n_{jm}) \leq st_{jm} + \mathcal{B}x_{jim}, \qquad \forall j, i \in \mathcal{J}_k\ (j > i),\ \forall m \tag{6.10}$$

Figure 6.2: Maintenance/production scheduling constraints in period k for $Z_k = |\mathcal{J}_k|$.

- Constraints (6.4) ensure that maintenance activities on machines requiring maintenance at time period k, $y^m_k = 1$, are scheduled within the length of the time period where $\mathcal{B}$ is a big value.

- Constraints (6.5), (6.6), and (6.7) define the relationships between the binary decision variables b_{jm} and the maintenance decisions. Respectively, the constraints guarantee that: if a job is processed before maintenance ($b_{jm} = 1$), then its processing is finished before maintenance is started; if a job is processed after maintenance ($b_{jm} = 0$), then maintenance is performed before processing the job is started; if a machine does not require maintenance, $y^m_k = 0$, all jobs are processed before maintenance, $b_{jm} = 1$.

- Since M is the last machine, Constraints (6.8) define whether job j in time period k is lost or not. If a job is not finished before or at its due date, it is then lost.

- Constraints (6.9) and (6.10) are disjunctive constraints ensuring that all jobs on a machine form a total ordering, meaning that no two jobs execute at the same time.

Therefore, the optimization problem can be written as:

$$\min_{\mathcal{Y}_1, \mathcal{Y}_2, \ldots} E_{\mathbb{X}}[\sum_{k=1}^{\infty} C(\mathcal{X}_k, \mathcal{Y}_k)\rho^{k-1} | \mathcal{X}_1]$$
$$\text{s.t. } C(\mathcal{X}_k, \mathcal{Y}_k) = \sum_{m=1}^{M} E_{X^m_{st_{pm}}}[\tau^m(X^m_{st_{pm}})]y^m_k + h\sum_{j \in \mathcal{J}_k} u_j,$$
$$\text{Constraints (6.2) to (6.10)},$$
$$y^m_k, u_j, x_{jim}, b_{jm} \in \{0, 1\}, \qquad \forall j, i \in \mathcal{J}_k,\ \forall m,\ \forall k$$
$$st_{jm}, p_{jm}, st_{pm} \in \mathbb{Z}^+ \cup \{0\}, \qquad \forall j \in \mathcal{J}_k,\ \forall m,\ \forall k$$

Figure 6.3: The optimization model.

Since the deterioration of each machine independently follows a continuous time Markov chain and the demand is also an independent and identically distributed random variable, the above optimization problem is a constrained Markov Decision Process model with infinite countable state and action spaces. Both state and action spaces are prohibitively large and there is no close-form expression for the cost of single period given the state and the action taken. Solving the model in Figure 6.3 as a single MDP is therefore computationally intractable. In the next section, we first decompose the problem and then present two different solution approaches to approximately solve the decomposed problem.

6.2 Decomposing the Problem

At the beginning of each time period, there are two different decisions: assigning maintenance to machines and scheduling maintenance and production jobs. Therefore, we decompose the global problem in Figure 6.3 into two sub-problems: a maintenance planning problem (MPP) and a production scheduling problem (PSP) addressing the former and the latter decisions, respectively.

The decomposition approach is shown in Figure 6.4. The MPP is first solved to determine the maintenance policy which is a decision rule prescribing either performing maintenance or not on a machine given the machine state and the demand. Then, at the beginning of each period, the sequence of the events is as follows: the system state is observed, the maintenance policy is used to determine the machines for maintenance, and the PSP problem is solved to find maintenance and production scheduling decisions for the current period. The planned maintenance and production schedule is executed. The real cost of the period and the new system state are observed and the procedure repeats.

In this section, we define each sub-problem.

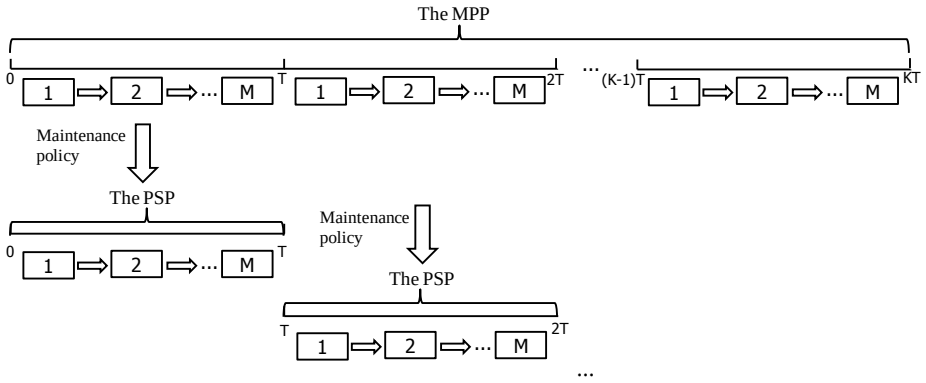


Figure 6.4: Schematic representation of the decomposition approach.

6.2.1 The Maintenance Planning Problem (MPP)

In the maintenance planning problem (MPP), the maintenance/production scheduling problem (PSP) is abstracted. More specifically, the following is assumed:

1. All production jobs on machine $m \in \{1, \ldots, M\}$ are assumed to have the same processing time equal to $\frac{1}{r^m(i_m)}$ where i_m is the state of machine m at the beginning of the period. It is further assumed that all production jobs are due at the end of the time period ($d_j = T$).

2. Maintenance, if any, is performed at the beginning of the time period and it has a negligible duration ($t_p^m = 0$). More specifically, the action performed on machine m at time 0, a_0^m, equals 1 if maintained ($y_k^m = 1$) and equals 0, otherwise ($y_k^m = 0$). Therefore, $y_k^m = a_0^m$.

Although there is negative economic dependency between machines, implying that there is a limit on the number of machines maintained in each time period, in the MPP machines are considered independently. A Markov decision process (MDP) model is used for each machine independently to find the optimal maintenance policy over an infinite horizon such that the total expected discounted cost of maintenance and a lower bound on the lost production is minimized. In the MDP model of each machine, one decision is made at the beginning of each period: whether to maintain the machine or not. To consider the maintenance capacity limit in the MDP, the conditions of the machines should be represented as a vector of size M with its elements being the state of machines which results in $\prod_{m=1}^{M}(N_m + 1)$ different levels of system deterioration. Therefore, solving one MDP to make the maintenance decisions of all machines is computationally intractable due to the size of the state space. In this section, the index of machine, i.e., m, is excluded from the notation for ease of reading.

In the MDP, the following information is required: the production rate of a machine at each state, $r(i)$; the maintenance cost of a machine at each state, $\tau(i)$; the transition probability that a machine changes its state in a time interval with length T; and the demand distribution.

The machine transition probability, $p_{ik}^{a_0}$, represents the probability that the machine is in state k at the beginning of next time period given its current state is i and action a_0 is taken at the beginning of the current period. Since the deterioration process of the machine follows a homogeneous continuous time Markov chain, the transition probabilities do not vary in time. Taking the behavior of deterioration process into account, we have:

$$p_{ik}^0 = \Pr(X_T = k | X_0 = i, a_0 = 0) = \begin{cases} p_{ik} & 0 \le i \le k \le N, \\ 1 & i = N, k = N, \\ 0 & 0 \le k < i \le N, \end{cases}$$

$$\begin{aligned} p_{ik}^1 = \Pr(X_T = k | X_0 = i, a_0 = 1) &= \sum_{l=0}^{N} \Pr(X_{0^+} = l | X_0 = i, a_0 = 1) \cdot \Pr(X_T = k | X_{0^+} = l, a_{0^+} = 0) \\ &= \sum_{l=0}^{N} R_{il} p_{lk}^0, \end{aligned}$$

where p_{ik} is the probability of changing the state from i to k as a result of production within a period of T time units. Using matrix notation, $P^0 = [p_{ik}^0] = e^{QT}$ and $P^1 = [p_{ik}^1] = R \times P^0$ where $R = [R_{ik}]$.

We define the maintenance cost, $\tau(i, a_0)$, and the production rate, $r(i, a_0)$, when machine is in state i and action a_0 is taken as follows.

$$\tau(i,0) = 0,\ \tau(i,1) = \tau(i),$$
$$r(i,0) = r(i),\ r(i,1) = \sum_{l=0}^{N} R_{il} r(l).$$

Assuming the machine is in state i, demand is z, and action a_0 is taken, the total maintenance and lost production cost of a single period, $C(i, z, a_0)$, is defined below where the first and the second terms denote maintenance cost and lost production cost, respectively. The number of products produced by the machine equals $Tr(i, a_0)$, and since in a series manufacturing all demand needs to be produced by each machine, $(z - Tbr(i, a_0))^+ = \max(0, z - Tr(i, a_0))$ denotes the number of lost products.

$$C(i, z, a_0) = \tau(i, a_0) + h(z - Tr(i, a_0))^+.$$

We define $V_n(i, z, a_0)$ as the total discounted expected maintenance and lost production cost when the machine state is i, the demand is z, action a_0 is taken at the beginning of the period, and n time periods are remaining. $V_n(i, z)$, the minimal discounted cost, $V_n(i, z) = \min_{a_0 \in \{0,1\}} (V_n(i, z, a_0))$, can be found by solving the following recursive equation.

$$V_n(i, z) = \min_{a_0 \in \{0,1\}} [C(i, z, a_0) + \rho \sum_{k=0}^{N} p_{ik}^{a_0} \sum_{\delta=0}^{\infty} g(\delta) V_{n-1}(k, \delta)]. \tag{6.11}$$

where $V_0(i, z) = 0,\ \forall i, z$. Recall that ρ is the discount factor and $g(\delta)$ is the probability that the demand in the next period equals δ. For the optimality Equation (6.11), Theorem 6.2.10 in Puterman (1994, p. 154) guarantees that there exists an optimal stationary policy because the state space is countable, the costs are bounded and stationary, i.e., they do not change from one decision point to another, the transition probabilities are stationary, and the action space for each state is finite.

The infinite-horizon equivalent of Equation (6.11) can be written as:

$$V(i, z) = \min_{a_0 \in \{0,1\}} [C(i, z, a_0) + \rho \sum_{k=0}^{N} p_{ik}^{a_0} \sum_{\delta=0}^{\infty} g(\delta) V(k, \delta)]. \tag{6.12}$$

The following Lemma shows that there is a solution to Equation (6.12).

Lemma 6.1. *$V(i, z) = \lim_{n\to\infty} V_n(i, z)$ exists for $\forall i,\ \forall z$.*

Proof. Consider π as a policy and let $W_\pi(i, z)$ denote the expected value of this policy when the initial machine state is i and the demand is z. Based on Theorem 8-13 in Heyman and Sobel (1984), if $W_\pi(i, z) < \infty$ for $\forall i,\ \forall z$, then $V(i, z) = \lim_{n\to\infty} V_n(i, z)$ exists for $\forall i,\ \forall z$. We consider π as a policy where we always maintain the machine. Since single period value function, $V_1(i, z) = C(i, z, 1)$, is bounded and the discount factor, ρ, is less than 1, $W_\pi(i, z) < \infty$ for $\forall i,\ \forall z$ which completes the proof. □

The solution to Equation (6.12) finds the maintenance decision for a given machine state and the demand (see Section 6.3.1).

6.2.2 The Production Scheduling Problem (PSP)

As already mentioned, at the beginning of time period k, the state of each machine and the demand are observed. Let assume that i_m is the state of machine m and $|\mathcal{J}_k|$ is the number of customer orders (demand). The decision rule identified in the MPP is then used for each machine, identifying the set of machines requiring maintenance denoted as Q. Since the maintenance capacity limit is not considered in the MPP, the number of machines requiring maintenance might be more than the maintenance limit, i.e., $|Q| > C$. To adjust the maintenance plan, C machines need to be selected for maintenance. We define the penalty cost $\varphi_m = V(i_m, |\mathcal{J}_k|, 0) - V(i_m, |\mathcal{J}_k|, 1),\ \forall m \in Q$ denoting the cost of deviating from the optimal long-term plan for machine $m \in Q$. The MIP model for the PSP problem in time period k is shown in Figure 6.5 assigning start times to both maintenance and production activities such that the sum of the actual lost production cost and the deviation cost from the optimal long-term plan is minimized.

$$\min h \sum_{j=1}^{|\mathcal{J}_k|} u_j + \sum_{m \in Q} \varphi_m (1 - y_k^m) \tag{6.13}$$

s.t. Constraints (6.3) to (6.10)

$$\sum_{m \in Q} y_k^m = \min(C, |Q|) \tag{6.14}$$

$$b_{jm}, y_k^m \in \{0,1\}, \quad st_{pm} \in \mathbb{Z}^+ \cup \{0\}, \qquad \forall j \in \mathcal{J}_k,\ \forall m \in Q$$

$$u_j, x_{jim} \in \{0,1\}, \quad st_{jm} \in \mathbb{Z}^+ \cup \{0\}, \qquad \forall j, i \in \mathcal{J}_k,\ \forall m$$

Figure 6.5: The PSP model for time period k.

The first term in the objective function (6.13) is the lost production cost for unsatisfied demand where each late job corresponds to an unsatisfied customer order. The second term represents the penalty cost for deviating from the long-term plan. Constraints (6.3) to (6.10) are detailed above in Section 6.1. Constraint (6.14) ensures the maintenance capacity limit.

6.3 Solving the Decomposed Problem

We use two approaches called *MDP-MIP* and *Myopic-EDD* to solve the decomposed problem. In the first approach, the policy improvement algorithm and the mixed integer programming (MIP) are used to solve the MPP and the PSP. In the second approach, the MPP and the PSP are both solved heuristically.

In this section, we discuss solution approaches for solving the MPP and the PSP.

6.3.1 MPP

Many solution approaches are available to solve the dynamic programming models (Heyman and Sobel, 1984; Puterman, 1994), each with specific advantages and disadvantages. To solve the recursive equation (6.12), we use the policy improvement algorithm (Heyman and Sobel, 1984, p 145), which is also applied by Sloan (2004). The policy improvement algorithm and the structure of the optimal maintenance policy in the MPP are discussed in this section. We do not utilize the developed structural properties to design an algorithm for solving the MDP faster because we aim at presenting a framework which is applicable to any Markovian deterioration process. Furthermore, the number of machine states in our experimental study is small (see Section 6.5.1). However developing a crafted algorithm based on the structural properties is a promising direction for decreasing the computational time especially as the size of the state space increases.

In this section, we also present a heuristic approach for finding the maintenance decisions when the information on machine conditions is not available.

6.3.1.1 MDP Approach

The policy improvement is defined in Algorithm 1 (Heyman and Sobel, 1984, p 145).

Algorithm 1 Policy improvement algorithm

(a) Denote the state and the action spaces as $\mathbb{S} = \{0, \dots, N\} \times \mathbb{Z}^+$ and $\mathbb{A} = \{0, 1\}$, respectively.
(b) Let $n = 1$ and choose any stationary policy, $\pi_1 = [\pi_1(i, z)]$. Note that $\pi_1(i, z)$ defines the action that we take at state (i, z).
(c) For each state (i, z), compute the cost function vector for policy π_n: $[W_n(i, z)]$. Note that $W_n(i, z)$ refers to the cost that incurs taking action $\pi_n(i, z)$ at state (i, z).
(d) For each state (i, z), compute the difference between the cost for action $\pi_n(i, z)$ and the minimal cost for actions $\mathbb{A}\backslash\pi_n(i, z)$ as follows:

$$\Delta_n(i, z) = \min_{a_0 \in \{\mathbb{A}\backslash\pi_n(i,z)\}} (C(i, z, a_0) + \rho \sum_{k=0}^{N} p_{ik}^{a_0} \sum_{\delta=0}^{\infty} g(\delta) W_n(k, \delta)) - W_n(i, z),$$

if $\Delta_n(i, z) \geq 0$, then $\pi_n(i, z)$ is the action minimizing the cost, so let $\pi_{n+1}(i, z) = \pi_n(i, z)$. Otherwise, let $\pi_{n+1}(i, z)$ be any action $a_0 \in \mathbb{A}$ where

$$C(i, z, a_0) + \rho \sum_{k=0}^{N} p_{ik}^{a_0} \sum_{\delta=0}^{\infty} g(\delta) W_n(k, \delta) - W_n(i, z) < 0.$$

(e) If $\pi_{n+1}(i, z) = \pi_n(i, z),\ \forall (i, z)$, then π_n is optimal. Otherwise replace $n + 1$ with n and return to (c).

The main result on the structural property of the optimal maintenance policy is stated in Theorem 6.1. The index of machine, m, is omitted from the notation in this section.

Theorem 6.1. *B4, B5, B6, B7 and the following two conditions guarantee that for each $z \in \mathbb{Z}^+$, there exists a threshold state, $\hat{i}_z$, such that the optimal maintenance policy maintains the machine in state (i, z) if $i \geq \hat{i}_z$ and does not maintain if $i < \hat{i}_z$. Furthermore, the threshold state $\hat{i}_z$ is non-increasing in z.*

- *A1: $C(i, z, 0) - C(i, z, 1)$ is non-decreasing in $i,\ \forall z$.*
- *A2: $\sum_{l \leq k \leq i} R_{ik} \geq \sum_{l \leq k \leq i+1} R_{(i+1)k},\ \forall l \leq i$.*

Theorem 6.1 guarantees a switching curve optimal maintenance policy for a machine. An example of a switching curve policy is shown in Figure 6.6. Furthermore, the following are worth mentioning:

- Condition A1 intuitively means that as the machine condition gets worse, the difference between the single period total cost of not-maintaining and maintaining decreases more for a given demand.
- Condition A2 intuitively means that the probability of being worse than a specific state after performing maintenance does not increase as the machine gets worse.
- Conditions A2 and B7 together imply that $R_{ii} = R_{(i+1)i} + R_{(i+1)(i+1)}$ and $R_{ij} = R_{(i+1)j},\ \forall j \leq (i-1)$ in maintenance probability matrix, $R = [R_{ik}]$.

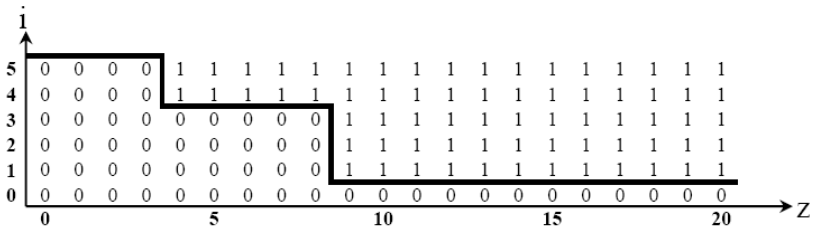


Figure 6.6: An example of a switching curve policy for a machine with six states.

Three maintenance probability matrices are shown in Figure 6.7 where the conditions on maintenance probabilities hold true in R^1 and R^3 matrices while do not hold true in R^2 since $R^2_{11} \neq R^2_{21} + R^2_{22}$ $(0.5 \neq 0.75 + 0)$.

$$R^1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \quad R^2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0.5 & 0.5 & 0 & 0 \\ 0.25 & 0.75 & 0 & 0 \\ 0.25 & 0.25 & 0.5 & 0 \end{pmatrix} \quad R^3 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0.5 & 0.5 & 0 & 0 \\ 0.5 & 0.5 & 0 & 0 \\ 0.5 & 0.5 & 0 & 0 \end{pmatrix}$$

Figure 6.7: Three examples of maintenance probability matrices.

While the previously studied conditions on these type of problems (Sloan and Shanthikumar, 2000; Sloan, 2004, 2008) are on transition probability matrix, $P^{a_0} = [p^{a_0}_{ik}]$, the main difference of our work is that we derive the sufficient conditions on the transition rate matrix, $Q = [q_{ik}]$, and the maintenance probability matrix, $R = [R_{ik}]$, to guarantee the optimal switching curve policy. The conditions on transition rate matrix which guarantee a monotone Markov model are similar to those found in the literature (Keilson and Kester, 1977; Lindqvist, 1987). The conditions on the maintenance probability matrix are however novel where the effect of maintenance on the production is considered uncertain such that maintenance does not make the machine new with probability 1. Moreover, in our problem the demand is a state variable since it is known at the beginning of each period. In the literature the demand becomes known at the end of the period.

The conditions stated in Theorem 6.1 are sufficient conditions to guarantee that an optimal threshold maintenance policy exists. There might be situations where the optimal maintenance policy has a threshold type though some of the conditions of Theorem 6.1 do not hold. For example, assume that the condition A1 does not hold true, meaning that for a given increase in machine deterioration, the decrease in lost production cost is less than the increase in maintenance cost. If the probability of going to a better state after maintenance also increases as the machine deteriorates, i.e., $R_{(i+1)j} > R_{ij},\ \forall j \leq (i-1)$, the extra spending on maintenance might trade off with the benefit of probabilistic improvement in machine conditions in the long term. However, the precise characterization of the situations where the trade-off occurs is hard.

The steps that we take to prove Theorem 6.1 are as follows: first, we prove that B6, B7, and A2 result in three conditions on the transition probability matrix denoted as C1, C2, and C3 which are stated below. These conditions along with B4, B5, and A1 are then used to prove that $V(i,z)$ is non-decreasing in i and in z, and finally we prove the theorem.

- C1: $\Pr(X_t \geq l | X_0 = i, a_0 = 0) \leq \Pr(X_t \geq l | X_0 = i+1, a_0 = 0)$.
- C2: $\Pr(X_t \geq l | X_0 = i, a_0 = 1) \leq \Pr(X_t \geq l | X_0 = i+1, a_0 = 1)$.
- C3: $\Pr(X_t \geq l | X_0 = i, a_0 = 0) - \Pr(X_t \geq l | X_0 = i, a_0 = 1) \leq$
  $\Pr(X_t \geq l | X_0 = i+1, a_0 = 0) - \Pr(X_t \geq l | X_0 = i+1, a_0 = 1)$.

Conditions C1 and C2 indicate that as the machine gets worse, it is more likely to be in a worse state after t units of time regardless of the action taken in the current time 0. Condition C3 means that the likelihood of going to a worse state after performing maintenance decreases more when the machine gets worse.

The three conditions are represented using $p_{ij}^{a_0}$ as below where $t = T$.

- C1: $\sum_{k=l}^{N} p_{ik}^{0} \leq \sum_{k=l}^{N} p_{(i+1)k}^{0}$.
- C2: $\sum_{k=l}^{N} p_{ik}^{1} \leq \sum_{k=l}^{N} p_{(i+1)k}^{1}$.
- C3: $\sum_{k=l}^{N} p_{ik}^{0} - \sum_{k=l}^{N} p_{ik}^{1} \leq \sum_{k=l}^{N} p_{(i+1)k}^{0} - \sum_{k=l}^{N} p_{(i+1)k}^{1}$.

Lemma 6.2 shows that B6, B7, and A2, the conditions on the transition rate matrix and on the maintenance probability matrix, guarantee that C1, C2, and C3, the conditions on the transition probability matrix, hold true.

Lemma 6.2. *B6, B7, and A2 guarantee that C1, C2, and C3 hold true.*

Proof. To prove Lemma 6.2, we take three steps. First, Lemma 6.3 shows that B6 guarantees C1. Second, Lemma 6.4 shows that B6 and B7 guarantee C2. Finally, Lemma 6.5 shows that B6 and A2 guarantee C3 which completes the proof. □

Lemma 6.3. *B6 guarantees C1. That is, if B6:* $\sum_{k \geq l} q_{ik} < \sum_{k \geq l} q_{(i+1)k}, \forall l \geq (i+2)$ *is true, then C1:* $\sum_{k=l}^{N} p_{ik}^{0} \leq \sum_{k=l}^{N} p_{(i+1)k}^{0},\ \forall l$ *holds true.*

Proof. The proof is based on induction.

The *first step* is to show $\Pr(X_\Delta \geq l|X_0 = i, a_0 = 0) \leq \Pr(X_\Delta \geq l|X_0 = i+1, a_0 = 0),\ \forall l$ for a small $\Delta \geq 0$. We discuss the following two cases:

1. If $l \leq i+1$, then $\Pr(X_\Delta \geq l|X_0 = i+1, a_0 = 0) = 1$ and the inequality is obvious.

2. If $l \geq i+2$, then

$$\Pr(X_\Delta \geq l|X_0 = i+1, a_0 = 0) - \Pr(X_\Delta \geq l|X_0 = i, a_0 = 0) =$$
$$\sum_{k\geq l}(q_{(i+1)k}\Delta + o_{(i+1)k}(\Delta)) - \sum_{k\geq l}(q_{ik}\Delta + o_{ik}(\Delta)) =$$
$$\Delta[\sum_{k\geq l} q_{(i+1)k} - \sum_{k\geq l} q_{ik} + \frac{o_{i+1}(\Delta) - o_i(\Delta)}{\Delta}],$$

where $\sum_{k\geq l} o_{(i+1)k}(\Delta) = o_{i+1}(\Delta)$ and $\sum_{k\geq l} o_{ik}(\Delta) = o_i(\Delta)$. Since as $\Delta \to 0$, $\frac{o_{i+1}(\Delta)-o_i(\Delta)}{\Delta} \to 0$ and $\sum_{k\geq l} q_{(i+1)k} - \sum_{k\geq l} q_{ik} > 0$, there exists a small Δ_0 such that $\forall \Delta \leq \Delta_0$, $\Pr(X_\Delta \geq l|X_0 = i+1, a_0 = 0) - \Pr(X_\Delta \geq l|X_0 = i, a_0 = 0) \geq 0$.

We choose $K > 0$ and big such that $\Delta = \frac{t}{K}$ is very small. Therefore, given 1 and 2 we have shown the first step which is

$$\Pr(X_\Delta \geq l|X_0 = i, a_0 = 0) \leq \Pr(X_\Delta \geq l|X_0 = i+1, a_0 = 0),\ \forall l.$$

The *induction assumption* is $\Pr(X_{(j-1)\Delta} \geq l|X_0 = i, a_0 = 0) \leq \Pr(X_{(j-1)\Delta} \geq l|X_0 = i+1, a_0 = 0),\ \forall l$. The *last step* is to show $\Pr(X_{j\Delta} \geq l|X_0 = i, a_0 = 0) \leq \Pr(X_{j\Delta} \geq l|X_0 = i+1, a_0 = 0),\ \forall l$.

We have

$$\Pr(X_{j\Delta} \geq l|X_0 = i, a_0 = 0) = \sum_{k=0}^{N} \Pr(X_\Delta = k|X_0 = i, a_0 = 0) \cdot \Pr(X_{j\Delta} \geq l|X_\Delta = k, a_\Delta = 0),$$
$$\Pr(X_{j\Delta} \geq l|X_0 = i+1, a_0 = 0) = \sum_{k=0}^{N} \Pr(X_\Delta = k|X_0 = i+1, a_0 = 0) \cdot \Pr(X_{j\Delta} \geq l|X_\Delta = k, a_\Delta = 0).$$

Defining $f(k) = \Pr(X_{j\Delta} \geq l|X_\Delta = k, a_\Delta = 0)$ and $\Pr(Y_i = k) = \Pr(X_\Delta = k|X_0 = i, a_0 = 0)$, we have

$$\Pr(X_{j\Delta} \geq l|X_0 = i, a_0 = 0) = E[f(Y_i)],$$
$$\Pr(X_{j\Delta} \geq l|X_0 = i+1, a_0 = 0) = E[f(Y_{i+1})].$$

In the first step, we showed that

$$\Pr(X_\Delta \geq l|X_0 = i, a_0 = 0) \leq \Pr(X_\Delta \geq l|X_0 = i+1, a_0 = 0),\ \forall l.$$

Using the definition of $\Pr(Y_i = k)$, we have

$$\Pr(Y_i \geq l) \leq \Pr(Y_{i+1} \geq l),$$

meaning that Y_i is stochastically smaller than Y_{i+1}, $Y_i \leq_{st} Y_{i+1}$. Since $f(k)$ is non-decreasing in k because of the induction assumption, $E[f(Y_i)] \leq E[f(Y_{i+1})]$.[2] Letting $K = j$ and $t = T$ where $K\Delta = t$, the proof is complete. □

Lemma 6.4. *B6 and B7 guarantee C2. That is, if*

(i) B6: $\sum_{k\geq l} q_{ik} < \sum_{k\geq l} q_{(i+1)k},\ \forall l \geq i+2$, *and*

(ii) B7: $\sum_{k\leq l} R_{ik} \geq \sum_{k\leq l} R_{(i+1)k},\ \forall l$ *are true,*

then C2: $\sum_{k=l}^{N} p^1_{ik} \leq \sum_{k=l}^{N} p^1_{(i+1)k},\ \forall l$ *holds true.*

Proof. We have

$$\begin{aligned}
\Pr(X_t \geq l | X_0 = i, a_0 = 1) &= \sum_{k=0}^{N} \Pr(X_{0^+} = k | X_0 = i, a_0 = 1) \cdot \Pr(X_t \geq l | X_{0^+} = k, a_{0^+} = 0) \\
&= \sum_{k=0}^{N} R_{ik} \Pr(X_t \geq l | X_{0^+} = k, a_{0^+} = 0) = E[f(Y_i)], \\
\Pr(X_t \geq l | X_0 = i+1, a_0 = 1) &= \sum_{k=0}^{N} \Pr(X_{0^+} = k | X_0 = i+1, a_0 = 1) \cdot \Pr(X_t \geq l | X_{0^+} = k, a_{0^+} = 0) \\
&= \sum_{k=0}^{N} R_{(i+1)k} \Pr(X_t \geq l | X_{0^+} = k, a_{0^+} = 0) = E[f(X_{i+1})],
\end{aligned}$$

where $R_{ik} = \Pr(Y_i = k) = \Pr(X_{0^+} = k | X_0 = i, a_0 = 1)$ and $f(k) = \Pr(X_t \geq l | X_{0^+} = k, a_{0^+} = 0)$. Since B7 indicates that $\Pr(Y_i \leq l) \geq \Pr(Y_{i+1} \leq l)$, we conclude that $Y_i \leq_{st} Y_{i+1}$. Furthermore, in the proof of Lemma 6.3, it is shown that $f(k)$ is non-decreasing in k. Therefore, $E[f(Y_i)] \leq E[f(Y_{i+1})]$. Letting $t = T$, the proof is complete. □

Lemma 6.5. *B6 and A2 guarantee C3. That is, if*

(i) B6: $\sum_{k\geq l} q_{ik} < \sum_{k\geq l} q_{(i+1)k},\ \forall l \geq i+2$, *and*

(ii) A2: $\sum_{l\leq k\leq i} R_{ik} \geq \sum_{l\leq k\leq i+1} R_{(i+1)k},\ \forall l \leq i$ *are true,*

then C3: $\sum_{k=l}^{N} p^0_{ik} - \sum_{k=l}^{N} p^1_{ik} \leq \sum_{k=l}^{N} p^0_{(i+1)k} - \sum_{k=l}^{N} p^1_{(i+1)k},\ \forall l$ *holds true.*

[2] A real random variable, A, is stochastically smaller than a random variable, B, if and only if for all non-decreasing functions u, $E[u(A)] \leq E[u(B)]$.

Proof. The proof is based on induction.

The *first step* is to show that:

$$\Pr(X_\Delta \geq l|X_0 = i, a_0 = 0) - \Pr(X_\Delta \geq l|X_0 = i, a_0 = 1) \leq$$
$$\Pr(X_\Delta \geq l|X_0 = i+1, a_0 = 0) - \Pr(X_\Delta \geq l|X_0 = i+1, a_0 = 1),\ \forall l.$$

To do so, we first need to prove

$$\Pr(X_{0^+} \geq l|X_0 = i, a_0 = 0) - \Pr(X_{0^+} \geq l|X_0 = i, a_0 = 1) \leq$$
$$\Pr(X_{0^+} \geq l|X_0 = i+1, a_0 = 0) - \Pr(X_{0^+} \geq l|X_0 = i+1, a_0 = 1),\ \forall l. \tag{6.15}$$

Let us discuss the following two cases:

1. if $l \geq i+1$, then the inequality (6.15) is obvious as $0 - 0 \leq 1 - c$ where $0 \leq c \leq 1$.

2. if $l \leq i$, based on A2, we have

$$\Pr(X_{0^+} \geq l|X_0 = i, a_0 = 1) \geq \Pr(X_{0^+} \geq l|X_0 = i+1, a_0 = 1).$$

By adding $\Pr(X_{0^+} \geq l|X_0 = i, a_0 = 0)$ to the above inequality, we have:

$$\Pr(X_{0^+} \geq l|X_0 = i, a_0 = 0) - \Pr(X_{0^+} \geq l|X_0 = i, a_0 = 1) \leq$$
$$\Pr(X_{0^+} \geq l|X_0 = i, a_0 = 0) - \Pr(X_{0^+} \geq l|X_0 = i+1, a_0 = 1). \tag{6.16}$$

In the proof of Lemma 6.3, we have shown $\Pr(X_{0^+} \geq l|X_0 = i, a_0 = 0) \leq \Pr(X_{0^+} \geq l|X_0 = i+1, a_0 = 0)$; therefore, we can write (6.16) as:

$$\Pr(X_{0^+} \geq l|X_0 = i, a = 0) - \Pr(X_{0^+} \geq l|X_0 = i, a = 1) \leq$$
$$\Pr(X_{0^+} \geq l|X_0 = i+1, a = 0) - \Pr(X_{0^+} \geq l|X_0 = i+1, a = 1).$$

Given 1 and 2, we have shown that the inequality (6.15) holds true. Then, we have

$$\begin{aligned}
&\Pr(X_\Delta \geq l|X_0 = i, a_0 = 0) - \Pr(X_\Delta \geq l|X_0 = i, a_0 = 1) \\
&= \sum_{k=0}^{N} \Pr(X_{0^+} = k|X_0 = i, a_0 = 0) \cdot \Pr(X_\Delta \geq l|X_{0^+} = k, a_{0^+} = 0) \\
&- \sum_{k=0}^{N} \Pr(X_{0^+} = k|X_0 = i, a_0 = 1) \cdot \Pr(X_\Delta \geq l|X_{0^+} = k, a_{0^+} = 0).
\end{aligned}$$

Defining $f(k) = \Pr(X_\Delta \geq l | X_{0^+} = k, a_{0^+} = 0)$, $\Pr(Y_i = k) = \Pr(X_{0^+} = k | X_0 = i, a_0 = 0)$, and $\Pr(Z_i = k) = \Pr(X_{0^+} = k | X_0 = i, a_0 = 1)$, we have

$$\Pr(X_\Delta \geq l | X_0 = i, a_0 = 0) - \Pr(X_\Delta \geq l | X_0 = i, a_0 = 1) = E[f(Y_i)] - E[f(Z_i)].$$

Following the same as above, we have

$$\Pr(X_\Delta \geq l | X_0 = i+1, a_0 = 0) - \Pr(X_\Delta \geq l | X_0 = i+1, a_0 = 1) = E[f(Y_{i+1})] - E[f(Z_{i+1})].$$

Knowing that $E[f(Y)] = f(0) + \sum_{j\geq 1}(f(j) - f(j-1)) \cdot \Pr(Y \geq j)$, we then have

$$E[f(Y_i)] - E[f(Z_i)] = \sum_{j\geq 1}(f(j) - f(j-1)) \cdot (\Pr(Y_i \geq j) - \Pr(Z_i \geq j)),$$
$$E[f(Y_{i+1})] - E[f(Z_{i+1})] = \sum_{j\geq 1}(f(j) - f(j-1)) \cdot (\Pr(Y_{i+1} \geq j) - \Pr(Z_{i+1} \geq j)).$$

Based on inequality (6.15), we have

$$\Pr(Y_i \geq j) - \Pr(Z_i \geq j) \leq \Pr(Y_{i+1} \geq j) - \Pr(Z_{i+1} \geq j).$$

Considering that f is non-decreasing based on the proof of Lemma 6.3, we have

$$E[f(Y_i)] - E[f(Z_i)] \leq E[f(Y_{i+1})] - E[f(Z_{i+1})].$$

which completes the proof of the first step. Let us choose $j > 0$ and big such that $\Delta = \frac{T}{j}$ is small enough. The *induction assumption* is $\Pr(X_{(j-1)\Delta} \geq l | X_0 = i, a_0 = 0) - \Pr(X_{(j-1)\Delta} \geq l | X_0 = i, a_0 = 1) \leq \Pr(X_{(j-1)\Delta} \geq l | X_0 = i+1, a_0 = 0) - \Pr(X_{(j-1)\Delta} \geq l | X_0 = i+1, a_0 = 1),\ \forall l$. The *last step* is to show $\Pr(X_{j\Delta} \geq l | X_0 = i, a_0 = 0) - \Pr(X_{j\Delta} \geq l | X_0 = i, a_0 = 1) \leq \Pr(X_{j\Delta} \geq l | X_0 = i+1, a_0 = 0) - \Pr(X_{j\Delta} \geq l | X_0 = i+1, a_0 = 1),\ \forall l$ where $T = j\Delta$.

The proof of the last step is similar to the first step where Δ and $j\Delta$ replace 0^+ and Δ, respectively. □

In the following, we use three conditions C1, C2, and C3 with B4 and B5 to show that the value function $V(i, z)$ is non-decreasing in i and in z.

Lemma 6.6. *$V(i, z)$ is non-decreasing in i for $\forall z \in \mathbb{Z}^+$.*

Proof. We use the induction to show the lemma. We first show that the single period value function, $V_1(i, z)$ is non-decreasing in i for $\forall z$.

Based on B5, the maintenance cost $\tau(i, a_0)$ is non-decreasing in i for $\forall a_0 \in \{0, 1\}$. B4 ensures that $r(i, 0) = r(i)$ is non-increasing in i. B4 and B7 also guarantee that $r(i, 1) = \sum_{k=0}^{N} R_{ik} r(k)$ is non-increasing in i (the proof is exactly the same as Lemma 6.4 with the only difference that $f(k) = r(k)$ is non-increasing in k, consequently $r(i, 1)$ is non-increasing). Therefore, lost production cost, $h(z - Tr(i, a_0))^+$

is non-decreasing in i for $\forall a_0,\ \forall z$. Finally, $C(i,z,a_0)$ as the sum of maintenance and lost production cost is non-decreasing in i for $\forall a_0,\ \forall z$ which concludes that the single period value function $V_1(i,z)$ is non-decreasing in i for $\forall z$.

The induction assumption is that $V_{(n-1)}(i,z)$ is non-decreasing in i. Based on C1, C2 and the induction assumption, $\sum_{k=0}^{N} p_{ik}^{a_0} \sum_{\delta=0}^{\infty} g(\delta) V_{(n-1)}(k,\delta)$ is non-decreasing in i (the proof is exactly the same as Lemma 6.4 where $R_{ik} = p_{ik}^{a_0}$ and $f(k) = \sum_{\delta=0}^{\infty} g(\delta) V_{(n-1)}(k,\delta)$.). Since we already showed that $C(i,z,a_0)$ is non-decreasing in i, $V_n(i,z) = \min_{a_0 \in \{0,1\}} [C(i,z,a_0) + \sum_{k=0}^{N} p_{ik}^{a_0} \sum_{\delta=0}^{\infty} g(\delta) V_{(n-1)}(k,\delta)]$ is therefore non-decreasing in i. By Lemma 6.1, we have $V(i,z) = \lim_{n\to\infty} V_n(i,z)$. Thus, $V(i,z)$ is non-decreasing in i which completes the proof. □

Lemma 6.7. *$V(i,z)$ is non-decreasing in z for $\forall i \in \{0,\dots,N\}$.*

Proof. We have $V(i,z) = \min_{a_0 \in \{0,1\}} (\tau(i,a_0) + h(z - Tr(i,a_0))^+ + \sum_{k=0}^{N} p_{ik}^{a_0} \sum_{\delta=0}^{\infty} g(\delta) V(k,\delta))$. Since the single period cost is only a function of the current demand z, it is obvious that $V(i,z)$ is non-decreasing in z for $\forall i$. □

Finally, we prove Theorem 6.1.

Proof of Theorem 6.1:

Proof. Assume that the optimal action in state $(\hat{i}_z, z)$ is $a_0 = 1$; therefore, we have

$$V(\hat{i}_z,z,0) \geq V(\hat{i}_z,z,1) \Leftrightarrow C(\hat{i}_z,z,0) + \rho \sum_{k=0}^{N} p_{\hat{i}_z k}^{0} \sum_{\delta=0}^{\infty} g(\delta) V(k,\delta) - C(\hat{i}_z,z,1) - \rho \sum_{k=0}^{N} p_{\hat{i}_z k}^{1} \sum_{\delta=0}^{\infty} g(\delta) V(k,\delta) \geq 0. \tag{6.17}$$

Based on A1: $C(\hat{i}_z,z,0) - C(\hat{i}_z,z,1) \leq C(\hat{i}_z+1,z,0) - C(\hat{i}_z+1,z,1)$. (6.18)

Based on C3: $\sum_{k=0}^{N} p_{\hat{i}_z k}^{0} - \sum_{k=0}^{N} p_{\hat{i}_z k}^{1} \leq \sum_{k=0}^{N} p_{(\hat{i}_z+1)k}^{0} - \sum_{k=0}^{N} p_{(\hat{i}_z+1)k}^{1}$.

Using the same reasoning as Lemma 6.4 where $R_{\hat{i}_z k} = p_{\hat{i}_z k}^{0} - p_{\hat{i}_z k}^{1}$ and $f(k) = \sum_{\delta=0}^{\infty} g(\delta) V(k,\delta)$, we have

$$\sum_{k=0}^{N} (p_{\hat{i}_z k}^{0} - p_{\hat{i}_z k}^{1}) \sum_{\delta=0}^{\infty} g(\delta) V(k,\delta) - \leq \sum_{k=0}^{N} (p_{(\hat{i}_z+1)k}^{0} - p_{(\hat{i}_z+1)k}^{1}) \sum_{\delta=0}^{\infty} g(\delta) V(k,\delta).$$

By summing the above inequality and inequality (6.18), we have

$$\begin{aligned} &V(\hat{i}_z+1,z,0) - V(\hat{i}_z+1,z,1) \\ &= C(\hat{i}_z+1,z,0) - C(\hat{i}_z+1,z,1) + \rho \sum_{k=0}^{N} p_{(\hat{i}_z+1)k}^{0} \sum_{\delta=0}^{\infty} g(\delta) V(k,\delta) - \rho \sum_{k=0}^{N} p_{(\hat{i}_z+1)k}^{1} \sum_{\delta=0}^{\infty} g(\delta) V(k,\delta) \end{aligned}$$

$$\geq C(\hat{i}_z, z, 0) - C(\hat{i}_z, z, 1) + \rho \sum_{k=0}^{N} p^0_{\hat{i}_z k} \sum_{\delta=0}^{\infty} g(\delta) V(k, \delta) - \rho \sum_{k=0}^{N} p^1_{\hat{i}_z k} \sum_{\delta=0}^{\infty} V(k, \delta) \geq 0.$$

The last inequality follows from (6.17). Since $V(\hat{i}_z + 1, z, 0) - V(\hat{i}_z + 1, z, 1) \geq 0$, the optimal action in state $(\hat{i}_z + 1, z)$ is $a_0 = 1$. Therefore, we proved that for $\forall z$, there is a threshold state $\hat{i}_z$ such that in all states (i, z) where $i \geq \hat{i}_z$, the optimal action is $a_0 = 1$.

Similarly, we can show that for $\forall i$, there is a threshold demand $\hat{z}_i$ where it is optimal to maintain the machine in state (i, z) if $z \geq \hat{z}_i$ and not to maintain if $z < \hat{z}_i$. Let assume that the optimal action in state $(i, \hat{z}_i)$ is $a_0 = 1$, we therefore have:

$$V(i, \hat{z}_i, 0) - V(i, \hat{z}_i, 1) = C(i, \hat{z}_i, 0) - C(i, \hat{z}_i, 1) = h(\hat{z}_i - Tr(i, 0))^+ - h(\hat{z}_i - Tr(i, 1))^+ - \tau(i, 1) \geq 0.$$

By discussion on all possibilities, we can show that $C(i, \hat{z}_i + 1, 0) - C(i, \hat{z}_i + 1, 1) \geq C(i, \hat{z}_i, 0) - C(i, \hat{z}_i, 1)$. We then have:

$$V(i, \hat{z}_i + 1, 0) - V(i, \hat{z}_i + 1, 1) = C(i, \hat{z}_i + 1, 0) - C(i, \hat{z}_i + 1, 1) \geq C(i, \hat{z}_i, 0) - C(i, \hat{z}_i, 1) \geq 0,$$

therefore, the optimal action in state $(i, \hat{z}_i + 1)$ is $a_0 = 1$ proving the existence of the threshold demand $\hat{z}_i$ for $\forall i$.

We showed that: (i) for $\forall z$, there is $\hat{i}_z$ where the optimal action in state (i, z) with $i \geq \hat{i}_z$ is to maintain the machine and (ii) for $\forall i$, there is $\hat{z}_i$ where the optimal action in state (i, z) with $z \geq \hat{z}_i$ is to maintain the machine.

Let assume that $\hat{i}_z$ and $\hat{i}_{z+1}$ are the threshold states for z and $z + 1$, respectively. If $\hat{i}_{z+1} \geq \hat{i}_z$, then the optimal action in state $(\hat{i}_{z+1}, z)$ is to maintain the machine which contradicts our assumption that $\hat{i}_{z+1}$ is the threshold state for $z + 1$. Therefore, we can conclude that $\hat{i}_z$ is non-increasing in the demand z. □

6.3.1.2 Heuristic Approach

In the heuristic approach, we use a myopic policy to solve the MPP where machines are maintained only if they are in state N_m (i.e., the failed state where there is no production) at the beginning of the period.

6.3.2 PSP

We present three approaches to solve the PSP including mixed integer programming (MIP), constraint programming (CP), and a dispatch rule.

6.3.2.1 MIP Approach

As noted in Section 6.1, the processing time of job $j \in \mathcal{J}_k$ on machine m is random with the expected value $\mathcal{P}^e_{jm}(st_{jm}, n_{jm})$ which is a function of its start-time and its nominal processing time. Calculating this expected value is analytically intractable because (i) the probability that the machine is in a specific state depends on the time of performing maintenance and the job's start time which are both decision

variables, and (ii) several transitions might happen during the processing of the job with their probabilities dependent on time. We therefore approximate $\mathcal{P}^e_{jm} = n_{jm} + \frac{1}{A^m(i_m)} - \frac{1}{r^m(0)}$ where $A^m(i_m)$ is the average production rate of machine m during the time period given its state is i_m at the beginning of the period. To approximate the average production rate of a machine, we simply assume that it is the average between the expected production rate of machine m at the beginning and at the end of the period. We distinguish between the following two cases:

- $m \notin Q$: If machine m does not need maintenance, the expected production rate of machine m at the beginning of the time period equals $r^m(i_m)$. Since the state of machine m is known at the beginning of the time period, its production rate is not random and is known. Therefore, its expected value equals $r^m(i_m)$. The expected production rate at the end of the time period equals $\sum_{k=0}^{N_m} p^{m0}_{i_m k} r^m(k)$ where $p^{m0}_{i_m k}$ is the transition probability that machine m changes its state from i_m to k within T units of time given it has not been maintained. Therefore, the average production rate of machine m can easily be calculated as follows:

$$A^m(i_m) = \frac{1}{2}[r^m(i_m) + \sum_{k=0}^{N_m} p^{m0}_{i_m k} r^m(k)], \quad \text{if } m \notin Q. \tag{6.19}$$

- $m \in Q$: If machine m needs maintenance, the probability that the machine is in a given state in the start-time of job j is dependent on both the state of the machine at the beginning of the period and on the time of performing maintenance. Although the start-time of maintenance is a decision variable, to approximate the expected value, we need to make an assumption on maintenance start-time. We make the same assumption as in the MPP, that maintenance is performed at the beginning of the period with a negligible time. Assuming that machine m is instantaneously maintained at the beginning of the time period, its state changes from i_m to k with probability $R^m_{i_m k}$. We then have the same problem as when machine m does not need maintenance with the only difference that the machine's initial state is k. The average production rate can therefore be approximated as follows:

$$A^m(i) = \sum_{k=0}^{N_m} R^m_{i_m k} \frac{1}{2}[r^m(k) + \sum_{j=0}^{N_m} p^{m0}_{kj} r^m(j)], \quad \text{if } m \in Q. \tag{6.20}$$

  As already mentioned, $R^m_{i_m k}$ is the probability that machine m changes its state from i_m to k as a result of maintenance.

We have also developed a method for calculating the exact average production rate of machine m using more rigorous probability analysis in both cases of $m \notin Q$ and $m \in Q$. Note that, in the latter case, we have made the same assumption that maintenance is performed at the beginning of the period with a negligible duration. However, we do not use the exact method in our experimental study because the approximation model's error is very small and the exact method is not computationally efficient as the number of machine states and the length of the scheduling horizon increase. The details and analysis of the exact method are provided in Appendix C.

Replacing $\mathcal{P}^e_{jm}(st_{jm}, n_{jm})$ with its approximation, we rely on the default branch-and-bound search in the IBM ILOG CPLEX 12.3 solver, a state-of-the-art commercial MIP solver, to solve the MIP model in Figure 6.5.

6.3.2.2 CP Approach

To formulate a CP model of the PSP, we define interval decision variables act_{jm} and act_{pm} for the production job j and maintenance operation on machine m. The size of each interval decision variable equals the activity's processing time (Laborie, 2009) which is $\mathcal{P}^e_{jm}(st_{jm}, n_{jm})$ and t^m_p for production and maintenance activities, respectively. The start and the end of the interval variable correspond to the start-time and the end-time of the activity (CP Optimizer, 2011).

We use the same approximation as in the previous section for the expected processing time of job j on machine m. The CP model is given in Figure 6.8.

$$
\begin{array}{lll}
\min & \text{Objective (6.13)} & \\
\text{s.t.} & \text{endBeforeStart}(act_{jm}, act_{j(m+1)}) & \forall j \in \mathcal{J}_k,\ \forall m(m \neq M) \quad (6.21)\\
& \text{IfThen}(y^m_k = 1, \text{EndOf}(act_{pm}) \leq T) & \forall m \quad (6.22)\\
& \text{NoOverlap}(act_{1m}, \ldots, act_{|\mathcal{J}_k|m}, act_{pm}) & \forall m \quad (6.23)\\
& \text{IfThen}(\text{EndOf}(act_{jM}) > d_j, u_j = 1) & \forall j \in \mathcal{J}_k \quad (6.24)\\
& \text{Constraint (6.14)} & \\
& u_j \in \{0,1\},\ \text{IntervalVar}\ act_{jm} & \forall j \in \mathcal{J}_k,\ \forall m\\
& y^m_k \in \{0,1\},\ \text{IntervalVar}\ act_{pm} & \forall m \in Q
\end{array}
$$

Figure 6.8: The CP model of the PSP for time period k.

The details of the CP model are summarized as follows:

- Constraints (6.21) ensure the precedence constraints and are equivalent to Constraints (6.3).
- Constraints (6.22), like Constraints (6.4), guarantee that if machine m is maintained, $y^m_k = 1$, its maintenance activity is scheduled within the period, $\text{EndOf}(act_{pm}) \leq T$.
- Constraints (6.23) are the equivalent of Constraints (6.5), (6.6), (6.9) and (6.10) ensuring that all activities, including production and maintenance, on a machine form a total ordering, meaning that no two activities execute at the same time.
- Constraints (6.24) define whether job j is lost or not, similar to Constraints (6.8).

We use the default search of IBM ILOG CP Optimizer 12.3 to solve the problem. Our early experimentation on 60 problem instances[3] with time limit of 600 seconds showed that the average run-time is

[3]In the problem instances, the number of machines is set at $\{3, 4, 5, 6\}$, the number of jobs is chosen from three uniform distributions $[4, 6]$, $[8, 12]$, and $[12, 18]$. Five instances for each combination of the number of machines and the number of jobs are generated resulting into 60 problem instances.

252.4 (sec) for MIP and is 530.04 (sec) for CP, that the number of instances solved to optimality is 38 for MIP and is 7 for CP, and that the number of best found solutions by the time limit is 58 for MIP and is 59 for CP. Since MIP outperforms CP in terms of the run-time, we do not use the CP model further in our experimental study.

6.3.2.3 Heuristic Approach

To solve the PSP heuristically, the dispatching policy Earliest Due Date (EDD) is used where the customer orders are processed in non-decreasing order of their due dates on machines. The EDD dispatching rule minimizes the maximum lateness in a single machine problem when all jobs are available at time zero (Pinedo, 2005).

6.4 Execution of the Planned Schedule

In both MDP-MIP and Myopic-EDD approaches, we solve the problem in real time, therefore, to compare the performance of the two solution approaches, we estimate the total discounted expected cost through simulation. After the maintenance plan and the production/maintenance schedule are determined at the beginning of each period, the schedule is executed and the real cost of the period is observed. Given the start-times assigned to both production jobs and the maintenance job on each machine in the PSP, we first determine the sequence of the jobs on each machine. We then start from the first machine, iterate through the jobs processing each at the earliest available time.

The production rates of machines, the processing times of the jobs, the maintenance cost and the effect of maintenance on machines are dependent on machine states. We therefore simulate the state of each machine at each time point during a period, i.e., $X_t^m,\ \forall m,\ \forall t\ (0 \leq t \leq T)$. To simulate the states of machines at every time point, we need to simulate the next time that the machine leaves its current state, called *transition time*, and the new state that the machine transitions into. The state of machine m at the beginning of the period is known as i_m. Since each machine deterioration process follows a continuous time Markov chain, the time that machine m leaves its state, t_n, has an exponential distribution with parameter $-q^m_{i_m i_m}$ and with probability density function $h(t|i_m)$. Furthermore, the probability that the machine transitions into state j after leaving state i_m equals $\frac{q_{i_m j}}{-q_{i_m i_m}}$ (Ross, 2010, p.384). Therefore, the state that machine m transitions into after leaving its current state is a random variable, $X^m_{t_n}$, with probability mass function, $\varphi(j|i_m) = \Pr(X_{t_n} = j|i_m) = \frac{q_{i_m j}}{-q_{i_m i_m}}$. The pseudocode for simulating the state of machine m during a period is given in Algorithm 2.

Knowing the state of each machine at each time point, we can simulate the processing times of the production jobs to find the completion time of each job on each machine. Algorithm 3 shows the pseudocode for simulating the execution of production job j on machine m started at time t given the next transition time of machine m is t_n. Recall that n_{jm} denotes the processing time of job j on machine m in its best state. We further define inc_{jm} denoting the increase in the processing time which is dependent on the machine states. Note that $\{x|y\}$ in Algorithm 3 denotes that the remaining processing time of the job is x if the machine is in state y.

Algorithm 2 Simulating states of machine m at each time point within a time period.

current state $\leftarrow i_m$
current time $\leftarrow 0$
next transition time (t_n) $\leftarrow$ current time + generate a random number with probability density function $h(t|\text{current state})$
$X_t^m = i_m, \ \forall \ 0 \leq t \leq t_n$
while $t_n < T$ **do**
  current time $\leftarrow t_n$
  $X_{t_n}^m \leftarrow$ generate a random number with probability mass function $\varphi(j|\text{current state})$
  current state $\leftarrow X_{t_n}^m$
  next transition time (t_n) $\leftarrow$ current time + generate a random number with probability density function $h(t|\text{current state})$
  $X_t^m = X_{t_n}^m, \ \forall$ current time $< t \leq t_n$
end while

Algorithm 3 Simulating execution of production job j on machine m started at time t.

current time $\leftarrow t$
current state $\leftarrow X_t^m$
$inc_{jm} = \frac{1}{r^m(X_t^m)} - \frac{1}{r^m(0)}$
remaining processing time $\leftarrow \{n_{jm}|0\} + inc_{jm}$
completion time $\leftarrow$ current time + remaining processing time
next transition time (t_n) $\leftarrow$ given by Algorithm 2
while completion time $> t_n$ **do**
  the state of machine at next transition time ($X_{t_n}^m$) $\leftarrow$ given by Algorithm 2
  $inc_{jm} = \frac{1}{r^m(X_{t_n}^m)} - \frac{1}{r^m(\text{current state})}$
  remaining processing time $\leftarrow \{(\text{completion time} - t_n)|\ \text{current state}\} + inc_{jm}$
  current time $\leftarrow t_n$
  current state $\leftarrow X_{t_n}^m$
  completion time $\leftarrow$ current time + remaining processing time
  next transition time (t_n) $\leftarrow$ given by Algorithm 2
end while
completion time of job j on machine $m \leftarrow$ completion time

If the completion time of job j on machine m returned by Algorithm 3 exceeds its due date, d_j, it is not executed on the downstream machines and is added to the list of late jobs.

Algorithm 4 defines the pseudocode for simulating the execution of a maintenance job at time t on machine m where the maintenance cost up to time t is denoted as C. When maintenance is performed, we need to simulate the state that the machine transitions into. Recall that R_{ij}^m is the probability that machine m changes its state from i to j after maintenance, therefore the state that machine m transitions into is a random variable with probability mass function $\xi(j|i) = R_{ij}^m$.

After the execution of the jobs on the last machine, M, is finished, the size of the late job list is multiplied by h to determine the lost production cost. The sum of the maintenance cost and the lost production cost defines the total cost of the executed schedule in the period.

The other details of simulating the achieved schedule are summarized below:

Algorithm 4 Simulating execution of maintenance job j on machine m started at time t.

current time $\leftarrow t$
current state $\leftarrow X_t^m$
maintenance cost $\leftarrow C$
maintenance cost $\leftarrow$ maintenance cost $+\tau^m(X_t^m)$
maintenance time $\leftarrow t_p^m$
completion time $\leftarrow$ current time + maintenance time
current time $\leftarrow t + t_p^m$
$X_{t+\eta^m}^m \leftarrow$ generate a random number with probability mass function $\xi(j|\text{current state})$
current state $\leftarrow X_{t+t_p^m}^m$

1. Job j is executed on machine m at its earliest available time considering its precedence constraints. More specifically, production job j cannot be executed on machine m unless its execution on machine $(m-1)$ is finished and machine m is also free.

2. Assume that the current time is t and the next transition time of machine m is t_n. This means that machine m leaves its current state after $(t_n - t)$ units of time processing the production jobs. Therefore, the idleness of machine m, waiting for the jobs to be finished on upstream machines, is not included in the remaining time to next transition from the current state.

6.5 Computational Study

In this section we discuss the results of our computational experiments to investigate the performance of two solution approaches. The next sub-section describes the problem instances and the experimental details. We then compare the performance of the solution approaches.

6.5.1 Experimental Setup

In our problem instances, the number of machines is set at $\{3, 4, 5\}$ where each machine has five states. The demand of each period is generated from the integer uniform distributions $U[4, 6]$ and $U[8, 12]$. Five different deterioration factors are considered numbered from 1 to 5. As the deterioration factor increases, the mean time to failure for machines decreases. Table 6.2 shows the range of the mean time to failure (MTTF) for different deterioration factors. These ranges are chosen to reflect the range of the real mean time to failure of different machines used in real industrial applications (see Section 6.6 for more details). Two instances for each combination of the parameters are generated yielding 60 problem instances.

The length of the time period is set at 50 and at 100 in problem instances with the number of customer orders generated from $U[4, 6]$ and $U[8, 12]$, respectively. The details of the other parameters such as transition rates, maintenance probabilities, production rates, maintenance cost, maintenance duration, nominal processing times and due dates of customer orders are explained in Appendix D.

Deterioration Factor	MTTF
1	$[10^5, 10^7]$
2	$[10^4, 10^6]$
3	$[10^3, 10^5]$
4	$[10^2, 10^4]$
5	$[10, 10^3]$

Table 6.2: The range of mean time to failure (MTTF) for different deterioration factors.

The policy improvement algorithm, heuristic policies, and the simulation are implemented in C++. The MIP formulation of the PSP in the MDP-MIP approach is solved using CPLEX 12.3. In the MDP-MIP approach, the time limit for solving the problem in each time period is 600 seconds. If the optimal solution is not found within the time limit, the best feasible schedule found by the time limit is executed.

6.5.2 Experimental Results

In this section we present our results comparing the performance of the two solution approaches. For each problem instance we compute the following quantities:

1. $C_{\text{MDP-MIP}}$, estimated total discounted cost of maintenance and lost production of the MDP-MIP approach: We simulate each time period using the maintenance plan and the production/maintenance schedule given by the MDP-MIP approach and obtain a value for the maintenance and lost production cost of the period. The number of time periods, K, are chosen such that $\rho^K > 10^{-4}$ where ρ is the discount factor.[4] The total cost of each run equals the discounted sum of the costs over K periods. The total number of simulation runs is set at 20. Finally, $C_{\text{MDP-MIP}}$ equals the average of the discounted costs over the simulation runs.

2. $C_{\text{Myopic-EDD}}$, estimated total discounted cost of maintenance and lost production of the Myopic-EDD approach: It is achieved following the same approach as $C_{\text{MDP-MIP}}$.

The difference between the normalized total discounted costs for each instance is calculated as $\frac{C_{\text{MDP-MIP}} - C_{\text{Myopic-EDD}}}{C_{\text{Myopic-EDD}}}$. Table 6.3 and Figure 6.9 show the mean and the standard deviation of the difference between the normalized total discounted costs for five deterioration factors and four discount factors.[5]

Tables 6.4 and 6.5 show the mean and the standard deviation of the run-time of the PSP problem per period, and the percentage of the periods where the PSP times out in the MDP-MIP approach. The run-times of the MDP and the simulation are not included in the time reported in Tables 6.4 and 6.5 since they are very short, less than a second, and are the same in all periods. It is worth mentioning that the average run-time to find $C_{\text{MDP-MIP}}$ approximately equals the multiplication of the times in Tables 6.4 and 6.5 by K and by the number of simulation runs. The Myopic-EDD heuristic approach finds a solution per-period almost instantaneously.

[4]The number of time periods , K, equals 6, 14, 42, and 180 for discount factors of 0.2, 0.5, 0.8, and 0.95, respectively.

[5]Because of the high computational time to find $C_{\text{MDP-MIP}}$, there are no results for the case where the number of customer orders is generated from $U[8, 12]$ and the discount factor is 0.95. Therefore, the mean and the standard deviation for $\rho = 0.95$ in Table 6.3 and Figure 6.9 are calculated over the demand situation of $U[4, 6]$.

	$\rho = 0.2$		$\rho = 0.5$		$\rho = 0.8$		$\rho = 0.95$	
Deterioration Factor	mean	std	mean	std	mean	std	mean	std
1	-0.03	0.10	0.05	0.07	0.14	0.12	0.23	0.18
2	-0.02	0.11	0.05	0.07	0.16	0.12	0.30	0.16
3	0.02	0.14	0.09	0.08	0.24	0.09	0.42	0.13
4	0.20	0.23	0.26	0.16	0.32	0.07	0.32	0.06
5	0.33	0.15	0.22	0.14	0.1	0.02	0.08	0.04
{1,2,3,4,5}	0.14	0.32	0.22	0.19	0.24	0.14	0.27	0.17

Table 6.3: The mean and the standard deviation (std) of the difference between the normalized total discounted costs.

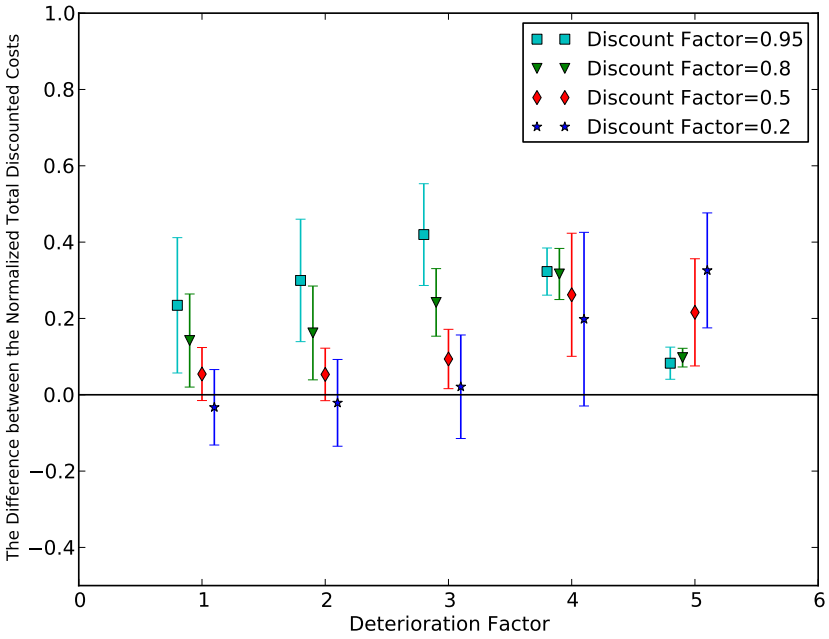


Figure 6.9: The mean and the standard deviation of the difference between the normalized total discounted costs for different deterioration factors and discount factors.

Figure 6.9 indicates the clear superiority of the MDP-MIP approach over the Myopic-EDD approach since the difference between the normalized total discounted costs is positive for the majority of problem instances. As shown in Table 6.3, the MDP-MIP approach decreases the mean of the total discounted cost by 21% over all deterioration factors and discount factors.

We further make the following observations:

- The performance of the MDP-MIP approach increases as the discount factor increases for all deterioration factors except 5. The long-term impact of the per-period decision significantly increases as the discount factor approaches 1. Since the MDP-MIP approach incorporates the long-term effect of the current decisions in the model for determining the optimal maintenance policy, its performance, as expected, improves for higher discount factors.

- At deterioration factor 5 when machines deteriorate very quickly, the mean time between failures

	$\rho = 0.2$						$\rho = 0.5$					
	$U[4,6]$			$U[8,12]$			$U[4,6]$			$U[8,12]$		
	MDP-MIP			MDP-MIP			MDP-MIP			MDP-MIP		
Deterioration Factor	mean	std	timed-out	mean	std	timed-out	mean	std	timed-out	mean	std	timed-out
1	0.04	0.08	0	223.99	262.63	19	0.02	0.01	0	305.18	252.19	26
2	0.04	0.08	0	218.72	259.81	21	0.02	0.01	0	304.88	253.90	27
3	0.04	0.08	0	220.91	262.48	25	0.02	0.02	0	296.69	253.23	27
4	0.04	0.07	0	211.72	260.11	20	0.02	0.02	0	207.76	244.86	19
5	0.02	0.03	0	10.18	58.30	0	0.01	0.01	0	16.45	83.48	2

Table 6.4: The mean and the standard deviation (std) of the PSP run-time (sec) per period and the percentage of timed-out periods in the MDP-MIP approach for $\rho = 0.2$ and $\rho = 0.5$.

	$\rho = 0.8$						$\rho = 0.95$		
	$U[4,6]$			$U[8,12]$			$U[4,6]$		
	MDP-MIP			MDP-MIP			MDP-MIP		
Deterioration Factor	mean	std	timed-out	mean	std	timed-out	mean	std	timed-out
1	0.03	0.05	0	204.06	235.62	16	0.05	0.08	0
2	0.03	0.05	0	196.39	232.49	14	0.05	0.08	0
3	0.03	0.04	0	163.52	224.01	14	0.03	0.06	0
4	0.02	0.02	0	59.58	155.22	5	0.02	0.02	0
5	0.01	0.01	0	0.22	2.65	0	0.01	0.01	0

Table 6.5: The mean and the standard deviation (std) of the PSP run-time (sec) per period and the percentage of timed-out periods in the MDP-MIP approach for $\rho = 0.8$ and $\rho = 0.95$.

in our experimental setup equals 148. Recall that the length of the scheduling horizon is 50 or 100. Therefore, we expect that at the beginning of many periods, machines are in the failed state and both MDP-MIP and Myopic-EDD almost always make the same maintenance decisions, i.e., maintaining the failed machines. A closer look to the data shows that the average per-period maintenance costs for the deterioration factor 5 in the MDP-MIP and in the Myopic-EDD approaches, shown in Tables 6.6 and 6.7, are very close. The performance difference between the algorithms mainly results from using an optimization model in the MDP-MIP for solving the PSP rather than a heuristic dispatch rule. It is worth mentioning that as shown in Figure 6.9, their performance difference due to different production scheduling decisions is significant for lower discount factors.

- Tables 6.6 and 6.7 show that as the deterioration factor increases, the average per-period maintenance cost and lost production cost respectively increases and decreases for the MDP-MIP approach compared to the Myopic-EDD. The only exception is that both per-period costs decrease for the deterioration factor 4 and the discount factors $\rho = 0.8$ and $\rho = 0.95$. We expect that the savings on the lost production cost would be higher than the spending on maintenance cost for medium deterioration factors, i.e., 3 and 4, resulting in a better performance for the MDP-MIP approach. In the extreme low or high deterioration factors, the machines are frequently either in a very good or in a very bad conditions. Therefore, both approaches make similar maintenance decisions, either not performing maintenance or performing maintenance. Furthermore, because

the processing times of the production jobs is either very short or very long in extreme cases, all customer orders are met or lost regardless of using a complete or a heuristic approach for the PSP. Figure 6.9 shows that the performance of the MDP-MIP approach, as expected, improves for medium deterioration factors especially as the discount factor increases. The MDP-MIP has the highest performance at the deterioration factor 3 and the discount factor 0.95.

	$\rho = 0.2$				$\rho = 0.5$			
	MDP-MIP		Myopic-EDD		MDP-MIP		Myopic-EDD	
Deterioration Factor	maintenance	lost	maintenance	lost	maintenance	lost	maintenance	lost
1	10.87	2246.86	0.00	3557.09	3.16	2533.12	0.00	3526.06
2	11.68	2285.50	0.15	3548.72	3.40	2558.53	0.04	3593.70
3	11.68	2339.26	0.97	3783.91	5.79	2724.17	1.08	4185.52
4	20.58	3159.57	12.08	4947.67	23.81	3936.07	26.78	6171.17
5	79.11	5241.55	87.53	5871.12	95.43	6098.96	105.86	6466.76

Table 6.6: The mean per-period maintenance cost (maintenance) and the mean per-period lost production cost (lost) for different approaches, different deterioration factors, and discount factors 0.2 and 0.5.

	$\rho = 0.8$				$\rho = 0.95$			
	MDP-MIP		Myopic-EDD		MDP-MIP		Myopic-EDD	
Deterioration Factor	maintenance	lost	maintenance	lost	maintenance	lost	maintenance	lost
1	2.55	3385.35	0.02	5171.26	1.15	1838.68	0.01	2726.71
2	2.80	3473.41	0.22	5424.99	1.27	1884.99	0.12	2966.59
3	5.33	3884.93	2.84	6269.57	3.91	2310.99	2.87	3705.36
4	29.41	5385.31	68.57	7017.75	26.27	3059.35	104.99	3908.54
5	113.92	6923.48	118.35	7072.82	100.55	3843.79	126.20	3934.89

Table 6.7: The mean per-period maintenance cost (maintenance) and the mean per-period lost production cost (lost) for different approaches, different deterioration factors, and discount factors 0.8 and 0.95.

6.6 Discussion

The experimental results demonstrate that utilizing machine condition information is beneficial particularly for high discount factors and medium deterioration factors. In this section, we first provide a background on the real failure rate data and then discuss the practical relevance of our results.

6.6.1 Real Failure Rate Data

There are several handbooks of real failure rate data for different equipment categories such as EIREDA (European Industry Reliability Data Handbook for Electrical Power Plants), MIL-HDBK 217F (Military Handbook for Electronic Equipment), OREDA (Offshore Reliability Data Handbook), and SRS (System Reliability Service of UK) (Smith, 1985). Given the available data, we have considered four different equipment categories of mechanical, electronic, safety, and semiconductor. The mean time to failure (MTTF) for various components in each category is shown in Table 6.8. The data for Table 6.8 is extracted from the papers by Green (1969) and Wright (1984) and the books by Smith (1985) and

Bently (1999). In all data banks, MTTF is given in hours, however, as already mentioned in Section 5.3, we have defined a time unit correspond to 15 minutes and the data in Table 6.8 is therefore converted to the time unit defined in this dissertation. For example, the lowest MTTF of 1000 time units for generator in Table 6.8 means that its MTTF is 250 hours. Furthermore, Figure 6.10 shows the spread of MTTF for some components (Green, 1969). Carter (1986) provided Table 6.9 summarizing Figure 6.10. For each component, the lowest and the highest MTTF are reported demonstrating the range of values found in various sources. As shown, for some components, the MTTF value range is two orders of magnitude wide indicating that there is no agreement among different data banks. In the reliability literature, it is known that four main factors including quality, temperature, environment, and stress can affect the failure rate value, the inverse of MTTF, by several orders of magnitudes (Smith, 1985; Bently, 1999).

Equipment Category	Component	Lowest MTTF	Highest MTTF
Mechanical	Compressor	1.33×10^4	4×10^4
	Pumps	1.33×10^3	4×10^5
	Valves	10^5	4×10^7
	Pipes	2×10^7	2×10^7
	Filters	1.33×10^5	4×10^6
	Joints	4×10^6	2×10^7
	Turbines	5×10^4	1.33×10^5
	Belts	10^4	10^6
	Heat Exchanger	10^5	4×10^6
Electronic	Generator	10^3	4×10^6
	Computer	5×10^2	2×10^5
	Cables	2.67×10^5	8×10^6
	Lamps	4×10^5	8×10^7
	Printer (line)	5×10^4	4×10^5
	Electricity Supply	3.64×10^4	3.64×10^4
	Motor	1.60×10^5	10^7
	Transformers	10^6	2×10^7
	Switches	6.67×10^5	4×10^8
Safety	Fire Pumps	1×10^4	4×10^5
	Detectors	5.33×10^4	2×10^7
Semiconductor	Diodes	1.33×10^7	4×10^8
	Transistors	4×10^6	1.33×10^8

Table 6.8: MTTF of different components in quarter hour time units (Green, 1969; Wright, 1984; Smith, 1985; Bently, 1999).

As shown in Figure 6.10 and Tables 6.8 and 6.9, the real MTTF for different equipments varies between 4×10 to 4×10^{11} time units. The range of values chosen for deterioration factors in the experiments (Table 6.2) therefore include a wide range of the real MTTF data indicating that our results are practically relevant. In the next section, we summarize the relevance of the results for different industries.

6.6.2 Practical Relevance of the Experimental Results

We summarize our results as follows:

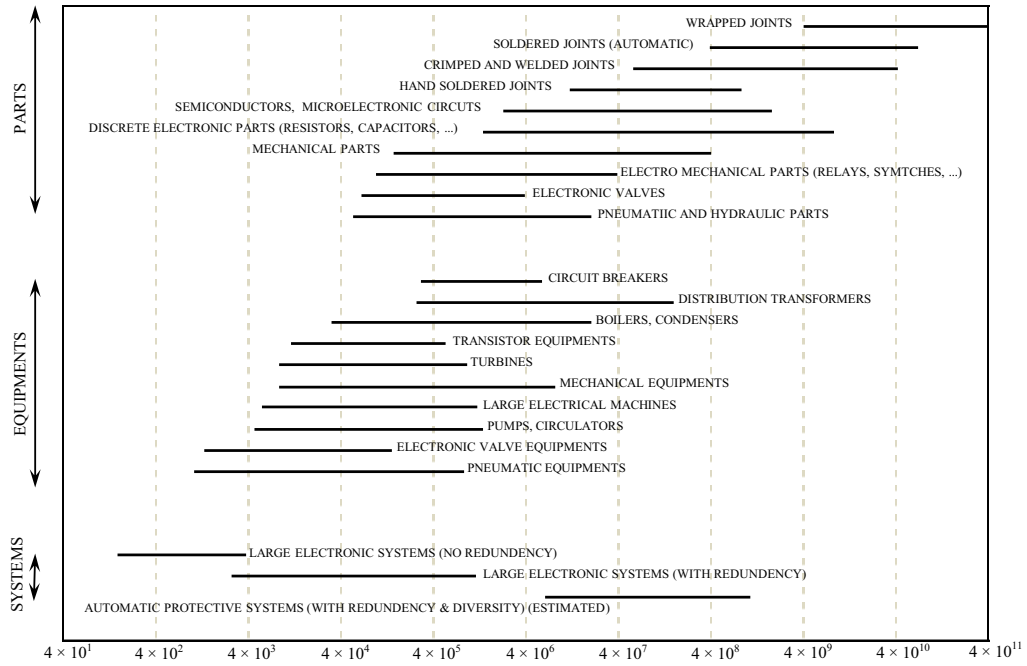


Figure 6.10: MTTF for parts, equipments, and systems in quarter hour time units (Green, 1969).

1. In low failure industries, those that have MTTF of 10^4 to 10^7 time units as seen more frequently in machinery based on for example safety, semi-conductor, circuit breakers, distribution transformers, boilers, and condensers equipments, our results demonstrate that utilizing the online machine condition information in maintenance and production scheduling decisions decreases the mean total discounted cost 13% compared to a greedy heuristic. It is also shown that in low failure industries where mean time to failure of machines is significantly longer than the length of the scheduling horizon, the benefit of using machine condition information increases as the discount factor increases.

2. For industries with medium MTTF of 10^2 to 10^5 time units, those that are mostly based on mechanical, electrical, transistors, turbines, pumps, or circulators equipments, our results demonstrate the highest decrease in the mean total discounted cost, 30% on average, compared to low and high failure industries. In these industries, the frequency that machine conditions change is not very low or very high, the maintenance decisions therefore have a more significant impact on the production scheduling decisions within each period yielding a higher benefit. The superiority of utilizing machine condition information also improves as discount factor approaches 1 since the long-term weight of short-term decisions increases.

3. In high failure industries with MTTF of 10 to 10^3 units of time which are perhaps using some kind of electronic or pneumatic equipment, our results show the decrease of 19% in the total discounted cost. In such industries where machine conditions change very quickly, the current decisions

Item	Lowest MTTF	Highest MTTF
Mechanical components	$4 \times 10^{4.6}$	4×10^{8}
Electro-mechanical components	$4 \times 10^{4.4}$	4×10^{7}
Boilers and condensers	$4 \times 10^{3.9}$	$4 \times 10^{6.8}$
Turbines	$4 \times 10^{3.4}$	$4 \times 10^{6.5}$
Mechanical equipment	$4 \times 10^{3.5}$	$4 \times 10^{6.5}$
Pumps and circulators	4×10^{3}	$4 \times 10^{5.4}$
Pneumatic equipment	$4 \times 10^{2.3}$	$4 \times 10^{6.5}$

Table 6.9: MTTF of different engineering items summarized from Figure 6.10 in quarter hour time units (Carter, 1986).

have a higher impact in the short term and as shown in Figure 6.9, the benefit of incorporating information on machine conditions decreases as the discount factor increases.

6.7 Conclusion

In this chapter, we addressed the interdependency between maintenance and production scheduling in a multi-machine production system where each machine deterioration process is modeled using a continuous time Markov chain. Machine conditions are characterized by a discrete set of states and can be partially controlled, that is, performing maintenance on machines stochastically improves their conditions. At the beginning of a period, the state of each machine is observed and the customer orders (demand) become known. The machines that need maintenance are then determined and a start-time is assigned to each production and maintenance activity within the period. The goal is to minimize the total discounted cost of maintenance and lost production in the long term.

To solve the problem, we decompose the global problem into maintenance planning and production scheduling sub-problems. A Markov decision process model is developed in the maintenance planning sub-problem to determine the maintenance plan for each machine individually where the scheduling combinatorics are abstracted. More specifically, all customer orders are assumed similar and due at the end of the period. After the machines for maintenance are determined using the maintenance plan, a mixed-integer programming model is solved in the production scheduling sub-problem to find the schedule of maintenance and production activities within the period incorporating all scheduling combinatorics. The planned schedule is then executed, the real cost of the period is realized, the new machine states and the customer orders are observed and the same procedure repeats.

We have derived sufficient conditions in the maintenance planning sub-problem guaranteeing that the optimal maintenance plan has a switching curve structure which is monotone in both machine state and the demand.

The computational results demonstrate that utilizing online machine condition information in maintenance and scheduling decisions decreases the total discounted cost on average 21% compared to a greedy heuristic approach. It is also shown that the benefit of incorporating long-term information in making short-term decisions increases for high discount factors and medium failure industries where

the long-term impact of short-term decisions is higher and where the maintenance decisions effect on short-term production scheduling decisions is more significant.

In this chapter, we continued addressing the interdependency between maintenance and production scheduling where there is a partial control over machine conditions and where each machine deterioration is modeled using a stochastic process. In the next chapter, we discuss future directions for studying the interdependency between maintenance and production problems.

Chapter 7

Future Work

In this dissertation, we addressed the interdependency between production and maintenance in the following three areas:

- maintenance and production planning with partial control over machine conditions,
- maintenance planning and production scheduling with no control over machine conditions, and
- maintenance planning and production scheduling with partial control over machine conditions.

In this chapter, we first present future research directions to extend the work in Chapters 3 to 6. We then discuss two general directions for further progress in studying the relationship between maintenance and production in real-world applications. We finally explain the relevance of our results to other integrated decisions in supply chain management and indicate broader areas for future work.

7.1 Maintenance & Production Planning with Partial Control over Machine Conditions

The problem of integrated maintenance and production planning where machines can be maintained before failure focuses on determining the optimal joint production and maintenance policies. In Chapter 3, we studied a simpler combined problem in the context of a periodic review production system assuming that the production policy is fixed. We analyzed the problem, determining the optimal maintenance policy.

In this section, we outline possible extensions of the problem studied in Chapter 3.

7.1.1 Different Assumptions

Non-stationary Demand: As mentioned in Section 3.1, we have assumed that the demand of each period is independent of the other periods. However, in real applications, the demand is affected by prevailing business environment (Papachristos and Katsaros, 2008). For example, the demand of a product could

be dependent on the technology status, society status, consumer wealth, and weather conditions. Assuming a non-stationary demand allows the modeling of period to period variation and a more informed maintenance policy can be, therefore, found. Given the fixed production quantity assumption, we would intuitively expect that the firm invests more money in maintenance in a period where the demand is stochastically larger. It would also be interesting to derive conditions that guarantee the optimality of a threshold maintenance policy. The threshold value is likely to be a function of the inventory level, the expected demand, and the variance of the demand.

Advanced Demand Information: In Chapter 3, investing in process improvement projects such as preventive maintenance is introduced as a strategy for partially controlling the sources of uncertainty internal to the production process. However, external sources of uncertainty like demand also exist in a production system. Advanced demand information strategies where the firm uses information technologies and sale techniques to collect accurate information about customer orders is shown to be effective in better management of external uncertainties (Özer and Wei, 2004). Studying the problem of Chapter 3 assuming both strategies of investing in process improvement projects and investing in obtaining more accurate demand information is a promising future research direction. More specifically, at the beginning of each period, the firm determines the amount of investment in each strategy. Modeling the effect of investing in information technologies on demand is challenging. One possible idea is to assume that the demand of period i, i.e., Z_i, is a stochastic function of the amount of investment. For example, there are several possible investment options where each results in a specific distribution for the demand. The mean of the demand is identical across all investment options, however, the variance of the demand is lower in an option with higher investment. Analyzing the problem in the presence of investing in both process improvement projects and demand information technologies would be harder than the problem of Chapter 3 since the demand distribution of each period is dependent on the investment in information technologies. We would intuitively expect that there exists a threshold value such that if the amount of inventory on hand is bigger, it is optimal not to invest in any of the investment strategies. Characterizing the optimal investment decisions in case of lower inventory on hand would be however challenging.

Multi-product Production Systems: We studied a single product system in Chapter 3. One direction for extending the problem is to consider a multi-product system following the paper by Hsu and Bassok (1999). They assumed that there is one raw material as input, producing N different products where the demands and the yields of the products are random and different. Addressing a multi-product system in the context of the problem in Chapter 3 means that the firm should make two decisions at the beginning of each period: (i) how to optimally allocate the given production quantity among different products and (ii) how much to invest in maintenance. Since it is hard to analytically deal with allocation decisions in the framework of Markov decision process, exploring mathematical programming approaches such as stochastic programming, also used by Hsu and Bassok (1999), is more appropriate to solve the problem.

Multi-stage Production Systems: As indicated in Section 2.2.2.1, the majority of the models integrating maintenance and production planning decisions are developed for single-stage production processes. The results on multi-stage systems which consist of M machines in series are scarce and mostly limited to $M = 2$. Extending the problem of Chapter 3 to a multi-stage setting is one possible future direction

where the fixed production quantity u is the input to the first stage and the random yield of stage k is then the input to the $(k+1)$-th stage. The decisions of the firm at the beginning of each period include the amount of investment in maintenance and its division among different stages.

Carry-over of the Budget: The problem of Chapter 3 is studied assuming that the total budget available at the beginning of each period is determined a priori and the remaining budget of a period cannot carry over to the next period. This assumption provides a sub-optimal solution for the case where the allocation of the total budget available at the beginning of the planning horizon to n periods is also a decision variable. In Section 3.1, we mentioned that analyzing the problem in this case is hard, however, it is interesting to investigate the validity of the results obtained in Chapter 3. Particularly, the following questions can be investigated:

1. Does the money invested in maintenance decrease if more inventory is available at the beginning of a period?

2. Does the amount of investment increase as the number of remaining periods decreases?

7.1.2 Efficient Algorithms

In this section, we discuss algorithmic ideas for solving the multiple period problem of Chapter 3 and for solving the original problem of integrated maintenance and production planning where the goal is to simultaneously determine the amount of investment and the production quantities.

Multiple Period Problem of Chapter 3: In Section 2.2.2, we reviewed a body of work focused on understanding when a myopic policy results in a solution that is close to optimal for general random yield problems and on designing efficient algorithms to solve the multiple period problem. The main idea in the majority of the techniques is to transfer the random yield problem to a newsvendor problem which has a myopic optimal base stock policy. Under a myopic policy, the optimal solution of the multiple period problem is equal to the optimal solution of several single period problems. Therefore, the problem can be solved easily without knowledge about future periods. Bollapragada and Morton (1999) studied the single item periodic review inventory problem with random yield and demand. They represented the cost function in terms of the inventory position at the end of the period and showed that the random yield problem is identical to the newsvendor problem with a demand distribution dependent on the quantity ordered. They then developed heuristic approaches to calculate the optimal production quantity. Li et al. (2008) studied the same problem where they derived upper and lower bounds for both the optimal order quantity and the order threshold value by solving various newsvendor problems. They used the bounds to design a heuristic approach. It is interesting to represent the problem of Chapter 3 in terms of the end of period inventory position or to investigate deriving bounds. However, since we have not considered a stochastically proportional yield model, it seems that the transformation or finding bounds would be difficult. The easiest next step would be to adopt the existing heuristics and compare them experimentally with the optimal solution of the backward dynamic programming procedure. The experimental evaluation can then provide insight in the promising heuristic to pursue theoretically.

Joint Maintenance and Production Policies: One idea to address the original single period problem is to use an iterative two-step approach. First, the production quantity u_1 is chosen based on the historical data, and the optimal amount of investment a_1 is found by the techniques discussed in Section 3.2.2. Second, the techniques in random yield literature reviewed in Section 2.2.2 are used to find a new production quantity u_2 where the yield distribution is fixed at a_1.[1] For the new production quantity u_2, a new investment value a_2 is found and the procedure repeats until the total cost converges. The main challenge of using such a technique is proving its convergence.

7.2 Maintenance Planning & Production Scheduling with No Control over Machine Conditions

To address the relationship between maintenance planning and production scheduling where machines are only maintained at failure, a technique that can efficiently incorporate the known stochastic information about machine conditions into scheduling decisions needs to be developed. In Chapter 4, we dealt with this challenge in the context of a military repair shop scheduling problem and developed a dynamic scheduling technique to solve the problem. In Section 4.5, we discussed a number of future research directions to better model the uncertainty on aircraft failures including leaving some availability slack on repair resources, two-stage stochastic programming, and multi-stage dynamic programming. We also presented some ideas for studying a more complex scheduling problem. In this section, we present two other ideas for extending the work in Chapter 4.

7.2.1 Competitive Ratios of the Developed Algorithms

Based on the definition given by Hall et al. (2009), in a semi-online scheduling problem, the data for the currently available jobs is known and new jobs might arrive only at known discrete future times. The dynamic repair shop scheduling problem of Chapter 4 is therefore a semi-online scheduling problem where the pre-flight and the post-flight checks are the discrete times at which new jobs might arrive to the repair shop. A semi-online scheduling problem interpolates between the classical offline scheduling where all the data on jobs is known at the beginning of the scheduling horizon and the classical online scheduling where jobs may arrive at any time and their data becomes known only on arrival (Hall et al., 2009).

The common performance measure in online or semi-online scheduling problems is the competitive ratio which informally implies how much one loses by not knowing the complete information in the worst case (Vestjens, 1997). To formally define the competitive ratio, let $Z^*(P)$ denote the optimal solution of instance P given by an offline algorithm where all data on jobs is known in advance. Further assume that $Z^A(P)$ is the solution achieved by the online algorithm A. In a maximization problem, the competitive ratio of algorithm A equals $R_A = \inf\{\frac{Z^A(P)}{Z^*(P)} | \forall P,\ \ Z^*(P) > 0\}$ (Hoogeveen et al., 2000). To calculate the competitive ratio of any online algorithms for the dynamic repair shop scheduling problem,

[1] The optimal policy in this case is a base stock policy where the production quantity equals the difference between the inventory threshold value and the inventory on hand.

assume that there is one aircraft ready for the pre-flight check and there is no aircraft in the repair shop. The number of waves already scheduled is W, each requiring one aircraft to carry the flight. Consider a situation where an online algorithm A assigns the aircraft to the first wave, the aircraft fails in the pre-flight check, the new job arriving to the repair shop has a very long processing time that cannot be processed before other waves. If no failures happen in the future pre- and post-flight checks, the coverage of algorithm A and the optimal coverage equals 0 and $(W-1)$, respectively, resulting in $R_A = 0$. On the other hand, assume that algorithm A waits and does not assign the aircraft to the first wave. If failure is not detected in the first pre-flight check and is detected in all future checks, the coverage of the algorithm A equals 0 while the optimal coverage equals 1 and $R_A = 0$, again. Therefore, any online algorithms can perform very poorly in the worst case for the dynamic repair shop problem.

The drawback of using competitive ratio to measure the quality of an algorithm is that it focuses on the worst-case instance and so fails to find online algorithms that work well in practice (Vestjens, 1997). One approach to deal with this issue is to use probabilistic analysis to characterize the average-case performance of the algorithm assuming certain distributions of the problem data (Vestjens, 1997). However, Coffman and Lueker (1991) mentioned that the probabilistic analysis of an algorithm, even a simple one, is very challenging and usually asymptotic. In other words, the average-case performance can be found when the problem size is very large. For the dynamic repair shop scheduling problem, the experimental results on the mean observed coverage up to flight 28 in Table 4.3 showed that the average performance of the three online algorithms is in range of $[0.52, 0.77]$. Performing probabilistic analysis to theoretically quantify the average-case performance measure for any of the three approaches would be however extremely challenging.

One possible idea to provide theoretical insight is to consider quantifying the average-case performance measure of a simple dispatching heuristic for a simple version of the repair shop scheduling problem. An example of a simple problem can be the one that reduces from a single machine scheduling problem with the objective of maximizing the weighted number of early jobs and with a common due date (see Section 4.2). A dispatching heuristic that processes the jobs in non-decreasing order of $\frac{p_j}{c_j}$ can be considered as a simple algorithm.[2] The literature on the average-case performance of bin-packing problem can be the starting point (Simchi-Levi et al., 2005).

7.2.2 Scheduling Horizon and Rescheduling Frequency

All three rescheduling policies for the dynamic repair shop scheduling problem in Section 4.3.2 are characterized by the length of the scheduling horizon and the frequency of rescheduling. Our experimental results showed that the P_{31} policy yields a higher mean observed coverage using both complete approaches for scheduling the static problem. As noted in Section 4.3.2, three is chosen as the length of the scheduling horizon since three waves are usually scheduled daily in the real-world application of the problem. However, the optimal length of the scheduling horizon i^* is the minimum number of waves

[2] p_j and c_j are the processing time and the capacity requirement of job j, respectively.

where the data on the waves after wave i^* has no effect on the optimal decisions for the current wave.[3]

The frequency of rescheduling has the highest possible value in the P_{31} policy since the rescheduling is done after each wave. Making a new schedule after each wave is computationally expensive, therefore, accounting the computational cost, it might not be optimal to revise the schedule at the earliest possible time.

As a future research direction, it is interesting to develop an optimization model where the length of the scheduling horizon and the frequency of rescheduling are also decision variables. Bidot (2005) and Bidot et al. (2009) consider these two challenges in studying scheduling problems under uncertainty. More specifically, they propose different algorithmic frameworks that incrementally generate the complete schedule by generating and maintaining robust[4] partial and flexible schedules. However, their study is done on a conceptual level providing a basis for combining the predictive and reactive approaches to construct the schedule in real time. We believe that dynamic programming and optimal control frameworks are the tools to theoretically determine the length of the scheduling horizon and the rescheduling frequency, however, solving the models resulting from these tools is challenging due to very large state and action spaces.

7.3 Maintenance Planning & Production Scheduling with Partial Control over Machine Conditions

Integrated maintenance planning and production scheduling where machines can be preventively maintained considers the conflict of maintenance and production in the short term and addresses the problem of simultaneously scheduling maintenance and production to minimize the total maintenance and lost production cost in the long run. In Chapters 5 and 6, we tackled this interdependency in a multi-machine production system where two different approaches are used to model machine deterioration. In Chapter 5, we assumed that the speed of the machine is a deterministic function of the number of periods since maintenance and in Chapter 6, we characterized machine conditions by a set of discrete numbers where machines stochastically degrade.

In this section, we first list some ideas for extending the work in Chapter 5 and then for Chapter 6.

7.3.1 Extensions of Chapter 5

In this section, we outlines several ideas for extending the work in Chapter 5.

7.3.1.1 The Length of the Maintenance Planning Horizon

In Section 5.1, we assumed that at the beginning of each period, the set of production jobs is known for the next L periods and L is a known parameter. One possible extension of the problem is to consider that the set of production jobs is only known for the current period and the firm has the option of buying

[3]The question of how far into the future the decision maker should forecast to make the optimal current decision is the concern of the horizon and forecast research literature (Sethi and Sorger, 1991; Chand et al., 2002).

[4]A robust schedule is a schedule whose quality is maintained during the execution (Bidot, 2005).

information for future periods. Therefore, the firm needs to also decide whether to pay for information in future periods and for how many periods. This extension of the problem in Chapter 5 is similar to the idea discussed in Section 7.2.2. Our computational experiments in Section 5.3.2 showed that the Short-term approach with $L = 1$ outperforms the Integrated approach with $L = 4$ on a small number of problem instances even in case that there is no cost associated with the information in future periods. Therefore, reasoning for a longer future is not always beneficial and it is interesting to consider the length of the maintenance planning horizon as a decision variable.

7.3.1.2 Developing Efficient Algorithms for the Production Scheduling Problem

Our computational results in Section 5.3.2 and the discussion in Section 5.4 provided evidence that solving the production scheduling problem (PSP) of each period to optimality can improve the performance of the Integrated approach. The existing literature on the flowshop scheduling problem with the objective of minimizing the number of tardy jobs (Ho and Gupta, 1995; Jr et al., 2004; Bülbül et al., 2004; Gupta and Jr, 2006; Shabtay, 2012) can be investigated to tighten the relaxation of the PSP in the maintenance planning problem, to design a stronger cut, and to develop more efficient dominance properties decreasing the run-time of the PSP.

7.3.1.3 Modeling Machine Failures

In Chapter 5, machine deterioration is modeled as a deterministic function of the number of time periods since maintenance. Adopting this approach does not model random machine breakdowns, therefore an interesting extension of the problem is to incorporate machine failures in the model.

Besides the deterioration over time modeled in Chapter 5, assume that machine m is subject to random breakdowns where the probability of failure increases as the machine age increases. The age of the machine equals the sum of its total up-times since previous maintenance. Time to failures of machine m are random variables. At each failure, machine m is minimally repaired where the repair times are also random variables. Minimal repair makes the machine as good as right before the failure, that is, the speed of the machine after repair is exactly the same as it was before failure.

Given the machine breakdown within a period, machine m is not continuously available during the whole period to process the jobs. Let $\mathcal{V}^m(T, x_0)$ denote the expected up-time of machine m in a period with length T and an initial age of x_0 at the beginning of the period. One approach to consider machine failures is to replace T with $\mathcal{V}^m(T, x_0)$ in the models of maintenance planning and production scheduling problems in Figures 5.5 and 5.6. To find the expected up-time for machine m, the model presented by Dagpunar and Jack (1993) can be extended. The drawback of this approach is modeling the uncertainty on machine conditions in the weakest form, as expectation.[5] To deal with this weakness and build a solution that is hedged against various uncertain situations, the same idea as proposed by Beck and Wilson (2007) can be investigated in future work.

Beck and Wilson (2007) studied the problem of minimizing makespan in a jobshop scheduling

[5]For a discussion on weaknesses of modeling the uncertainty as expectation, see Section 4.5.

problem where the processing time of each job on each machine is represented by an independent random variable with a known mean and variance. They defined the best solutions as ones which have a high probability of achieving a good makespan. Following the same idea, we can define $\mathcal{H}^m(c; T, x_0)$ as the probability that the up-time of machine m in a period with length T and the initial age of x_0 is greater than c. Therefore, the goal of the problem is not only minimizing the total cost of maintenance and lost production, but also constructing a schedule where the probability of the up-time being greater than c is high.

7.3.2 Extensions of Chapter 6

In this section, we discuss extensions of the problem studied in Chapter 6.

7.3.2.1 Different Assumptions

The Start-Time and the Duration of Maintenance: In Section 6.2.1, it is assumed that maintenance is performed at the beginning of a period and is instantaneous. Relaxing this assumption to represent the time of performing maintenance as a random variable over the interval $[0, T]$ with a general probability distribution and/or to consider the maintenance duration of $t_p^m > 0$ is a challenging setting to investigate the existence of the switching curve maintenance policy. More specifically, it is interesting to characterize the set of the probability distributions for maintenance start-time that guarantees a monotone maintenance policy.

Non-stationary Demand: In Section 6.1, it is assumed that the demands of the time periods are identically distributed random variables. Relaxing this assumption where the demand of period i is a function of time can capture prevailing business environment as explained in Section 7.1.1. Understanding the structure of the optimal maintenance policy in this case is a direction to pursue in the future.

7.3.2.2 Different Modeling Approaches

We discuss two different approaches for modeling the problem of Chapter 6, below. The first approach is to model the maintenance planning problem (MPP) as a restless bandit problem and the second one is to incorporate the decision about when to perform maintenance into the MPP.

Modeling the MPP as a Restless Bandit Problem: We first define the classical restless bandit problem. Assume that there are a set of N projects. Project $n \in \{1, \ldots, N\}$ can be in one of a finite number of states $i_n \in \mathbb{S}_n$ where $\mathbb{S}_n$ is the state space of project n. At the beginning of each period $k \in \{1, 2, \ldots\}$, exactly M projects ($M < N$) must be set active. If project n in state i_n is set active, an active reward $R_{i_n}^1$ is earned and the project state changes to j_n with probability $p_{i_n j_n}^1$ at the beginning of the next period. If the project in state i_n is set passive, a passive reward $R_{i_n}^0$ is received and the state changes to j_n with probability $p_{i_n j_n}^0$. The challenge is to find a Markovian scheduling policy that sets projects either active or passive at the beginning of each period such that the total expected discounted reward is maximized over an infinite horizon (Bertsimas and Niño-Mora, 2000).

The maintenance planning problem in Section 6.2.1 can be modeled as a restless bandit problem where the machines represent the projects and performing maintenance or not on a machine corresponds to setting a project active or passive. The maintenance capacity limit, C, is the equivalent of the number of active projects, M, in each period. However there is a subtle difference. In a restless bandit problem, the number of active projects must be M at each time period, in the maintenance planning problem, however, the number of machines that are maintained must be at most C.

Bertsimas and Niňo-Mora (2000) used a mathematical programming approach called performance region to formulate a series of N linear programming relaxations for the restless bandit problem. They then developed a priority-index heuristic in terms of the optimal dual variables of the first-order relaxation and experimentally showed that the heuristic is significantly accurate. One possible future research direction is to formulate the first-order linear relaxation of the maintenance planning problem determining the priority-index heuristic. It is interesting to compare the performance of an approach where the maintenance decisions are found using the priority-index heuristic with the other two approaches discussed in Section 6.3.1 where the policy improvement algorithm and the myopic heuristic are used to determine the maintenance decisions.

Incorporating the Decision about When to Perform Maintenance into the MPP: The MPP in Chapter 6 determines whether a machine needs maintenance or not. The exact time of maintenance is then determined in the production scheduling problem along with the schedule of the production activities. It would be interesting to change the MPP such that it determines both decisions on whether the machine needs maintenance and if so, the exact time of performing maintenance. In this case, the scheduling horizon with length T is discretized and the action space on machine m equals $\{0_0, 0_1, 1, 2, \ldots, T - t_p\}$ where 0_0, 0_1 and $i > 0$ mean that machine m is not maintained, is maintained at time 0, and is maintained at time $i > 0$, respectively. Note that t_p is the duration of maintenance activity. The disadvantage of this approach is that the solution space increases from $(N_m + 1)^2$ to $(N_m + 1)^{2+(T-t_p)}$ and solving the MPP problem using the policy improvement algorithm would be more challenging. Recall that $(N_m + 1)$ is the number of machine m's states.

7.4 General Future Research Directions on Integrated Maintenance and Production Decision

In this section, we first present conceptual and theoretical general future research directions on integrated maintenance and production decisions.

7.4.1 Conceptual Directions

In Chapters 5 and 6, we considered a production system composed of M machines where customers place their orders at the beginning of each period. Each order should be processed on each machine in sequence and has a specific due date. One of our decisions, which is also of concern in the maintenance literature, is to determine the optimal time to perform preventive maintenance on each machine.

To find the time to perform preventive maintenance, a typical maintenance model considers each of M machines independently and develops an optimization model based on the state of the machine, for example, its age or its deterioration level. In such a model, it is assumed that there is no limit on maintenance resources such as spares and manpower and that the time to perform maintenance is negligible. Furthermore, the effect of maintenance on temporary production capacity reduction is presented in terms of expected preventive and corrective maintenance costs which do not depend on the operational state of the system such as due dates of customer orders. The model then results in a static control rule that connects the maintenance decision to the state of the machine. This model has the following drawbacks:

1. The interactions between machines are not considered. For example, it is assumed that there is no limit on maintenance resources. However, in real-world setting, there are restrictions on the number of available spares and maintenance staff. Therefore, additional labor or spares might be required, incurring significant cost by considering each machine independently.

2. Maintenance decisions are represented as static rules dependent only on the state of the machine. However, in real applications, the state of production system is dynamic and new opportunities happen in real time. For example, delaying maintenance on a machine might result in satisfying the order of a customer resulting in significant gains.

3. The effect of maintenance on production system capacity is represented only in terms of machine state. The available information on the number of orders, their processing requirements and their due dates are not considered in maintenance decisions resulting in a less informed maintenance decision.

We took the initial steps in Chapters 5 and 6 addressing all three mentioned drawbacks and simultaneously determined the allocation of production capacity to maintenance and production activities. We believe in order to ensure the reliability of production systems to have three features of quality, on-time delivery, and safety, future research should focus on incorporating maintenance reasoning in operational scheduling decisions. More specifically, maintenance decisions based on the stationary information on machine conditions should be used only as a guide line. The exact time of performing maintenance should be determined considering the workload on each machine, the due dates, and the restrictions on available maintenance resources.

It would be interesting to quantify the benefit of adjusting maintenance decisions based on real-time changes. One idea is to consider a production system consisted of multiple machines, either flowshop or jobshop. Assume that the maintenance rule for each machine is available based on the recommendation given by the manufacturer of the machine. For example, machine i should be maintained after x hours of operations. The goal is to maximize the number of customer orders satisfied by their due dates ensuring the maintenance requirements of each machine. There are two different approaches to deal with maintenance rules. The first approach treats them as hard constraints while the second approach allows maintenance happen between $(x - \Delta x)$ and $(x + \Delta x)$ hours of operations where the exact time depends on the operational state of the production system. The experimental comparison of these two

approaches would be a solution for calculating the benefit of having flexibility on the exact time of performing maintenance.

7.4.2 Theoretical Directions

In this dissertation, we classified the production problems to production planning and production scheduling. We then studied the relationship between maintenance and production planning decisions in Chapter 3 and between maintenance and production scheduling decisions in Chapters 4, 5, and 6. In this section, we discuss future research directions on combining maintenance reasoning with production decisions.

As noted in Chapters 2 and 3, the problem of integrated maintenance and production planning aims at determining the optimal joint maintenance and production policies. Since both maintenance and production planning decisions are long-term and based on aggregate and stochastic information, stochastic optimization techniques such as dynamic programming are the primary approach for modeling these problems. The resulting models are usually prohibitively large and solving them is challenging. This research area has therefore mainly focused on developing heuristic approaches generally in three steps which are very problem-specific: simplifying the problem by relaxing the assumptions, deriving several structural properties to gain insight into the optimal solution of the simplified problem, and finally utilizing the derived properties to develop a heuristic approach for the real problem. The success of this research area in solving real-world integrated maintenance and production planning problems therefore appears to be dependent on developing techniques that can deal with the curse of dimensionality in dynamic programming techniques. Investigating the applicability of artificial intelligence techniques such as machine learning (Sutton and Barto, 1998) and approximate dynamic programming techniques (Powell, 2011) is an interesting direction to pursue in future. These techniques combine the ideas from dynamic programming, mathematical programming, simulation and statistics.

In Chapters 4, 5, and 6, we dealt with the interdependency between maintenance and production scheduling decisions. We focused on developing algorithmic techniques to incorporate maintenance reasoning in short-term combinatorial production scheduling decisions and showed that such techniques led to higher system performance. The examples are the scheduling-rescheduling approach in Chapter 4, the Integrated approach in Chapter 5, and the MDP-MIP approach in Chapter 6. As this dissertation represents the first work on integrated maintenance planning and combinatorial scheduling for a long-term horizon, the investigation of real-world maintenance planning and scheduling problems in future work is likely to inspire a variety of problem definitions, formulations, and solution approaches that may be complementary to and extend the work presented in this dissertation. As noted earlier, maintenance planning decisions are stochastic in nature and are the concern of stochastic optimization techniques. The scheduling decisions are however deterministic and combinatorial in nature and are modeled using mathematical programming approaches. The efficiency of the future work in addressing the real-world problems that are both stochastic and combinatoric in nature is then strongly tied to developing solution frameworks that can efficiently integrate stochastic and combinatorial optimization techniques.

In summary, to further progress on addressing realistic integrated maintenance and production prob-

lems whether planning or scheduling, the future work focus should be on developing algorithmic techniques that can unify the ideas of dynamic programming and mathematical programming.

7.5 Toward Integrated Decision Making

Utilizing both long-term and short-term information to make integrated maintenance and scheduling decisions is the central challenge addressed in this dissertation. In order to make decisions to determine which machines need maintenance and the time to perform maintenance, the detailed objective information such as the lost production cost from the solution to the scheduling problem is required. The common approach in the literature is to approximate the objective based on stochastic and aggregate information about machine conditions, production rates, and due dates. However, the true objective value depends on the short-term and combinatorial scheduling decisions including the allocation of machines to jobs. At the same time, scheduling decisions require the information about which machines are under maintenance and the maintenance duration. A similar relationship exists in other integrated decision-making problems such as inventory allocation and routing (Dror et al., 1985; Dror and Ball, 1987; Moin and Salhi, 2007); facility location-allocation and routing (Fazel-Zarandi and Beck, 2012; Fazel-Zarandi et al., 2013); inventory management and scheduling (Terekhov et al., 2012; Terekhov, 2013); lot-sizing and scheduling (Mateus et al., 2010); and capacity planning and scheduling (Megow et al., 2011).

Inventory Allocation and Routing: Inventory allocation decisions determine which customers to serve and the amount of inventory allocated to each customer. To make these decisions, an estimate of the delivery cost for each customer is used even though its exact value depends on short-term decisions of the vehicle routes. However, information about customer selection and inventory allocation is required to find the optimal route for each vehicle and to calculate the exact delivery cost (Dror et al., 1985; Dror and Ball, 1987; Moin and Salhi, 2007).

Facility Location-Allocation and Routing: The objective of the location-allocation problem is to select a set of facilities to open and to allocate customers to facilities. The cost of serving a client from a facility is usually abstracted as a known parameter, despite the fact that its exact value is a function of short-term decisions such as the type of vehicle assigned to the customer and the vehicle route. In contrast, the allocation of clients to facilities should be known to make routing decisions and to find the accurate cost of serving a customer from a facility (Fazel-Zarandi and Beck, 2012; Fazel-Zarandi et al., 2013).

Inventory Management and Scheduling: Inventory management addresses the decisions on the quantity delivered to a manufacturing facility and on the timing of this delivery. The inventory models discard the details of a production process and make decisions based on fixed production, holding and backlog costs. Due to abstraction, the delivery plan might lead to an infeasible production schedule where, for example, there are not enough components to produce the customer orders by their due dates. To find the inventory replenishment policy, the scheduling constraints such as the processing requirements and the due dates of the customer orders should be therefore considered. On the other hand, scheduling models assume a given delivery plan and construct the optimal schedule using the concepts of completion time,

earliness, and tardiness which can represent the cost of producing a product, the inventory cost of a finished product, and the cost of delay in delivery of a product (Terekhov et al., 2012; Terekhov, 2013).

Capacitated Lot-sizing and Scheduling: An estimate of the available production capacity is considered in lot-sizing models to find the production quantities. The exact capacity, however, is dependent on machine utilization which is determined by scheduling decisions. Production lots nevertheless constitute the jobs for scheduling and therefore are inputs to scheduling models (Mateus et al., 2010). An opportunity for investigating this interdependency is to extend the paper by Duan et al. (2012) where the challenge of applying automated negotiation between two self-interested agents (a manufacturer and a supplier), each solving their lot-sizing problems locally is studied. The goal for two agents is to achieve an agreement while optimizing their own objective functions. A fixed and known production capacity is considered in the lot-sizing problems of both agents that is private to the two agents. One does not know the others capacity. Representing the production capacity as a function of local scheduling problems of each agent is an interesting extension to pursue. More specifically, the research question is on how the final agreement changes with an accurate representation of the production capacity in local optimization problems of both agents.

Capacity Planning and Scheduling: In order to quantify the number of workers and other resources for a turnaround project in a power plant, the project duration information is required. While the exact duration of the project depends on the scheduling decisions considering all side constraints such as working shifts, resource capacities, and due dates, the number of available workers and resources is required to make detailed scheduling decisions (Megow et al., 2011).

The main idea of this dissertation for addressing this relationship is based on decomposition where long-term and short-term decisions are tackled in different, coupled stages. Investigating the applicability of the approach of this dissertation in other problem settings with a similar interdependency is a promising future direction.

7.6 Conclusion

In this chapter, we presented ideas for extensions of the work in Chapter 3 addressing the integrated maintenance and production planning with partial control over machine conditions, in Chapter 4 studying the interdependency between maintenance planning and production scheduling where machines are only correctively maintained, and in Chapters 5 and 6 tackling the maintenance planning relationship with production scheduling decisions where machines are both correctively and preventively maintained. We then discussed general ideas to further progress on solving the real-world integrated maintenance and production problems. Finally, we noted the relevance of the ideas developed in this dissertation to other integrated decision making problems.

In the next chapter, we summarize the contributions of this dissertation and conclude.

Chapter 8

Conclusion

The importance of timely and continuous production, quality improvement, and fast delivery has forced production and delivery processes to be highly reliable. Maintenance improves the reliability by reducing the occurrence of breakdowns, however, it results in planned periods of process unavailability that could be otherwise utilized for production. Therefore, the coordination of maintenance and production decisions is necessary to trade off the increase in planned production capacity reduction for the decrease in the number of unexpected interruptions. In this dissertation, we created a novel framework based on the type of production decision, maintenance strategy and the length of decision horizon to capture possible interdependencies between maintenance and production. More specifically, different combinations of two production decisions of planning and scheduling with two maintenance strategies of corrective and preventive over a short- or long-term decision horizon define the possible relationships between maintenance and production. In three of these relationships within the contexts of a periodic review production system, a dynamic military aircraft repair shop, and a multi-machine production system, we showed that integrating maintenance and production decisions enhances efficiency by increasing the yield, the utilization of resources, and the on-time deliveries. The integrated decisions addressed in each context are as follows:

- Periodic review production system: integrated maintenance and production planning with partial control over machine conditions.

- Dynamic military aircraft repair shop: integrated maintenance planning and production scheduling with no control over machine conditions.

- Multi-machine production system: integrated maintenance planning and production scheduling with partial control over machine conditions.

In this chapter, we summarize the work in each area and re-state the contributions of this dissertation.

8.1 Maintenance & Production Planning with Partial Control over Machine Conditions

In Chapter 3, we considered a production system which produces one product in a single-stage process over multiple periods to meet customer demands at the end of each period. Due to the internal causes including machine deteriorations and breakdowns, the quantity produced is not equal to the input production quantity. The firm has an interest to commit resources for improving machine conditions and consequently increasing the yield. The problem at the beginning of each time period is to simultaneously determine the input production quantity and the investment in maintenance to minimize the total discounted expected cost over multiple periods. This is an example of an integrated maintenance and production planning problem where machine conditions can be partially controlled by performing maintenance.

Because analyzing the problem with joint decisions is intractable due to the non-convexity of the cost function, we studied a simpler problem where the input production quantity is fixed. Since yield losses are due to internal causes, the output production quantity is not proportional to the input. To model the random yield of each period, we assumed a more general form than the stochastically proportional yield model (Yano and Lee, 1995). Further, we introduced two different cases of positive and expected positive maintenance to model the effect of investment in maintenance on the random yield. In case of positive maintenance, the yield does not decrease as the amount of investment increases. In case of expected positive maintenance, the expected yield does not decrease as more money is invested in maintenance. Finally, we assumed that the budget available for making an investment at the beginning of each period is determined a priori.

We focused on understanding the structure and the properties of the optimal maintenance (investment) policy. Our main results in case of positive maintenance are summarized below:

- If yield is a linear function of the amount of investment, the optimal maintenance policy over multiple periods is a single critical level type of the inventory level.
- The optimal amount of investment in maintenance does not increase as the inventory on hand increases.
- The inventory threshold value does not decrease if there is more budget for investment at the beginning of the period.

If maintenance is expected positive, our analysis shows that the threshold maintenance policy is optimal, though only over single period, if the yield is linear in investment value and the demand density function is non-increasing, having, for example, an exponential or a uniform distribution. However, the maintenance policy over multiple periods does not necessarily have a threshold structure. It is also shown that all single-period results hold true when yield is concave in the investment value and there is no holding cost.

We had a strategic perspective for studying the relationship between maintenance and production in Chapter 3 where both decisions of production quantity and maintenance investment are long-term.

Assuming that long-term decisions are determined, we took an operational view in Chapters 4 to 6 to determine the optimal allocation of resources to either production or maintenance.

8.2 Maintenance Planning & Production Scheduling with No Control over Machine Conditions

In Chapter 4, we studied a dynamic military aircraft repair shop where a number of flights, each with a requirement for a specific number and type of aircraft, are scheduled over a long horizon. Aircraft are checked for failure before and after each flight: if failure is detected in an aircraft, it enters the repair shop and waits until its repair operations are performed. The goal is to assign aircraft to flights and schedule repair activities while considering the flight requirements, repair capacity, and aircraft failures to maximize the flight coverage. This problem is an example of an integrated maintenance and production scheduling problem with no control over machine conditions where machine failures (aircraft breakdowns) limit their availabilities for production (undertaking the flights) and where machines are only reactively maintained upon failures.

To solve the problem, we viewed the dynamic repair shop as linked static repair scheduling sub-problems. The solution of the static problem allocates the aircraft to flights and constructs the repair schedule maximizing the flight coverage. When a failed aircraft enters the repair shop while the previous repair schedule is still under execution, we reschedule the repair activities by solving a new static sub-problem. We designed five different approaches including mixed integer programming, constraint programming, logic-based Benders decomposition, a dispatching heuristic, and a simple hybrid to solve the static sub-problems. We then defined three rescheduling policies, distinguished based on the length of the scheduling horizon and the frequency of rescheduling, to connect the static sub-problems.

Computational experiments demonstrate that the approach that uses logic-based Benders decomposition to solve the static sub-problems, incorporates the known information on aircraft failures into the repair schedule, schedules over a longer horizon, and reschedules as soon as a failed aircraft enters the repair shop increases the flight coverage on average 10% compared to the other approaches tested. It is also shown that this approach balances against different uncertain scenarios of aircraft breakdowns.

8.3 Maintenance Planning & Production Scheduling with Partial Control over Machine Conditions

We continued addressing the integration of maintenance planning and production scheduling in Chapters 5 and 6 where machines can be partially controlled by performing maintenance both before and at failure. We considered a multi-machine flowshop production system that produces multiple products over multiple periods. As machines are used for production, they deteriorate and the production capacity therefore decreases. Maintenance improves machine conditions and restores the production capacity but results in temporary production unavailability. The challenge is how to use the available information on machine conditions to simultaneously schedule maintenance and production activities. At the

beginning of each time period, two decisions are made: which machines are maintained, if any, and when each maintenance and each production activity start on each machine to minimize the total cost of maintenance and lost production. We addressed this problem from the perspective of the scheduling and the maintenance research literatures in Chapters 5 and 6, respectively.

In Chapter 5, we assumed that a machine's speed deterministically decreases as the number of periods since maintenance increases. To precisely model the production capacity as a function of both machine conditions and scheduling constraints, we designed a coupled two-stage algorithm. The first stage contains the abstraction of the scheduling problem where all customer orders are considered to require the same production capacity and to be due at the same time. It determines the assignment of maintenance to machines and time periods minimizing the sum of maintenance cost and the lower bound on the lost production cost over multiple periods. The second stage finds the maintenance and the production schedule of the current period given the specified maintenance plan. The real lost production cost of the current period is then communicated to the first stage by a constraint. The solution of the first stage can then be revised if it is no longer optimal given more detailed lost cost information. The iteration between two stages continues until the lower bound and the actual lost production cost of the current period converges.

We compared the integrated approach experimentally with three other approaches: a hierarchical approach where there is no feedback between two stages, an integrated short-term approach where maintenance planning and production scheduling are done together for each period, and a heuristic approach. The computational results demonstrate that the integrated approach yields lower total cost. It is also shown that the benefit of integrated decision making and long-term reasoning increases for lower and higher maintenance cost relative to lost production cost.

In Chapter 6, we studied the same problem as Chapter 5 from the perspective of maintenance research literature. We assumed that a set of discrete states characterizes machine conditions and each machine deteriorates stochastically following a continuous time Markov chain. To find the maintenance plan and the schedule of maintenance and production activities, we decomposed the global problem into two sub-problems. The first sub-problem utilizes a Markov decision process model to find the optimal decision rule for performing maintenance, abstracting the scheduling combinatorics. The decision rule identifies machines for maintenance based on their states and the number of customer orders. The sufficient conditions are also derived to prove that the optimal decision rule has a switching curve structure which is monotone in both machine state and the number of customer orders. After the machines for maintenance are determined using the decision rule, the second sub-problem uses mixed integer programming model to schedule maintenance, if any, and production activities in the current period, incorporating scheduling combinatorics. The planned maintenance and production schedule is executed, the real maintenance and lost production cost is realized, the new machine states and the number of customer orders are observed, and the same procedure repeats.

We experimentally compared the designed algorithm with a heuristic approach where both maintenance planning and production scheduling sub-problems are solved using dispatching policies. Our results show that incorporating accurate information on machine deterioration into maintenance and

production scheduling decisions decreases the total discounted maintenance and lost production cost on average 21% over the heuristic approach. It is further shown that the benefit of integrated maintenance and production scheduling decisions increases for high discount factors and industries with moderate mean time to failures.

8.4 Summary of Contributions

The main contributions of this dissertation are:

- We are the first to theoretically identify the set of conditions that guarantee the existence of an optimal threshold type maintenance policy in a periodic review production system with random yield. The results will help managers to decide how much money should be invested to improve the state of the production system.
- We provide several managerial insights, including how the amount of investment in maintenance changes as the inventory or the total available budget increases. Understanding these relationships is useful to coordinate maintenance and production planning decisions.
- We are the first to develop optimization techniques that can effectively reason about both stochastic and combinatorial challenges in the context of maintenance and production scheduling decisions over a long-time horizon. Our techniques are all based on the idea of decomposition where the stochastic and the combinatorial challenges are addressed in different, coupled stages.
- We design an integrated technique to create a repair schedule for a dynamic military aircraft repair shop problem and show that adjusting the repair schedule as new short-term information becomes known significantly increases flight coverage. The integrated technique is based on a novel logic-based Benders decomposition approach which is four times faster than a novel mixed integer programming model on average which in turn is two orders of magnitude faster than an existing mixed integer programming model in the literature.
- We are the first to explicitly model the effect of machine deterioration and restoration on the processing times of customer orders in integrated maintenance and scheduling decisions.
- To precisely model the production capacity as a function of both machine state and the operational state of the system in a multi-machine production environment, we design appropriate solution techniques that depend on the deterioration process of machines. More specifically,
  - if machines deteriorate as the number of time periods since maintenance increases, we design a coupled two-stage integrated approach inspired by the idea of logic-based Benders decomposition; and
  - if machines deteriorate following a continuous Markov chain, we design a two-stage decomposed approach combining Markov decision process and mixed integer programming.

- We are the first to prove the conditions guaranteeing the monotonicity of a maintenance policy on both machine state and the number of customer orders when the effect of performing preventive maintenance on the production is not certain. More specifically, we consider preventive maintenance does not necessarily make the machine as good as new.

8.5 Conclusion

The central thesis of this dissertation is that integrating maintenance and production decisions increases efficiency by ensuring high quality production, effective resources utilization, and on-time deliveries. In this dissertation, we created a novel framework with three axes of the type of production problem, the maintenance strategy, and the length of decision horizon to capture possible interdependencies between maintenance and production. In our framework, different combinations of two production problems of planning and scheduling with two maintenance strategies of corrective and preventive over a short- or long-term decision horizon define areas where maintenance and production are interrelated. We investigated our thesis in three areas.

Firstly, we addressed the integrated problem of maintenance and production planning in the context of a periodic review production system where machines can be preventively maintained. Our analysis shows that the integrated decision-making ensures high quality production.

Secondly, we dealt with the problem of integrated maintenance planning and production scheduling with no control over machine conditions in the context of a dynamic military aircraft repair shop. Our results demonstrate that incorporating the known information on machine conditions into the repair schedule leads to the effective utilization of the valuable resources, i.e., aircraft.

Finally, we studied the integration of maintenance planning and production scheduling where machines can be maintained both before and at failure in the context of a multi-machine production system. It is shown that the precise representation of the production capacity as a function of both machine states and scheduling constraints decreases the total maintenance and lost production cost, increasing the number of on-time deliveries.

Appendix A

Proofs of Some Propositions in Chapter 3

In this appendix, the proofs of some propositions in Chapter 3 are provided.

A.1 Proofs of the Single Period Propositions

The proofs of several propositions discussed in Section 3.2 are given below.

A.1.1 Proof of Proposition 3.1

As stated in Section 3.2.1, Proposition 3.1 provides sufficient conditions such that the derivative of the function $g(t) = E[(V + Ut)^+]$ exists.

Proposition 3.1: If $g(t) = E[(V + Ut)^+]$ where V and U are random variables, $E[|U|] < \infty$, and $\Pr(V + Ut = 0) = 0$, then $g'(t) = E[UI(V + Ut > 0)]$. Note that $x^+ = \max(0, x)$.

Proof. We need to show that $g'(t) = \lim_{\Delta\to 0} \frac{g(t+\Delta)-g(t)}{\Delta}$ exists. Therefore, we have

$$g'(t) = \lim_{\Delta\to 0} \frac{E[(V + Ut + U\Delta)^+ - (V + Ut)^+]}{\Delta}.$$

Defining $Z_\Delta = \frac{(V+Ut+U\Delta)^+-(V+Ut)^+}{\Delta}$, we have with probability 1

$$\begin{aligned}\lim_{\Delta\to 0} Z_\Delta &= I(V + Ut > 0) \lim_{\Delta\to 0} Z_\Delta + I(V + Ut < 0) \lim_{\Delta\to 0} Z_\Delta + I(V + Ut = 0) \lim_{\Delta\to 0} Z_\Delta \\ &= I(V + Ut > 0) \lim_{\Delta\to 0} Z_\Delta + I(V + Ut < 0) \lim_{\Delta\to 0} Z_\Delta \quad .\end{aligned}$$

1. For $V + Ut > 0$ and $|\Delta| \neq 0$ small, we have $V + Ut + U\Delta > 0$; therefore,

$$I(V + Ut > 0) \lim_{\Delta\to 0} Z_\Delta = I(V + Ut > 0) \lim_{\Delta\to 0} [\frac{V + Ut + U\Delta - V - Ut}{\Delta}] = UI(V + Ut > 0).$$

2. For $V + Ut < 0$ and $|\Delta| \neq 0$ small, we have $V + Ut + U\Delta < 0$; therefore,

$$I(V + Ut < 0) \lim_{\Delta \to 0} Z_\Delta = I(V + Ut < 0) \lim_{\Delta \to 0} [\frac{0 - 0}{\Delta}] = 0.$$

Putting 1 and 2 together, we have with probability 1

$$\lim_{\Delta \to 0} Z_\Delta = UI(V + Ut > 0).$$

Knowing that $\forall X, Y \in \mathbb{R}$, $|(X + Y)^+ - X^+| \leq |Y|$, we have $|Z_\Delta| \leq |U|$. We can therefore use *the dominated convergence theorem* and

$$\begin{aligned} g'(t) &= \lim_{\Delta \to 0} \frac{E[(V + Ut + U\Delta)^+ - (V + Ut)^+]}{\Delta} = \lim_{\Delta \to 0} E[Z_\Delta] \\ &= E[\lim_{\Delta \to 0} Z_\Delta] = E[UI(V + Ut > 0)]. \end{aligned}$$

which completes the proof. □

A.1.2 Proof of Proposition 3.2

Proposition 3.2 presented in Section 3.2.1 states the sufficient conditions such that the derivative of the function $g(t) = E[Q(V + Ut)]$ exists.

Proposition 3.2: Let $g(t) = E[Q(V + Ut)]$ where V and U are random variables, $E[|U|] < \infty$, $\Pr(V + Ut = 0) = 0$, and $Q(x)$ is a CDF such that $Q(x) = 0$, $\forall x < 0$. Assume also, for simplicity, that $|Q(x + h) - Q(x)| \leq C|h|$ where C is a positive constant. Then $g'(t) = E[UQ'(V + Ut)]$.

Proof. The proof is similar to Proposition 3.1. We have

$$g'(t) = \lim_{\Delta \to 0} \frac{E[Q(V + Ut + U\Delta) - Q(V + Ut)]}{\Delta}.$$

Defining $Z_\Delta = \frac{Q(V+Ut+U\Delta)-Q(V+Ut)}{\Delta}$, we have

$$\begin{aligned} \lim_{\Delta \to 0} Z_\Delta &= \lim_{\Delta \to 0} [\frac{Q(V + Ut + U\Delta) - Q(V + Ut)}{\Delta}] I(V + Ut > 0) \\ &\quad + \lim_{\Delta \to 0} [\frac{0 - 0}{\Delta}] I(V + Ut < 0) = UQ'(V + Ut). \end{aligned}$$

From the condition stated in the proposition, we have $|Z_\Delta| = |\frac{Q(V+Ut+U\Delta)-Q(V+Ut)}{\Delta}| \leq C|U|$. This condition guarantees that $Q(x)$ is Lipschitz continuous on $\mathbb{R}$ and is the common simplest assumption that allows applying the *dominated convergence theorem* as below:

$$\begin{aligned} g'(t) &= \lim_{\Delta \to 0} \frac{E[Q(V + Ut + U\Delta) - Q(V + Ut)]}{\Delta} = \lim_{\Delta \to 0} E[Z_\Delta] \\ &= E[\lim_{\Delta \to 0} Z_\Delta] = E[UQ'(V + Ut)], \end{aligned}$$

which completes the proof. □

A.1.3 Supermodularity of the Total Expected Cost over One Period

To prove Propositions 3.3 and 3.4 in Section 3.2.3, we need to first prove the supermodularity of both $\pi(x, a)$ and $\Phi_1(x, y)$.

We first define the sumpermodular function and then prove the supermodularity of both the total expected cost, $\pi(x, a)$, and the optimal total expected cost, $\Phi_1(x, y)$ over single period.

Definition A.1. *Let f, $f : R^k \to R$, be a real-valued function. f is supermodular if*

$$f(x \wedge y) + f(x \vee y) \geq f(x) + f(y) \quad \forall x, y \in R^k,$$

where $x \wedge y$ and $x \vee y$ respectively denote the minimum and maximum of x and y componentwise (Definition 8-5 of Heyman and Sobel (1984)). If f is twice continuously differentiable, based on Topkis's Characterization Theorem (Milgrom and Roberts, 1990), f is supermodular if and only if $\forall x \in R^k$, and $\forall i \neq j$, $\frac{\partial^2 f}{\partial x_i \partial x_j} \geq 0$.

Proposition A.1. *$\pi(x, a)$ and $\Phi_1(x, y)$ are supermodular functions if*

(i) maintenance is positive, or

(ii) maintenance is expected positive and the demand density is non-increasing.

Proof. Given Propositions 3.1 and 3.2, we have

$$\frac{\partial^2 \pi}{\partial a \partial x} = BE[\dot{Y}_a q(x + Y_a)] \geq 0,$$

in both cases as discussed in proof of Theorem 3.1. Therefore $\pi(x, a)$ is a supermodular function. Letting $x_1 \leq x_2$ and $y_1 \leq y_2$, we define

$$\Phi_1(x_1, y_1) = \min_{a \leq y_1}(\pi(x_1, a)) = \pi(x_1, a_1),$$
$$\Phi_1(x_2, y_2) = \min_{a \leq y_2}(\pi(x_2, a)) = \pi(x_2, a_2).$$

Based on the above definition, $a_1 \leq y_1$ and $a_2 \leq y_2$. Therefore, $\Phi_1(x_2, y_1) \leq \pi(x_2, a_1)$ and $\Phi_1(x_1, y_2) \leq \pi(x_1, a_2)$. We discuss the following two cases:

- If $a_1 \leq a_2$, by supermodularity of $\pi(x, a)$ we have

$$\begin{aligned} \Phi_1(x_1, y_1) + \Phi_1(x_2, y_2) &= \pi(x_1, a_1) + \pi(x_2, a_2) \\ &\geq \pi(x_1, a_2) + \pi(x_2, a_1) \\ &\geq \Phi_1(x_1, y_2) + \Phi_2(x_2, y_1), \end{aligned}$$

  which proves the supermodularity of $\Phi_1(x, y)$.

- If $a_1 \geq a_2$, then $y_2 \geq y_1 \geq a_1 \geq a_2$ and we have $\Phi_1(x_1, y_2) \leq \pi(x_1, a_1)$ and $\Phi_1(x_2, y_1) \leq \pi(x_2, a_2)$. Therefore, we have

$$\begin{aligned}\Phi_1(x_1, y_1) + \Phi_1(x_2, y_2) &= \pi(x_1, a_1) + \pi(x_2, a_2) \\ &\geq \Phi_1(x_1, y_2) + \Phi_2(x_2, y_1),\end{aligned}$$

  which proves the supermodularity of $\Phi_1(x, y)$

Therefore, we prove that $\pi(x, a)$ and $\Phi_1(x, y)$ are supermodular functions in (x, a) and (x, y), respectively. □

A.1.4 Proof of Proposition 3.3

Proposition 3.3 in Section 3.2.3 states the relationship between the optimal investment and the inventory level.

Proposition 3.3: For a given budget y, if one of the following conditions holds true, then the optimal investment, a_1^*, is non-increasing in the inventory level, x.

(i) Maintenance is positive.

(ii) Maintenance is expected positive and the demand density is non-increasing.

Proof. Letting $a_1^*(x, y)$ be the optimal amount of investment with initial inventory x and the budget y and $x_2 \geq x_1$, we have

$$\begin{aligned}a_1^*(x_1, y) &= \arg\min_{a \leq y}(\pi(x_1, a)), \\ a_1^*(x_2, y) &= \arg\min_{a \leq y}(\pi(x_2, a)).\end{aligned}$$

To show $a_1^*(x_2, y) \leq a_1^*(x_1, y)$, we need to show $\pi(x_2, a) \geq \pi(x_2, a_1^*(x_1, y)),\ \forall a \geq a_1^*(x_1, y)$. Let consider (x_1, a) and $(x_2, a_1^*(x_1, y))$ where $x_2 \geq x_1$ and $a \geq a_1^*(x_1, y)$. Since π is supermodular in both conditions (i) and (ii) (Proposition A.1), we have

$$\pi(x_1, a_1^*(x_1, y)) + \pi(x_2, a) \geq \pi(x_1, a) + \pi(x_2, a_1^*(x_1, y)).$$

Furthermore, since $\pi(x_1, a) \geq \pi(x_1, a_1^*(x_1, y))$, we then have

$$\pi(x_1, a_1^*(x_1, y)) + \pi(x_2, a) \geq \pi(x_1, a_1^*(x_1, y)) + \pi(x_2, a_1^*(x_1, y)).$$

Therefore,

$$\pi(x_2, a) \geq \pi(x_2, a_1^*(x_1, y)),$$

which proves that $a_1^*(x_2, y) \leq a_1^*(x_1, y)$. □

A.1.5 Proof of Proposition 3.4

Proposition 3.4 given in Section 3.2.3 shows how the inventory threshold value changes as the total budget increases.

Proposition 3.4: For a given production quantity u, if the conditions of Theorem 3.1 hold true, then the inventory threshold value, $\bar{x}_1(u, y)$, is non-decreasing in the total budget, y.

Proof. In all four stated conditions of Theorem 3.1, the inventory threshold value exists. Assuming that $y_2 \geq y_1$, we define

$$\Phi_1(\bar{x}_1(u, y_1), y_1) = \pi(\bar{x}_1(u, y_1), 0),$$
$$\Phi_1(\bar{x}_1(u, y_2), y_2) = \pi(\bar{x}_1(u, y_2), 0).$$

To show that $\bar{x}_1(u, y_2) \geq \bar{x}_1(u, y_1)$, it is enough to show that $\pi(x, 0) \geq \Phi(x, y_2)$, $\forall x \leq \bar{x}_1(u, y_1)$. Let consider (x, y_2) and $(\bar{x}_1(u, y_1), 0)$ where $x \leq \bar{x}_1(u, y_1)$ and $0 \leq y_2$. Using Remark 3.1, we have $\Phi_1(\bar{x}_1(u, y_1), y_2) \leq \pi(\bar{x}_1(u, y_1), 0)$. Therefore,

$$\Phi_1(x, 0) + \pi(\bar{x}_1(u, y_1), 0) \geq \Phi_1(x, 0) + \Phi_1(\bar{x}_1(x, y_1), y_2).$$

Further, since Φ_1 is a supermodular function, we have:

$$\Phi_1(x, 0) + \Phi(\bar{x}_1(u, y_1), y_2) \geq \Phi_1(x, y_2) + \Phi_1(\bar{x}_1(u, y_1), 0).$$

The above two inequalities result in

$$\Phi_1(x, 0) + \pi(\bar{x}_1(u, y_1), 0) \geq \Phi_1(x, y_2) + \Phi_1(\bar{x}_1(u, y_1), 0).$$

Since $\Phi_1(x, 0) = \pi(x, 0)$ and $\Phi_1(\bar{x}_1(u, y_1), 0) = \pi(\bar{x}_1(u, y_1), 0)$, we have

$$\pi(x, 0) \geq \Phi_1(x, y_2),$$

which completes the proof that $\bar{x}_1(u, y_1) \leq \bar{x}_1(u, y_2)$. □

A.2 Proofs of the Multiple Period Propositions

In this section, we present the proofs of several propositions discussed in Section 3.3.1.

A.2.1 Proof of Proposition 3.5

Proposition 3.5 of Section 3.3.1 shows the convexity of total expected cost over one period.

Proposition 3.5: The expected cost over one time period, $\pi(x, a)$, is a jointly convex function given the conditions of Lemma 3.1 hold true on the yield function.

Proof. We discuss two cases:

1. If Y_a is linear in a, we have

$$\begin{aligned}
\frac{\partial^2 \pi}{\partial x^2} &= BE[q(x+Y_a)] \geq 0. \\
\frac{\partial^2 \pi}{\partial a^2} &= BE[\ddot{Y}_a Q(x+Y_a) + (\dot{Y}_a)^2 q(x+Y_a)] - pE[\ddot{Y}_a] = BE[(\dot{Y}_a)^2 q(x+Y_a)] \geq 0. \\
\frac{\partial^2 \pi}{\partial a \partial x} &= \frac{\partial^2 \pi}{\partial x \partial a} = BE[\dot{Y}_a q(x+Y_a)]. \\
\frac{\partial^2 \pi}{\partial x^2} \times \frac{\partial^2 \pi}{\partial a^2} - (\frac{\partial^2 \pi}{\partial a \partial x})^2 &= B^2 E[q(x+Y_a)]E[\ddot{Y}_a Q(x+Y_a)] + B^2 E[q(x+Y_a)]E[(\dot{Y}_a)^2 q(x+Y_a)] \\
&\quad - pBE[q(x+Y_a)]E[\ddot{Y}_a] - B^2 (E[\dot{Y}_a q(x+Y_a)])^2 \\
&= B^2 E[q(x+Y_a)]E[(\dot{Y}_a)^2 q(x+Y_a)] - B^2 (E[\dot{Y}_a q(x+Y_a)])^2 \geq 0,
\end{aligned}$$

   the last inequality follows from applying the Chauchy-Schwartz inequality in the integral form $|\int (f(x)g(x)dx)|^2 \leq \int |f(x)|^2 dx \times \int |g(x)|^2 dx$ where $f(x) = B\sqrt{q(x+Y_a)}$ and $g(x) = \dot{Y}_a \sqrt{q(x+Y_a)}$. The above inequalities prove the convexity of π in (x, a).

2. If Y_a is concave in a and $h = 0$, we have

$$\begin{aligned}
\frac{\partial^2 \pi}{\partial x^2} &= pE[q(x+Y_a)] \geq 0. \\
\frac{\partial^2 \pi}{\partial a^2} &= pE[\ddot{Y}_a Q(x+Y_a) + (\dot{Y}_a)^2 q(x+Y_a)] - pE[\ddot{Y}_a] \\
&= pE[\ddot{Y}_a (Q(x+Y_a) - 1) + (\dot{Y}_a)^2 q(x+Y_a)] \geq 0. \\
\frac{\partial^2 \pi}{\partial a \partial x} &= \frac{\partial^2 \pi}{\partial x \partial a} = pE[\dot{Y}_a q(x+Y_a)]. \\
\frac{\partial^2 \pi}{\partial x^2} \times \frac{\partial^2 \pi}{\partial a^2} - (\frac{\partial^2 \pi}{\partial a \partial x})^2 &= p^2 E[q(x+Y_a)]E[\ddot{Y}_a Q(x+Y_a)] + p^2 E[q(x+Y_a)]E[(\dot{Y}_a)^2 q(x+Y_a)] \\
&\quad - p^2 E[q(x+Y_a)]E[\ddot{Y}_a] - p^2 (E[\dot{Y}_a q(x+Y_a)])^2 \\
&= p^2 E[q(x+Y_a)]E[(\dot{Y}_a)^2 q(x+Y_a)] - p^2 (E[\dot{Y}_a q(x+Y_a)])^2 \\
&\quad + p^2 E[q(x+Y_a)]E[\ddot{Y}_a (Q(x+Y_a) - 1)] \geq 0,
\end{aligned}$$

   the last inequality follows from applying the Chauchy-Schwartz inequality as stated above. The above inequalities prove the convexity of π in (x, a).

Given 1 and 2, we complete the proof. □

A.2.2 Proof of Proposition 3.6

Proposition 3.6 in Section 3.3.1 proves the convexity of the optimal total expected cost over one period.

Proposition 3.6: $\Phi_1(x, y_n)$ is convex in x given the conditions of Lemma 3.1 hold true on the yield function.

Proof. We prove this proposition applying the same idea as Proposition B-4 of Heyman and Soble (1984). Recall that

$$\Phi_1(x, y_n) = \min_{0 \le a \le y_n} (\pi(x, a)).$$

Let $(x, a) = \lambda(x_1, a_1) + (1 - \lambda)(x_2, a_2)$. For $\forall \delta > 0$ and small, there is $a_i \le y_n$ such that $\Phi_1(x_i, y_n) + \delta \ge \pi(x_i, a_i)$. we have

$$\lambda\Phi_1(x_1, y_n) + (1 - \lambda)\Phi_1(x_2, y_n) \ge \lambda\pi(x_1, a_1) + (1 - \lambda)\pi(x_2, a_2) - \delta.$$

Since π is convex in (x, a) (Proposition 3.5), we have

$$\begin{aligned} &\ge \pi(x, a) - \delta \\ &\ge \Phi_1(x, y_n) - \delta. \end{aligned}$$

Letting $\delta \to 0$ proves the convexity of $\Phi_1(x, y_n)$ in x. □

A.2.3 Supermodularity of the Total Discounted Expected Cost over Multiple Periods

The next proposition shows that both $J_n(x, a, y_2, \ldots, y_n)$ and $\Phi_n(x, y_1, \ldots, y_n)$ are supermodular functions in (x, a) and in (x, y_1), respectively. This proposition is used in the proof of Propositions 3.8 and 3.9 in Section 3.3.2.

Proposition A.2. *$J(x, a, y_2, \ldots, y_n)$ and $\Phi_n(x, y_1, \ldots, y_n)$ are, respectively, supermodular functions in (x, a) and in (x, y_1) if maintenance is positive and the yield is linear in the investment value.*

Proof. To prove this proposition, we first state one property of a convex function.

Let consider u_1, u_2, u_3, u_4 where $u_2 \ge u_4$ and $u_1 - u_2 = u_3 - u_4 \ge 0$. If $f(u)$ is a convex function, then we have $f(u_1) - f(u_2) \ge f(u_3) - f(u_4)$.

Now, let $x_2 \ge x_1$ and $a_2 \ge a_1$. Since maintenance is positive, i.e., $Y_{a_2} \ge Y_{a_1}$ and $\Phi_{n-1}(x, y_2, \ldots, y_n)$ is convex in x (Proposition 3.7), using the stated property of the convex function, we have

$$\begin{aligned} &\Phi_{n-1}(x_2 + Y_{a_2} - Z, y_2, \ldots, y_n) - \Phi_{n-1}(x_1 + Y_{a_2} - Z, y_2, \ldots, y_n) \ge \\ &\Phi_{n-1}(x_2 + Y_{a_1} - Z, y_2, \ldots, y_n) - \Phi_{n-1}(x_1 + Y_{a_1} - Z, y_2, \ldots, y_n), \end{aligned} \tag{A.1}$$

where $u_1 = x_2 + Y_{a_2} - Z$, $u_2 = x_1 + Y_{a_2} - Z$, $u_3 = x_2 + Y_{a_1} - Z$, and $u_4 = x_1 + Y_{a_1} - Z$. Further we have

$$\begin{aligned} J_n(x_1, a_1, y_2, \ldots, y_n) + J_n(x_2, a_2, y_2, \ldots, y_n) = \pi(x_1, a_1) + \rho E[\Phi_{n-1}(x_1 + Y_{a_1} - Z, y_2, \ldots, y_n)] \\ + \pi(x_2, a_2) + \rho E[\Phi_{n-1}(x_2 + Y_{a_2} - Z, y_2, \ldots, y_n)], \end{aligned}$$

since $\pi(x, a)$ is supermodular when maintenance is positive (Proposition A.1), we have

$$\begin{aligned}
&\geq \pi(x_1, a_2) + \pi(x_2, a_1) \\
&+ \rho E[\Phi_{n-1}(x_1 + Y_{a_1} - Z, y_2, \ldots, y_n)] \\
&+ \rho E[\Phi_{n-1}(x_2 + Y_{a_2} - Z, y_2, \ldots, y_n)],
\end{aligned}$$

applying the inequality (A.1), we can write

$$\begin{aligned}
&\geq \pi(x_1, a_2) + \pi(x_2, a_1) \\
&+ \rho E[\Phi_{n-1}(x_1 + Y_{a_2} - Z, y_2, \ldots, y_n)] \\
&+ \rho E[\Phi_{n-1}(x_2 + Y_{a_1} - Z, y_2, \ldots, y_n)], \\
&\geq J_n(x_1, a_2, y_2, \ldots, y_n) + J_n(x_2, a_1, y_2, \ldots, y_n),
\end{aligned}$$

which completes the proof for supermodularity of $J_n(x, a, y_2, \ldots, y_n)$ in (x, a). Following the same proof as in Proposition A.1, the supermodularity of $\Phi_n(x, y_1, \ldots, y_n)$ in (x, y_1) follows. □

Appendix B

Structural Properties of the Production Scheduling Problem

In this appendix, we prove a number of dominance properties for the production scheduling problem (PSP) defined in Section 5.2.1.2. Our computational results show that the dominance properties do not have a significant impact on decreasing the run-time of the PSP problem.

The appendix is organized as follows: We first describe the dominance properties, and then present our experiments. We end with a conclusion.

B.1 Dominance Properties

Four dominance properties of an optimal production and maintenance schedule are proved in this section as conditional statements. If the predicate of the statement is true, the consequent is added as a new constraint to the PSP model. The PSP model refers to the model given in Figure 5.6 in Section 5.2.1.2.

Property 1: If the duration of maintenance (t_p^m) on a given machine is less than or equal to the sum of the minimum possible processing times of the jobs on the upstream machines ($\sum_{l=1}^{m-1} \min_j(p_{jl})$), it is always best to schedule maintenance first on the machine. Note that $p_{jl} = \frac{n_{jl}}{\nu^l_{s_l}},\ \forall l \notin Q$, and $p_{jl} = n_{jl},\ \forall l \in Q$.

$$\textbf{if } t_p^m \leq \sum_{l=1}^{m-1} \min_j(p_{jl}) \textbf{ then } st_{pm} = 0,\ \ \forall m \in Q, m \neq 1.$$

We perform maintenance on machine m during the otherwise idle time while it is waiting for the first job to be executed on the upstream machines, $\{1, 2, ..., m-1\}$. An example of this property is shown in Figure B.1 where there are three machines, each shown in one row, where the solid and dashed rectangles represent the production and maintenance jobs, respectively and where the numbers inside the rectangles indicate the duration of the jobs. As illustrated, it is best to perform maintenance on the third machine while waiting for the first job to be processed on the first two machines.

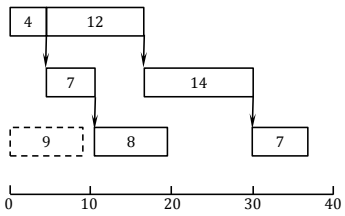


Figure B.1: An example of Property 1.

In the following three properties, we denote the increase in the processing time of job j on machine m if scheduled before maintenance as $\epsilon_{jm} = \frac{n_{jm}}{v^m_{sm}} - n_{jm}$.

Property 2: If the increase in the processing times of all jobs on a given machine is greater than or equal to the maintenance duration, then the schedule in which the maintenance is processed first is as good as or better than any other schedule.

$$\textbf{if } t^m_p \leq \epsilon_{jm}, \ \ \forall j \ \textbf{then } \ st_{pm} = 0, \ \ \forall m \in Q.$$

The time to perform maintenance is saved by the reduction in the processing time of any job processed after maintenance because the reduction in the processing times of all jobs is greater than the maintenance duration. Therefore, performing maintenance first in a schedule is as good as any other schedule.

Property 3: If the increase in the processing time of a job on a machine (ϵ_{jm}) is greater than or equal to the maintenance duration (t^m_p), the job is then scheduled after maintenance.

$$\textbf{if } t^m_p \leq \epsilon_{jm} \ \textbf{then } \ st_{pm} + t^m_p \leq st_{jm}, \ \ \forall j, \forall m \in Q.$$

Proof. Let $S_1 = \{\pi_1, f, \pi_2, p, \pi_3\}$ denote a feasible schedule of the jobs on machines in set Q where π_1, π_2, π_3 are partial schedules, p is the maintenance activity, and f is the first job whose processing time increase is greater than or equal to the maintenance duration. Note that the sequence of the jobs on different machines is not necessarily the same in S_1. To prove the property, it is enough to show that the schedule $S_2 = \{\pi_1, p, f, \pi_2, \pi_3\}$ where jobs only have a different sequence on machines in set Q dominates S_1. Let C^i_j denote the completion time of job j on the first machine in set Q (say m) in schedule S_i. Comparing the completion times of the jobs in the partial schedule π_1 in both S_1 and S_2 schedules, we have

$$C^1_j - C^2_j = 0, \ \forall j \in \pi_1.$$

Letting $|\pi_1|$ denote the index of the last job in π_1, we have

$$C_f^1 - C_f^2 = (C_{|\pi_1|}^1 + n_{fm} + \epsilon_{fm}) - (C_{|\pi_1|}^2 + t_p^m + n_{fm}) \geq 0.$$

The above inequality follows because of our assumption on the increase in the processing time of production job f. Assuming that l, j represent the indices of the jobs in π_2, we have

$$C_j^1 - C_j^2 = (C_f^1 + \sum_{l=1}^{l=j}(n_{lm} + \epsilon_{lm})) - (C_f^2 + \sum_{l=1}^{l=j} n_{lm}) \geq 0, \ \forall j \in \pi_2.$$

Denoting $|\pi_2|$ as the index of the last job in π_2 and l, j as the indices of the jobs in π_3, we have

$$C_j^1 - C_j^2 = (C_{|\pi_2|}^1 + t_p^m + \sum_{l=1}^{l=j} n_{lm}) - (C_{|\pi_2|}^2 + \sum_{l=1}^{l=j} n_{lm}) \geq 0, \ \forall j \in \pi_3.$$

As shown, the completion time of each job on machine m in schedule S_2 is smaller than or equal to its corresponding in schedule S_1. The same argument can be used to show that schedule S_2 finishes the processing of any job on any downstream machine, i.e., $\{m+1, ..., M\}$, no later than schedule S_1. Therefore, in schedule S_2, the number of lost jobs is less than or equal to the one in schedule S_1 which completes the proof. □

Property 4: If the maintenance duration on a given machine (t_p^m) is greater than the sum of the increase in the processing times of all possible combinations of l out of $|\mathcal{J}|$ jobs, the schedule in which maintenance is scheduled after $|\mathcal{J}| - s$ jobs such that $0 < s \leq l$ is not optimal.

$$\textbf{if } \ t_p^m > \sum_{i=1}^{i=l} \epsilon_{j_i m}, \ \forall j_1, j_2, ..., j_l \ \textbf{ then}$$
$$\sum_{j=1}^{|\mathcal{J}|} b_{jm} \leq (|\mathcal{J}| - l - 1)w + |\mathcal{J}|(1-w), \quad \forall m \in Q,$$
$$\sum_{j=1}^{|\mathcal{J}|} b_{jm} \geq |\mathcal{J}|(1-w), \quad \forall m \in Q.$$

Note that $|\mathcal{J}|$ is the number of jobs in the PSP and w is a binary variable. The consequence guarantees that maintenance is either scheduled last or scheduled after at most $(|\mathcal{J}| - l - 1)$ jobs.

Proof. Following the same reasoning as Property 3 where $\{\pi_1, p, \pi_2\}$ and $\{\pi_1, \pi_2, p\}$ represent the sequence of the jobs on machines $m \in Q$ in schedules S_1 and S_2, respectively, the property follows. Note that the number of jobs in partial schedule π_2 is equal to s. □

B.2 Empirical Study

The next sub-section describes the problem instances and the experimental details. We then investigate the effect of dominance properties on the speed of the solver.

B.2.1 Experimental Setup

All problem instances have $M \in \{3, 4, 5, 6\}$ machines and $|\mathcal{J}| \in \{5, 10, 15, 20\}$ jobs. As three of the properties in Section B.1 are defined based on the comparison between the increase in the processing times of the jobs and the maintenance duration, we define $\rho \in \{0.5, 1, 1.5\}$ as an indication of the ratio between the increase in the processing times of the jobs and the maintenance duration. Ten instances for each combination of parameters are generated, resulting in 480 instances. The maintenance duration for machine m, t_p^m, is drawn from the discrete uniform distribution $[5, 15]$. It is assumed that all machines need maintenance, i.e., $Q = \{1, 2, ..., M\}$. The processing time at the best state of machine m, n_{jm}, and the increase in the processing times of the jobs, ϵ_{jm} are generated from the discrete uniform distributions $[10, 20]$ and $[5\rho, 15\rho]$. We set the length of the scheduling horizon at $T = 1.2 \times LB$ where LB equals the sum of the first M biggest processing times of the jobs at the best state of machines. The due date of job j is set at $\min(T, f^d \times \sum_{m=1}^{M} n_{jm})$, where $f^d = 1.5$ is the due-date factor (Pinedo and Singer, 1999).

All experiments were run on an AMD 270 CPU with 1 MB cache per core, 4 GB of main memory, running Red Hat Enterprise Linux 4. The MIP solver is CPLEX 12.1. A time-limit of 900 seconds is used for each instance.

B.2.2 Computational Results

Figure B.2 shows scatter-plots of run-times of the PSP model with and without the dominance properties. Both axes are log-scale, and the points below the line $y = x$ indicate a lower run-time for the algorithm on the y-axis. The numbers in the boxes indicate the number of points below or above the line. Run-times are counted as equal if they differ less than 10%.

Although the graph illustrates more points below the line $y = x$, Table B.1 shows that both the mean run-time and the percentage of unsolved problems decrease only by 3% over all problem instances when the dominance properties are used.

Method	Mean	% Unsolved
Figure B.2: without DP	576.43	63
Figure B.2: with DP	559.23	61
Figure B.3: without DP ($\rho = 0.5$)	621.62	69
Figure B.3: with DP ($\rho = 0.5$)	614.77	68
Figure B.3: without DP ($\rho = 1$)	579.02	63
Figure B.3: with DP ($\rho = 1$)	569.11	63
Figure B.3: without DP ($\rho = 1.5$)	528.66	58
Figure B.3: with DP ($\rho = 1.5$)	493.80	54

Table B.1: The mean run-time and the percentage of unsolved problems.

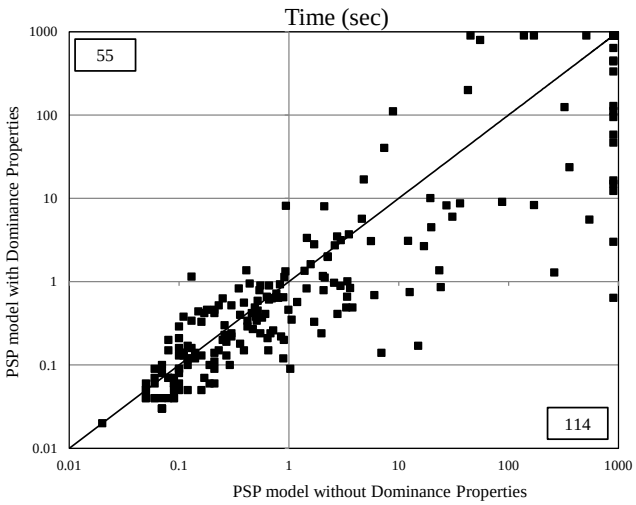


Figure B.2: Run-times of the PSP model with and without the dominance properties (DP).

Figure B.3 shows the same results as in Figure B.2 with the problem instances divided based on different values of ρ. As illustrated, the number of points below the line $x = y$ increases as ρ increases. Table B.1 shows that the mean run-time decreases by 1.1%, 1.7%, and 6.6%, while the percentage of unsolved problems decreases by 1.4%, 0%, and 6.9% as ρ increases.

When $\rho = 1.5$, the number of jobs whose processing time increase is greater than the maintenance duration is more. Therefore, Properties 2 and 3 are likely to rule out a large number of possible schedules, decreasing the run-time. To support our conjecture, Figure B.4 shows the difference between the mean run-times of PSP models with and without dominance properties for different ρ values. As illustrated, when ρ is higher, the average number of times the left-hand side (LHS) of dominance properties is triggered increases. Therefore, the dominance properties are in effect, decreasing the run-times.

In summary, using the dominance properties in the solver does not result in a significant speed-up. However, there is evidence showing that the properties can reduce the run-time if they are effective, i.e., their predicate is true.

B.3 Conclusion

In this appendix, several dominance properties are developed to provide some insight on the optimal schedule of the maintenance activities along with the production activities for the production scheduling problem (PSP) discussed in Chapter 5.

Our computational results show that incorporating the dominance properties does not significantly decrease the run-times. However, there is evidence that the properties can lead to a lower run-time if they are in effect.

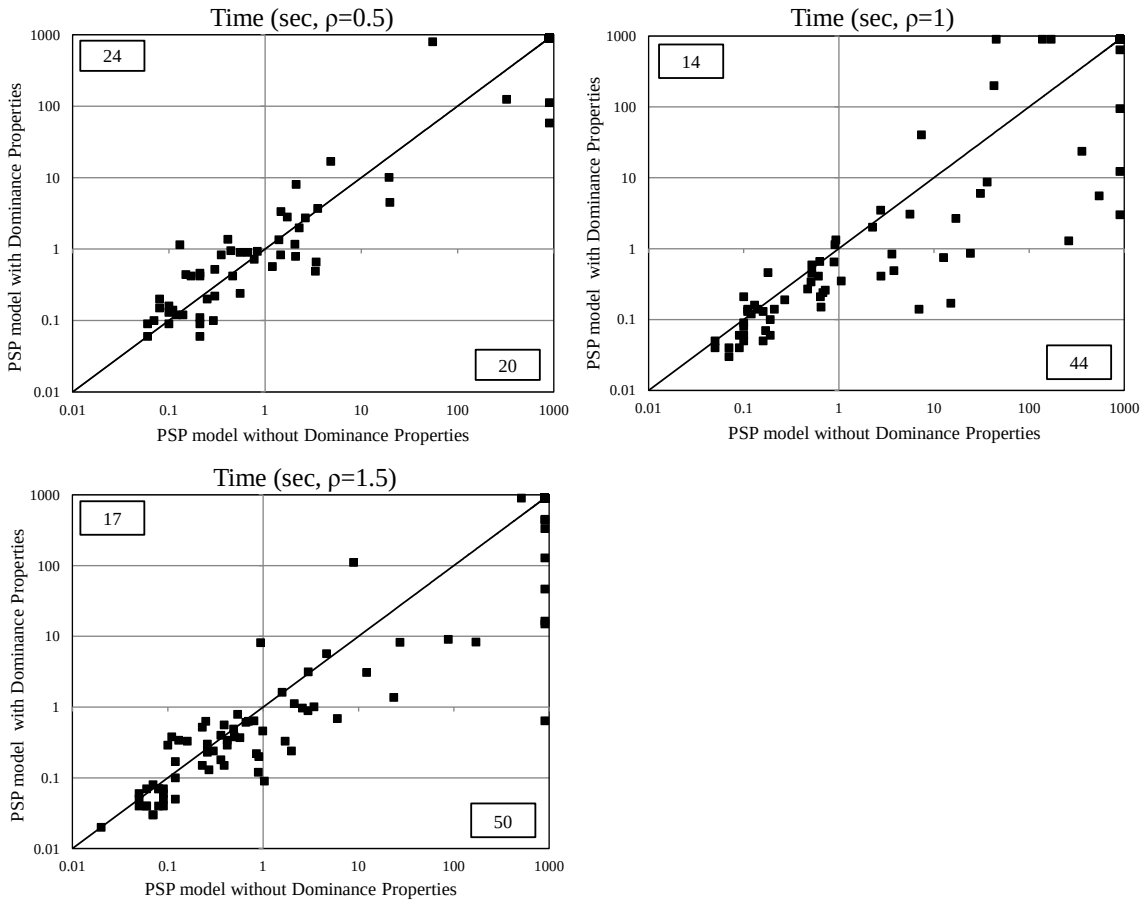


Figure B.3: Run-times of the PSP model with and without the dominance properties (DP) for different ρ values.

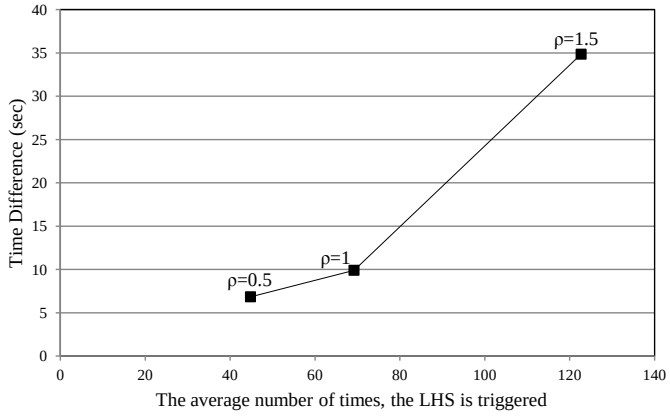


Figure B.4: Difference between mean run-times of the PSP models with and without the dominance properties for different ρ values.

Appendix C

Approximating the Average Production Rate

In Section 6.3.2.1, we discussed a simple approach to approximate the average production rate of a machine. In this appendix, we discuss an exact approach.

This appendix is organized as follows. We first present the exact method for calculating the average production rate of machine m within the time period. We then discuss the error of the approximation approach presented in Section 6.3.2.1. In Section C.3, we solve a numerical example to compare the two approaches and explain the reasons for using the approximation method in the experimental study of Section 6.5.2.

C.1 Exact Method

The main idea of the exact method is to define the production quantity (the number of products produced within the time period) as a random variable and to find its expected value. The average production rate then equals the expected production quantity per time unit.

In this section, we first calculate the average production rate of machine m in a time period with length t assuming that it does not need maintenance and then extend the result to the case when machine m needs maintenance. In the latter case we need to make an assumption on the time at which maintenance is performed. Similar to Section 6.3.2.1, we assume that maintenance is performed at the beginning of the interval and that maintenance duration is negligible.

C.1.1 Machine m does not Need Maintenance

In this section, we assume that machine m does not need maintenance. We do not include the index of machine in the notation for ease of reading. We use the following notation.

- $[0, t)$: The time period starting at time 0 and ending at time t.
- X_t: The state of machine m at time t.

- $r(i)$: The production rate of machine m at state i.
- y: The time of the first transition from the initial state, i.e., $X_0 = i$, with pdf $h(y|i)$ and CDF $H(y|i)$.
- P_{ik}: The probability of going to state k ($k \neq i$) given machine m has left its current state, i. It is equal to $\frac{q_{ik}}{-q_{ii}}$ where q_{ik} is the transition rate from state i to state k and where $\frac{1}{-q_{ii}}$ is the expected time to leave the state i.
- $V(t, i)$: The expected number of products produced by machine m in a time period with length t given the initial state is $X_0 = i$.
- $J(t, i)$: The average production rate of machine m in an interval with length t given the initial state i.

Let the random variable $N[0, t)$ denote the number of products produced by machine m in the interval $[0, t)$. By conditioning on the first time to leave the initial state, i.e., y, we have

$$N[0,t) = \begin{cases} r(X_0)t & y > t, \\ r(X_0)y + N[y,t) & y \leq t. \end{cases}$$

If the first time to transition from the initial state is greater than the length of the time period, i.e., $y > t$, the state of machine m does not change within the time interval. Machine m continues production at rate $r(X_0)$ and the number of products produced within t units of time equals $r(X_0)t$. However, if $y \leq t$, the number of products within the time period is then the sum of the following:

1. The number of products produced before time y in the interval $[0, y)$ which equals $r(X_0)y$.
2. The number of products produced in the interval $[y, t)$ which equals $N[y, t)$.

Therefore, the expected number of products produced in the interval with length t given that the initial state of the machine is i can be represented as:

$$\begin{aligned} V(t,i) &= E[N[0,t)|X_0 = i] \\ &= E[r(X_0)tI(y > t)|X_0 = i] + E[r(X_0)yI(y \leq t)|X_0 = i] + E[N[y,t)I(y \leq t)|X_0 = i] \\ &= r(i)E[\min(t,y)|X_0 = i] + E[E[N[y,t)I(y \leq t)|X_y = k, X_0 = i]|X_0 = i] \\ &= r(i)E[\min(t,y)|X_0 = i] + E[I(y \leq t)E[V(t-y,k)|X_0 = i]|X_0 = i]. \end{aligned} \tag{C.1}$$

The above equation can be written as:

$$V(t,i) = r(i)\int_0^t (1 - H(y|i))dy + \sum_{\substack{k=0 \\ k\neq i}}^{N} \int_0^t P_{ik}V(t-y,k)dH(y|i).$$

The first time to leave the initial state has an exponential distribution, i.e., $H(y|i) = 1 - e^{q_{ii}y}$ and we can write the above equation as follows:

$$V(t,i) = \int_0^t r(i)e^{q_{ii}y}dy + \sum_{\substack{k=0 \\ k\neq i}}^{N} \int_0^t q_{ik}V(t-y,k)e^{q_{ii}y}dy. \tag{C.2}$$

Finally, the average production rate equals

$$J(t,i) = \frac{V(t,i)}{t}. \tag{C.3}$$

Equation (C.2) can be written as a system of linear equations using Laplace transform since the time to first transition, y, has an exponential distribution. The details are given in the next section.

C.1.1.1 Solving Equation (C.2)

Denote the Laplace transform of the expected number of products as $\mathcal{F}(s,i) = \int_0^\infty e^{-st}V(t,i)dt$. Taking the Laplace transform from both sides of Equation (C.2), we have

$$\begin{aligned}
\mathcal{F}(s,i) &= \int_0^\infty e^{-st}\int_0^t r(i)e^{q_{ii}y}dydt + \int_0^\infty e^{-st}\sum_{\substack{k=0 \\ k\neq i}}^{N}\int_0^t q_{ik}V(t-y,k)e^{q_{ii}y}dydt \\
&= \frac{r(i)}{s(s-q_{ii})} + \sum_{\substack{k=0 \\ k\neq i}}^{N} q_{ik}\int_0^\infty e^{-(s-q_{ii})y}\int_y^\infty e^{-s(t-y)}V(t-y,k)dtdy \\
&= \frac{r(i)}{s(s-q_{ii})} + \frac{1}{s-q_{ii}}\sum_{\substack{k=0 \\ k\neq i}}^{N} q_{ik}\mathcal{F}(s,k).
\end{aligned} \tag{C.4}$$

Taking an iterative procedure, the solution to Equation (C.4) is:

$$\begin{aligned}
\mathcal{F}(s,i) = \frac{r(i)}{s(s-q_{ii})} + \frac{1}{s(s-q_{ii})}\Big[&\sum_{k_1}^{N}\frac{r(k_1)q_{ik_1}}{s-q_{k_1k_1}} + \sum_{k_1,k_2}^{N}\frac{r(k_2)q_{ik_1}q_{k_1k_2}}{(s-q_{k_1k_1})(s-q_{k_2k_2})} \\
&+ \sum_{k_1,k_2,k_3}^{N}\frac{r(k_3)q_{ik_1}q_{k_1k_2}q_{k_2k_3}}{(s-q_{k_1k_1})(s-q_{k_2k_2})(s-q_{k_3k_3})} \\
&+ \dots \\
&+ \sum_{k_1,k_2,\dots,k_n}^{N}\frac{r(k_n)q_{ik_1}q_{k_1k_2}\dots q_{k_{n-1}k_n}}{(s-q_{k_1k_1})(s-q_{k_2k_2})\dots(s-q_{k_nk_n})}\Big],
\end{aligned}$$

where $k_j = k_{j-1} + 1$, $k_1 = i + 1$, and $k_n = N$. Note that $\sum_{k_1,k_2}^{N} = \sum_{k_1}^{N}\sum_{k_2}^{N}$. However, to find the expected number of products, $V(t,i)$, we need the Laplace inverse, $V(t,i) = \mathcal{L}^{-1}(\mathcal{F}(t,i))$. Using the standard

Laplace inverse table,

$$V(t,i) = \frac{r(i)}{-q_{ii}}(1-e^{q_{ii}t}) + \sum_{k_1}^{N} r(k_1)q_{ik_1}[\frac{1}{q_{ii}q_{k_1k_1}} + \frac{e^{q_{ii}t}}{(q_{ii}-q_{k_1k_1})q_{ii}} + \frac{e^{q_{k_1k_1}t}}{(q_{k_1k_1}-q_{ii})q_{k_1k_1}}]$$
$$+ \dots$$
$$+ \sum_{k_1,k_2,\dots,k_n}^{N} r(k_n)q_{ik_1}\dots q_{k_{n-1}k_n}[\frac{1}{(-1)^{(n-i+1)}q_{ii}q_{k_1k_1}\dots q_{k_nk_n}} +$$
$$\frac{e^{q_{ii}t}}{(q_{ii}-q_{k_nk_n})\dots(q_{ii}-q_{k_1k_1})q_{ii}} + \dots +$$
$$\frac{e^{q_{k_nk_n}t}}{(q_{k_nk_n}-q_{k_{n-1}k_{n-1}})\dots q_{k_nk_n}}]. \tag{C.5}$$

C.1.2 Machine m Needs Maintenance

If machine m needs maintenance, assuming that maintenance is performed at the beginning of the time period with a negligible duration, we follow the same reasoning as the previous section and calculate $J(t,i)$ as below where R_{ik} is the probability that machine m changes its initial state, i, to state k as a result of maintenance. When machine m transitions into a new state k after performing maintenance, we have the same problem as the previous section with the only difference that the initial state of machine m is k.

$$J(t,i) = \sum_{k=0}^{N} R_{ik}J(t,k) = \frac{\sum_{k=0}^{N} R_{ik}V(t,k)}{t} \tag{C.6}$$

C.2 Error of the Approximation Method

Our early analysis showed that the best approach for calculating the error is to use matrix representations of the average production rate. In this section, we use the following notation:

- Q: Transition rate matrix
- $P(t) = e^{Qt}$: Transition probability matrix within t units of time without performing maintenance
- R: Maintenance probability matrix
- r: Production rate vector
- $V(t)$: The vector of expected number of products in the interval $[0,t)$
- $J(t)$: The vector of exact average production rate in the interval $[0,t)$
- $A(t)$: The vector of approximated average production rate in the interval $[0,t)$

Defining r_{t_1} as the random production rate at time t_1, the random number of products produced in the interval $[0, t)$ equals $\int_0^t r_{t_1} dt_1$. Therefore, we have

$$\begin{aligned} V(t) = E[\int_0^t r_{t_1} dt_1] &= \int_0^t E[r_{t_1}] dt_1 \\ &= \int_0^t P(t_1) r dt_1 \\ &= (\int_0^t e^{Qt_1} dt_1) r \\ &= (\int_0^t [I + \frac{Qt_1}{1!} + \frac{(Qt_1)^2}{2!} + \frac{(Qt_1)^3}{3!} + \ldots] dt_1) r \\ &= [It + \frac{(Qt)t}{2!} + \frac{(Qt)^2 t}{3!} + \frac{(Qt)^3 t}{4!} + \ldots] r. \\ J(t) = \frac{V(t)}{t} &= [I + \frac{Qt}{2!} + \frac{(Qt)^2}{3!} + \frac{(Qt)^3}{4!} + \ldots] r. \end{aligned}$$

Multiplying the right hand side by $(Qt)(Qt)^{-1}$, we have

$$\begin{aligned} &= [I + \frac{Qt}{2!} + \frac{(Qt)^2}{3!} + \frac{(Qt)^3}{4!} + \ldots](Qt)(Qt)^{-1} r \\ &= [\frac{Qt}{1!} + \frac{(Qt)^2}{2!} + \frac{(Qt)^3}{3!} + \ldots](Qt)^{-1} r \\ &= [e^{Qt} - I](Qt)^{-1} r. \end{aligned} \tag{C.7}$$

The matrix representation of the approximation method (Equation 6.19) is

$$A(t) = \frac{1}{2}[I + P(t)]r = \frac{1}{2}[I + e^{Qt}]r \tag{C.8}$$

Let us define the error vector of the approximation method in an interval with length t as $\epsilon(t) = J(t) - A(t)$. In the following sections, we bound $\epsilon(t)$ in case machine m does not need maintenance and in case it needs.

C.2.1 Machine m does not Need Maintenance

The error of the approximation method is

$$\begin{aligned} \epsilon(t) = J(t) - A(t) &= ([I + \frac{Qt}{2!} + \frac{(Qt)^2}{3!} + \frac{(Qt)^3}{4!} + \ldots] \\ &\quad - \frac{1}{2}[I + I + \frac{Qt}{1!} + \frac{(Qt)^2}{2!} + \frac{(Qt)^3}{3!} + \ldots]) r \\ &= ((Qt)^2[\frac{1}{3!} - \frac{1}{2 \times 2!}] + (Qt)^3[\frac{1}{4!} - \frac{1}{2 \times 3!}] + \ldots) r \\ &= \frac{-1}{2}[\sum_{k=2}^{\infty} \frac{k-1}{(k+1)!}(Qt)^k] r \end{aligned} \tag{C.9}$$

We can bound the norm[1] of the error vector as below.

$$\|\epsilon(t)\| = \|\frac{-1}{2}[\sum_{k=2}^{\infty} \frac{k-1}{(k+1)!}(Qt)^k]r\| \leq \frac{1}{2}[\sum_{k=2}^{\infty}(\|Q\|t)^k \times \frac{1}{(k)!}]\|r\| = \frac{1}{2}(e^{\|Q\|t} - \|Q\|t - 1)\|r\| \tag{C.10}$$

C.2.2 Machine m Needs Maintenance

When machine m needs maintenance, we have

$$\epsilon(t) = RJ(t) - RA(t) = R(\frac{-1}{2}[\sum_{k=2}^{\infty} \frac{k-1}{(k+1)!}(Qt)^k]r).$$

Therefore, the bound on the norm of the error vector is

$$\|\epsilon(t)\| \leq \frac{\|R\|}{2}(e^{\|Q\|t} - \|Q\|t - 1)\|r\|. \tag{C.11}$$

C.3 Numerical Example

In this section, we solve a numerical example to compare the average production rate of a machine using the approximation method and the exact method.

We assume that the machine has four working states and one failure state. The transition rate matrix, $Q = [q_{ik}]$, the maintenance probability matrix, $R = [R_{ik}]$, and the production rate vector, $r = [r(i)]$, are given below.

$$Q = \begin{pmatrix} -0.001 & 0.0005 & 0.0003 & 0.00015 & 0.00005 \\ 0 & -0.0008 & 0.0005 & 0.0002 & 0.0001 \\ 0 & 0 & -0.0007 & 0.0004 & 0.0003 \\ 0 & 0 & 0 & -0.0005 & 0.0005 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$R = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0.9 & 0.1 & 0 & 0 & 0 \\ 0.8 & 0.15 & 0.05 & 0 & 0 \\ 0.7 & 0.2 & 0.1 & 0 & 0 \\ 0.6 & 0.3 & 0.1 & 0 & 0 \end{pmatrix},$$

[1] Informally, a vector norm measures the magnitude of the vector. The p-norm of the vector $x = (x_1, x_2, \ldots, x_n)$ is defined as $\|x\|_p, \forall p = 1, 2, \ldots$ which equals $(\sum_{i=1}^{n} x_i^p)^{\frac{1}{p}}$.

$$r = \begin{pmatrix} 0.2 \\ 0.15 \\ 0.1 \\ 0.05 \\ 0 \end{pmatrix}.$$

Assuming that $T = 200$, the transition probability matrix when machine m does not need maintenance, $P(T) = [p_{ik}] = e^{QT}$, is:

$$P(200) = \begin{pmatrix} 0.8187 & 0.0835 & 0.0549 & 0.0297 & 0.0131 \\ 0 & 0.8521 & 0.0861 & 0.0386 & 0.0232 \\ 0 & 0 & 0.8694 & 0.0710 & 0.0597 \\ 0 & 0 & 0 & 0.9048 & 0.0952 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Table C.1 shows the average production rate of machine m if it does not need maintenance using the approximation and the exact methods.

Initial state	0	1	2	3	4
Approximation Method	0.1916	0.1442	0.0952	0.0476	0
Exact Method	0.1881	0.1432	0.0952	0.0476	0

Table C.1: The average production rate of machine m given its initial state is i, $\forall i \in \{0, 1, 2, 3, 4\}$ and it does not need maintenance using the approximation and the exact methods.

In the approximation method, the average production rate of machine m is approximated using Equation (6.19). In the exact method, we first solve the system of linear equations (Equation C.4) and then take the Laplace inverse finding the expected number of products, $V(T, i)$. Dividing $V(T, i)$ by T gives the average production rate as shown in the second row of Table C.1.

Table C.2 shows the same results as Table C.1 when machine m needs maintenance. The first and the second rows are calculated using Equations (6.20) and (C.6), respectively.

Initial state	0	1	2	3	4
Approximation Method	0.1916	0.1869	0.1797	0.1725	0.1677
Exact Method	0.1881	0.1836	0.1767	0.1698	0.1653

Table C.2: The average production rate of machine m given its initial state is i, $\forall i \in \{0, 1, 2, 3, 4\}$ and it needs maintenance using approximation and the exact methods.

Equations (C.10) and (C.11) result in the error bound of less than 0.0043 and 0.008 in case machine m does not need maintenance and in case it does, respectively.[2] Table C.1 and Table C.2 show that the maximum difference between the approximated average production rate and its exact value is 0.0035 in both cases of no maintenance and maintenance which is less than the corresponding theoretical achieved error bounds.

[2]We use 2-norm in our calculation.

The main question of our interest is to decide which method is appropriate to calculate the average production rate. We use the approximation method in our experimental study (Section 6.5.2) mainly because of the following:

- In our experimental setup (see Appendix D), the transition rate and the maintenance probability matrices have small norms. Equations (C.10) and (C.11) therefore result in error bounds close to 0 implying that the approximated average production rate is very close to the exact value.
- As the number of states increases, Equation (C.5) is not computationally efficient since the denominators of the fractions in the sums tend to 0. The alternative approach is to solve the integral Equation (C.2) using successive approximation procedure (Pogorzelski, 1966; Keffer, 1999). However, our previous experiment on the successive approximation procedure shows that its convergence speed is very slow specially as t increases. It is also worth mentioning that we do not use the matrix representation of the exact average production rate (Equation C.7) for calculation since the transition rate matrix Q is singular in the majority of our problem instances and the inverse matrix Q^{-1} does not exist.
- The approximated procedure is simple requiring less information and is fast.

Appendix D

Experimental Setup of Chapter 6

This appendix describes the detailed experimental setup of Chapter 6. We explain the simulation of the data related to time periods, machines, and jobs in the next three sections.

D.1 Time Periods

Discount Factor (ρ)		$\{0.2, 0.5, 0.8, 0.95\}$
Number of Time Periods (K)	6	if $\rho = 0.2$
	14	if $\rho = 0.5$
	42	if $\rho = 0.8$
	180	if $\rho = 0.95$
Lost Cost (h)		$U[50, 100]$
Demand (Z_k)		$U[4, 6]$, $U[8, 12]$
Length of Time Period (T)	50	if $Z_k \sim U[4, 6]$
	100	if $Z_k \sim U[8, 12]$
Maintenance Capacity (C)		$\lfloor \frac{M}{2} \rfloor$

Table D.1: The range of the data related to time periods where M is the number of machines.

D.2 Machines

Number of Machines: The number of machines varies between three and five, i.e., $M \in \{3, 4, 5\}$.

Number of States: The number of states for each machine, $N_m + 1$, equals 5.

Initial State: The initial state of each machine, i_m, equals $(X - 1)$ if $X < 2$ and $(X - 2)$, otherwise where X is generated from the discrete uniform distribution $U[1, N_m]$.

Maintenance Time: The maintenance duration for machine m, t_p^m, is drawn from discrete uniform distribution $U[0.05 \times T, 0.15 \times T]$ where T is the length of the time period.

Transition Rate: The state transition rate matrix of machine m is defined as $Q^m = [q_{ik}^m]_{(N_m+1)\times(N_m+1)}$ where $\sum_{k \geq l} q_{ik}^m < \sum_{k \geq l} q_{(i+1)k}^m, \forall l \geq (i + 2)$. To generate such a matrix for machine m, a value is first assigned to q_{00}^m corresponding to each deterioration factor such that $q_{00}^m = -V(\mathcal{DF} + 1)$, $\forall \mathcal{DF} \in$

$\{1, 2, 3, 4, 5\}$ where $V = [\frac{1}{5\times10^5}, \frac{1}{5\times10^4}, \frac{1}{5\times10^3}, \frac{1}{5\times10^2}, \frac{1}{5\times10^1}]$. As the deterioration factor increases, the mean time that the machine spends in its best state, $\frac{-1}{q^m_{00}}$, becomes shorter.

After generating q^m_{00}, the other elements of the first row of the matrix are generated following Algorithm 5. Since the sum of the elements of each row equals 0, we simulate how $-q^m_{00}$ is divided between the other elements. As shown in Algorithm 5, we first generate a random number from $U[1, N_m]$ representing the number of states that the machine transitions into leaving its best state. We then generate a vector containing a random permutation of the integers from 1 to the generated number to find the ratio based on which $-q^m_{00}$ is divided.

Algorithm 5 Simulating $q^m_{0i}, \forall i > 0$

1: $q^m_{0i} \leftarrow 0, \forall i > 0$
2: number of states to go, $n \leftarrow U[1, N_m]$
3: $\mathbb{D} \leftarrow randperm(\text{n})$
4: **for** $i = 1 : n$ **do**
5: $\quad q^m_{0i} \leftarrow \frac{-q^m_{00}\times\mathbb{D}(i)}{\sum\mathbb{D}}$
6: **end for**

We then need to generate the other elements of the matrix such that $\sum_{k\geq l} q^m_{ik} < \sum_{k\geq l} q^m_{(i+1)k}, \forall l \geq (i + 2)$. Algorithms 6 shows the procedure. As shown in Figure D.1, the stated condition requires that the sum of elements in the i-th row from the l-th column to the last column should be less than the sum of corresponding elements in the $(i + 1)$-th row, i.e., $s_1 < s_2 + q^m_{(i+1)l}$. Line 10 guarantees that the stated condition holds.

Algorithm 6 Simulating $q^m_{(i+1)l}, \forall i \geq 0, \ \forall l$

1: $q^m_{(i+1)l} \leftarrow 0, \forall i \geq 0, \ \forall l$
2: **for** $i = 0 : N_m - 2$ **do**
3: $\quad$ **for** $l = N_m : -1 : i + 2$ **do**
4: $\quad\quad s_1 \leftarrow \sum_{k=l}^{N_m} q^m_{ik}$
5: $\quad\quad$ **if** $l = N_m$ **then**
6: $\quad\quad\quad s_2 \leftarrow 0$
7: $\quad\quad$ **else**
8: $\quad\quad\quad s_2 \leftarrow \sum_{k=l+1}^{N_m} q^m_{(i+1)k}$
9: $\quad\quad$ **end if**
10: $\quad\quad q^m_{(i+1)l} \leftarrow \max(0, s_1 - s_2) + \frac{U[0,\mathbb{V}(\mathcal{DF})]}{2}$
11: $\quad$ **end for**
12: $\quad q^m_{(i+1)(i+1)} \leftarrow -\sum_{j=i+2}^{N_m} q^m_{(i+1)j}$
13: **end for**

Maintenance Probability: The maintenance probability matrix $R^m = [R^m_{ik}]$ should be generated such that

$$R^m_{ii} = R^m_{(i+1)i} + R^m_{(i+1)(i+1)}, \tag{D.1}$$

$$R^m_{ij} = R^m_{(i+1)j}, \ \forall j \leq (i - 1). \tag{D.2}$$

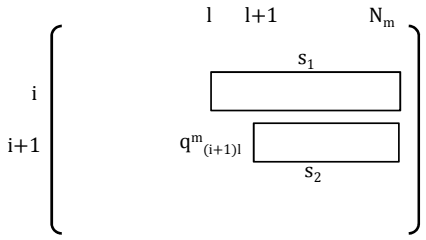


Figure D.1: Transition Rate Matrix

Algorithm 7 shows the procedure for generating maintenance probability matrix.

First, we assign 0 to $R^m_{N_m N_m}$, the probability of not leaving the failure state after maintenance. Second, we generate the other elements of the last row of the maintenance probability matrix. Since the sum of the elements in each row equals 1, we need to randomly divide $(1 - R^m_{N_m N_m})$ among the other elements. The idea is the same as the one used in generating the first row of the transition rate matrix. As shown in Algorithm 7, we generate a random number from $U[1, N_m]$ representing the number of states that the machine transitions into leaving its worst state after maintenance. We then generate a vector containing a random permutation of the integers from 1 to the generated number to find the ratio based on which $(1 - R^m_{N_m N_m})$ is divided. The other rows of the matrix are generated following both conditions (D.1) and (D.2) as shown in lines 10 and 12.

Algorithm 7 Simulating $\mathcal{R}^m$

1: $R^m_{ij} \leftarrow 0,\ \forall i, j$
2: number of states to go, n, $\leftarrow U[1, N_m]$
3: $\mathbb{D} \leftarrow randperm(n)$
4: index $\leftarrow 0$
5: **for** $j = N_m - 1 : -1 : N_m - n$ **do**
6: $\quad R^m_{N_m j} \leftarrow \frac{(1-R^m_{N_m N_m}) \times \mathbb{D}(n-\text{index})}{\sum \mathbb{D}}$
7: $\quad$ index $\leftarrow$ index + 1
8: **end for**
9: **for** $i = N_m - 1 : -1 : 1$ **do**
10: $\quad R^m_{ii} \leftarrow R^m_{(i+1)(i+1)} + R^m_{(i+1)i}$
11: $\quad$ **for** $j = 0 : i - 1$ **do**
12: $\quad\quad R^m_{ij} \leftarrow R^m_{(i+1)j}$
13: $\quad$ **end for**
14: **end for**
15: $R^m_{00} \leftarrow 1$

Transition Probability: The transition probability matrix of machine m, $P^{ma} = [p^{ma}_{ik}]$, is the probability of changing the state from i to k within T units of time given action a. Since $P^{m0} = e^{Q^m T}$, the matrix exponential function, *expm*, in MATLAB is used to calculate P^{m0}, i.e., $P^{m0} = expm(Q^m T)$. Then we have $P^{m1} = R^m \times P^{m0}$ where R^m is the maintenance probability matrix.

Production Rate: The production rate of machine m at state $i \in \{0, \ldots, N_m\}$ is $W(i + 1)$ where we set $N_m = 4$ and $W = [0.2, 0.15, 0.1, 0.05, 0]$. The production rate of machine m at state i given action a,

$r^m(i,a)$, is calculated as below where R^m_{il} is the maintenance probability.

$$r^m(i,0) = W(i+1),\ r^m(i,1) = \sum_{l=0}^{N_m} R^m_{il} W(l+1).$$

Maintenance Cost: We need to generate maintenance cost of machine m at state i, $\tau^m(i)$, such that condition (D.3) holds true.

$$\begin{aligned}\tau^m(i+1) - \tau^m(i) \leq\ & h(z - Tr^m(i+1,0))^+ - h(z - Tr^m(i+1,1))^+ \\ & - h(z - Tr^m(i,0))^+ + h(z - Tr^m(i,1))^+,\ \ \forall i,\ \forall z \end{aligned} \tag{D.3}$$

Moreover, $\tau^m(i+1) \geq \tau^m(i)$ implying that the left-hand side of condition (D.3) is greater than 0. Algorithm 8 shows the procedure that we use to generate the maintenance cost at each state for each machine where a and b are the lower and the upper bounds for demand. First maintenance cost at the worst state is generated from the uniform distribution $U[50,100]$. As shown in Line 6 of Algorithm 8, if the right-hand side of condition (D.3) is less than 0, the required condition does not hold and we re-initiate the procedure of generating the data from the beginning. Otherwise, as line 9 shows in Algorithm 8, $\tau^m(i)$ is generated such that condition (D.3) holds true.

Algorithm 8 Simulating $\tau^m(i),\ \forall i$

1: $\tau^m(N_m) \leftarrow U[50,100]$
2: **for** $i = N_m - 1 : -1 : 0$ **do**
3: **for** $z = a : b$ **do**
4: $c \leftarrow h(z - Tr^m(i+1,0))^+ - (z - Tr^m(i+1,1))^+ - h(z - Tr^m(i,0))^+ + (z - Tr^m(i,1))^+$
5: **if** $c < 0$ **then**
6: re-initiate simulating the data
7: **end if**
8: **end for**
9: $\tau^m(i) \leftarrow \tau^m(i+1)$
10: **end for**

D.3 Jobs

Nominal Processing Time: The nominal processing time of the production activity j on machine m, n_{jm}, is drawn from the discrete uniform distribution $U[1,9]$. It is worth mentioning that the nominal processing time distribution is chosen such that its mean equals $\frac{1}{r^m(0)} = 5$.

Due date: The due date of production activity j is set at $\min(T, f^d \times \sum_{m=1}^{M} n_{jm})$ where f^d, the due-date factor, is 2.5 and 3 for $T = 50$ and $T = 100$, respectively.

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