Online Appendix to "Compiling Optimal Numeric Planning to Mixed Integer Linear Programming", published at ICAPS2018

C. Piacentini[†], M. P. Castro[†], A. A. Cire[‡], J. C. Beck[†]
[†]Department of Mechanical and Industrial Engineering,
University of Toronto, Toronto, Canada, ON M5S 3G8
[‡]Department of Management,
University of Toronto Scarborough, Toronto, Canada, ON M1C 1A4

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1 State-change IP Model for Classical Planning

Consider $\Pi = \langle V_p, V_n, A, I, G\rangle$ with $V_n = \emptyset$ and $T \in \mathbb{Z}^+$. Let $\mathcal{T} = \{0, ..., T-1\}$ and $\tilde{\mathcal{T}} = \mathcal{T} \cup \{T\}$ be sets of time-steps. For each $p \in V_p$, let

- $pnd(p) = \{a \in A : p \in pre_p(a), p \notin del(a)\}$ be the set of actions that require and do not delete p,
- $anp(p) = \{a \in A : p \notin pre_p(a), p \in add(a)\}$ the set of actions that add and do not require p, and
- $pd(p) = \{a \in A : p \in pre_p(a), p \in del(a)/add(a)\}$ the set of actions that require and delete p.

Variable $u_{a,t} \in \{0,1\}\ \forall a \in A, \forall t \in \mathcal{T}$ indicates if a is applied at time-step t. Consider variables $u^a_{p,t}$, $u^{pa}_{p,t}$, $u^{pd}_{p,t}$ and $u^m_{p,t} \in \{0,1\}\ \forall p \in V_p, \forall t \in \tilde{\mathcal{T}}$.

- $u^a_{p,t}$ indicates whether p is added at time-step t but it is not required before.
- $u^{pa}_{p,t}$ indicates whether p is required and it is not deleted by any action at time-step t.
- $u^{pd}_{p,t} = 1$ if p is deleted and not added at time-step t but is required before.
- $u^m_{p,t} = 1$ if p is true at time-step t and is not required nor deleted.

The *state-change* model $\mathcal{SC}(\Pi, T)$ is as follows. Constraints (1) and (2) represent the initial state and goal conditions, respectively. Constraints (3)-(5) update the value of the state change variables. It should be noted that only one action at each time-step with a negative effect on the same proposition is allowed (5). Constraints (6)-(7) enforce actions preconditions and effects. Constraints (8)-(10) avoid the simultaneous application of conflicting actions. Constraint (11) propagates the value of the state change variables from one time-step to the next.

$$\min \sum_{a \in A, t \in \mathcal{T}} cost_a u_{a,t} \qquad (\mathcal{SC}(\Pi, T))$$

$$\text{s.t. } u^a_{p,0} = I(p) \qquad \forall p \in V_p \qquad (1)$$

$$u^a_{p,T} + u^{pa}_{p,T} + u^m_{p,T} \geq 1 \qquad \forall p \in G_p \qquad (2)$$

$$\sum_{a \in pnd(p)} u_{a,t} \geq u^{pa}_{p,t+1} \qquad \forall p \in V_p, \forall t \in \mathcal{T} \qquad (3)$$

$$\sum_{a \in anp(p)} u_{a,t} \geq u^a_{p,t+1} \qquad \forall p \in V_p, \forall t \in \mathcal{T} \qquad (4)$$

$$\sum_{a \in pd(p)} u_{a,t} = u^{pd}_{p,t+1} \qquad \forall p \in V_p, \forall t \in \mathcal{T} \qquad (5)$$

$$u_{a,t} \leq u^{pa}_{p,t+1} \qquad \forall p \in V_p, \forall a \in pnd(p), \forall t \in \mathcal{T} \qquad (6)$$

$$u_{a,t} \leq u^a_{p,t+1} \qquad \forall p \in V_p, \forall a \in anp(p), \forall t \in \mathcal{T} \qquad (7)$$

$$u^a_{p,t} + u^m_{p,t} + u^{pd}_{p,t} \leq 1 \qquad \forall p \in V_p, \forall t \in \tilde{\mathcal{T}} \qquad (8)$$

$$u^{pa}_{p,t} + u^m_{p,t} + u^{pd}_{p,t} \leq 1 \qquad \forall p \in V_p, \forall t \in \tilde{\mathcal{T}} \qquad (9)$$

$$u_{a,t} + u_{a',t} \leq 1 \qquad \forall a, a' \in \tilde{A} \text{ s.t. } a \neq a' \wedge del(a) \cap (add(a') \cup pre(a')) \neq \emptyset \forall t \in \tilde{\mathcal{T}} \qquad (10)$$

$$u^{pa}_{p,t+1} + u^m_{p,1} + u^{pd}_{p,t+1} \leq u^a_{p,t} + u^{pa}_{p,t} + u^m_{p,t} \qquad \forall p \in V_p \forall t \in \mathcal{T} \qquad (11)$$

2 MILP Model of Numeric Planning Tasks

This section presents our extension to numeric planning valid for $\mathcal{SC}(\Pi, T)$. We will refer to the extended model as $\mathcal{SCN}(\Pi, T)$. For modeling purposes, we partition the set of actions affecting a numeric variable $v \in V_n$ into:

- $se(v) = \{a \in A : (v \coloneqq v + k^{v,a}) \in num(a)\}$ the set of actions that change v via constant effects, and
- $le(v) = \{a \in A : (v \coloneqq \xi) \in num(a),\ a \notin se(v)\}$ the set of actions that change v via linear effects.

Given an action $a \in A$, we call $nmutex(a)$ (*numeric mutex of* a) the set of mutex actions of a due to an interference of some numeric variables.

Definition 2.1. Given actions $a, a' \in A$, a' is *numeric mutex* to a if there exists a variable $v \in V_n$ such that $(v \coloneqq \xi) \in num(a)$ and either: (i) v is used in one of the numeric effects of a', i.e., $\exists v' \in V_n$ such that $(v' \coloneqq \xi') \in num(a')$ and $v \in \xi'$, or (ii) v is part of a precondition of a', i.e., $\exists\ (c : \sum_{v \in V_n} w_v^c v + w_0^c \trianglerighteq 0) \in pre_n(a')$ with $w_v^c \neq 0$.

2.1 MILP Formulation

Consider parameters $m_{c,t} \in \mathbb{Q}, \forall c \cup C, \forall t \in \tilde{\mathcal{T}}$, $M_{v,t}^{step}, m_{v,t}^{step}, M_{v,t}^{a}, m_{v,t}^{a} \in \mathbb{Q}, \forall v \in V_n, \forall t \in \tilde{\mathcal{T}}$. Let $y_{v,t} \in \mathbb{Q}\ \forall v \in V_n, \forall t \in \tilde{\mathcal{T}}$ represent the value of the numeric variable v at time-step t. The constraints modeling numeric effects and conditions are:

$$y_{v,0} = I(v) \qquad \forall v \in V_n \quad (12)$$

$$\sum_{v \in V_n} w_v^c y_{v,T} + w_0^c \trianglerighteq 0 \qquad \forall c \in G_n \quad (13)$$

$$\sum_{v \in V} w_v^c y_{v,t} + w_0^c \trianglerighteq m_{c,t}(1 - u_{a,t}) \qquad \forall a \in A, \forall c \in pre_n(a), \forall t \in \mathcal{T} \quad (14)$$

$$y_{v,t+1} \leq y_{v,t} + \sum_{a \in se(v)} k^{v,a} u_{a,t} + M_{v,t+1}^{step} \sum_{a \in le(v)} u_{a,t} \qquad \forall v \in V_n, \forall t \in \mathcal{T} \quad (15)$$

$$y_{v,t+1} \geq y_{v,t} + \sum_{a \in se(v)} k^{v,a} u_{a,t} + m_{v,t+1}^{step} \sum_{a \in le(v)} u_{a,t} \qquad \forall v \in V_n, \forall t \in \mathcal{T} \quad (16)$$

$$y_{v,t+1} - \sum_{w \in V_n} k_w^{v,a} y_{w,t} \leq k^{v,a} + M_{v,t+1}^{a}(1 - u_{a,t}) \quad \forall v \in V_n, \forall a \in le(v), \forall t \in \mathcal{T} \quad (17)$$

$$y_{v,t+1} - \sum_{w \in V_n} k_w^{v,a} y_{w,t} \geq k^{v,a} + m_{v,t+1}^{a}(1 - u_{a,t}) \quad \forall v \in V_n, \forall a \in le(v), \forall t \in \mathcal{T} \quad (18)$$

$$u_{a,t} + u_{a',t} \leq 1 \qquad \forall a \in A, \forall a' \in nmutex(a) \forall t \in \mathcal{T} \quad (19)$$

Constraint (12) sets the variables to their initial state values, while constraint (13) enforces the numeric goal conditions. Constraint (14) ensures the satisfaction of numeric preconditions. Constraints (15)-(18) update the values of the numeric variables according to the action effects. Constraint (19) enforce the mutex action relation.

3 Proof of the Tighter Linear Relaxation

Proposition

Consider a planning task $\Pi = \langle V_p, V_n, A, I, G \rangle$ with $le(v) = \emptyset$ for all $v \in V_n$. $\mathcal{SCN}(\Pi, T)$ computes tighter LP relaxations when using constraints (15)-(16) instead of (17)-(18), where the set $le(v)$ is replaced by $se(v)$.

Proof. Note that constraints (17) and (18) can only update a numeric variable when there is only one action affecting the variable. Hence, we will restrict our proof to such case, i.e., $\sum_{a \in se(v)} u_{a,t} \leq 1$ for all $v \in V_n$, $t \in \mathcal{T}$.

Given $le(v) = \emptyset$ for all $v \in V_n$, (15) and (16) reduce to

$$y_{v,t+1} = y_{v,t} + \sum_{a \in se(v)} k_{v,a} u_{a,t} \qquad \forall v \in V_n, \forall t \in \mathcal{T}. \tag{i}$$

Similarly, (17) and (18) reduce to

$$y_{v,t+1} - y_{v,t} \leq k_{v,a} + M_{v,t}(1 - u_{a,t}), \tag{ii}$$
$$y_{v,t+1} - y_{v,t} \geq k_{v,a} + m_{v,t}(1 - u_{a,t}), \tag{iii}$$

for every $v \in V_n$, $t \in \mathcal{T}$ and $a \in se(v)$.

We will first show that every feasible solution (integer or continuous) respecting constraint (i) also satisfies (ii) and (iii). Consider a feasible solution $\langle \hat{y}, \hat{u} \rangle$ that satisfies (i), with $\hat{y} = \{\hat{y}_{v,t}, \ \forall \ v \in V_n, t \in \tilde{\mathcal{T}}\}$ and $\hat{u} = \{\hat{u}_{a,t} \ \forall \ a \in se(v), t \in \mathcal{T}\}$. In particular, we have that $\hat{y}_{v,t+1} - \hat{y}_{v,t} = \sum_{a \in se(v)} k_{v,a} \hat{u}_{a,t}$ for an arbitrary $v \in V_n$ and $t \in \mathcal{T}$. Notice that,

$$\begin{aligned} \sum_{a \in se(v)} k_{v,a} \hat{u}_{a,t} &= k_{v,a'} \hat{u}_{a',t} + \sum_{a \in se(v) \setminus \{a'\}} k_{v,a} \hat{u}_{a,t} \\ &\leq k_{v,a'} \hat{u}_{a',t} + \max_{a \in se(v)} \{k_{v,a}\} \sum_{a \in se(v) \setminus \{a'\}} \hat{u}_{a,t} \\ &\leq k_{v,a'} \hat{u}_{a',t} + M_{v,t}(1 - \hat{u}_{a',t}). \end{aligned}$$

Also,

$$\begin{aligned} \sum_{a \in se(v)} k_{v,a} \hat{u}_{a,t} &\geq k_{v,a'} \hat{u}_{a',t} + \min_{a \in se(v)} \{k_{v,a}\} \sum_{a \in se(v) \setminus \{a'\}} \hat{u}_{a,t} \\ &\geq k_{v,a'} \hat{u}_{a',t} + m_{v,t}(1 - \hat{u}_{a',t}) \end{aligned}$$

Since the inequalities hold for any $v \in V_n$, $t \in \mathcal{T}$ and $a' \in se(v)$, solution $\langle \hat{y}, \hat{u} \rangle$ satisfies (ii).

To complete the proof we need to show that there is a numeric example that satisfies (ii) and (iii), but not (i). Consider an instance of the counter domain with $V_n = \{v_0, v_1\}$, $V_p = \emptyset$, $I = \{v_0 = 1, v_1 = 1\}$, $G_n = \{v_1 - v_0 - 1 \geq 0\}$ and actions definitions presented in the table below. Let $M_{v_0,1} = M_{v_1,1} = 2$, $m_{v_0,1} = m_{v_1,1} = 0$ and $T = 1$.

action	pre_n	num	$cost$
a_1	$v_0 \leq 2$	$v_0 \coloneqq v_0 + 1$	1
a_2	$v_0 \geq 1$	$v_0 \coloneqq v_0 - 1$	1
a_3	$v_1 \leq 2$	$v_1 \coloneqq v_1 + 1$	1
a_4	$v_1 \geq 1$	$v_1 \coloneqq v_1 - 1$	1

Solution $\hat{u}_{a_1,0} = \hat{u}_{a_2,0} = \hat{u}_{a_4,0} = 0$, $\hat{u}_{a_3,0} = 0.5$, $\hat{y}_{v_0,0} = \hat{y}_{v_1,0} = \hat{y}_{v_0,1} = 1$ and $\hat{y}_{v_1,1} = 2$ is feasible for (12)-(14), (ii),(iii) and (19). However, it violates constraint (i), which implies that the linear relaxation when using (i) is tighter. $\square$

4 Correctness of the Encoding

Proposition 4.1

Given a numeric planning task Π and a plan π, a solution $S = \mathcal{M}(\pi)$ satisfies all the constraints of $IP(\Pi, |\pi| + 1)$.

Proof. We show the proof for constraints (12)-(19). Constraints (12) and (13) are satisfied by definition. For constraint (14), if an action a with numeric precondition c is not in π at time-step t, then $u_{a,t} = 0$ and the expression is satisfied since $m_{c,t}$ is a lower bound on the numeric expression in c. If action a is applied at time-step t, the action is applicable, since π is a feasible plan c is satisfied, $u_{a,t} = 1$, therefore constraint (14) is satisfied.

For constraints (15)-(18), given a time-step t, only one action a_t is applied. If an action a_t does not have any effect on a numeric state variable v, then the value of v is unchanged. Therefore, all the terms containing $u_{a,t}$ in (15)-(16) are equal to 0 and the constraints are satisfied. Constraints (17)-(18) are satisfied because the quantities $M/m^a_{v,t+1}$ are upper and lower bounds on the expression on the r.h.s.

If action a_t has a simple effect on a state variable v, then it changes the value of v by adding the quantity $k^{v,a}_t$. Constraints (17)-(18) are satisfied because every term containing $u_{a,t}$ $\forall a \neq a_t$ are equal to 0. Similarly, constraints (17)-(18) are satisfied, since $M/m^a_{v,t+1}$ are upper and lower bounds on the expression on the r.h.s.

If the action a_t has a linear effect on a numeric variable v, then constraints (15)-(16) are satisfied since $M/m^{step}_{v,t}$ are upper and lower bounds on expression $y_{v,t+1} - y_{v,t}$. Also, constraints (17)-(18) are satisfied because action a changes the value of the numeric state variable according to the values of the variables in the previous state. For each t, there is only one action such that $u_{a,t} = 1$, so constraint (19) is satisfied. $\square$

Proposition 4.2

Given a feasible solution S of $IP(\Pi, T)$, a plan $\pi = \tilde{\mathcal{M}}(S)$ is a feasible plan for Π.

Proof. We need to show that for the plan $\pi = \tilde{\mathcal{M}}(S)$: (i) state $\pi(I)$ satisfies all the goal conditions; (ii) every the action a in π at time-step t is applicable in $a_{t-1}(...(a_0(I)))$.

For condition (i) we show that the application of all the actions for which $u_{a,t} = 1$, then the values of variables $y_{v,t+1}$ change according the effect of actions a evaluated on the state $a_{t-1}(...(a_1(v)))$. Given a numeric state variable $v \in V_n$ in a time-step t, if no actions with effect on a numeric variables v has $u_{a,t} = 1$, then the value of v remains unchanged, due to constraints (15)-(16). If an action with simple effects on v has $u_{a,t} = 1$, then the value of v is only increased by the quantity of the applied action. Due to constraint (19) no other action with linear effects on v can be applied simultaneously. If an action with a linear effect on v has $u_{a,t} = 1$, then the values of the numeric state variables are unequivocally assigned to the linear expression representing the effect of the action, evaluated in the previous time-step. If two actions a and a' have $u_{a',t} = u_{a,t} = 1$, then $a'(a(...(I))) = a(a'(...(I)))$, because due to constraint (19), only variables corresponding to actions with no effects or a state-independent effect on the same numeric state variable can occur simultaneously. The effect of the application of the sequence of action induces a state for which the numeric goal conditions are satisfied.

For condition (ii) we show that every action in π is applicable in the predecessor state. Due to constraint (14), if only one action a has $u_{a,t} = 1$, then its numeric preconditions are satisfied by the numeric state variables at time t, which correspond to the predecessor state of a. If two actions a and a' have $u_{a,t} = u_{a',t} = 1$, then constraint (19) ensures that none of the numeric state variables appearing in the numeric precondition of one action are affected by the other action. Therefore, the order of application of does not affect the applicability of these two actions. $\square$