Appendix for Solving Domain-Independent Dynamic Programming Problems with Anytime Heuristic Search*

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DP Models

We present the DP models for a traveling salesperson problem with time windows (TSPTW) and a capacitated vehicle routing problem (CVRP). These models are adaptations of the models in Kuroiwa and Beck (2023a).

TSPTW

In TSPTW, a set of customers $N = \{0, ..., n\}$ is given, where 0 is the depot. Visiting customer j from i incurs the travel time c_{ij}, and the objective is to minimize the total travel time to visit each customer exactly once starting from and returning to the depot, where each customer i must be visited in the time window $[a_i, b_i]$.

Kuroiwa and Beck (2023a) used a DP model proposed by Dumas et al. (1995). In the model, c_{ij}, a_i, and b_i are numeric constants. A set variable U represents the set of unvisited customers, an element variable i represents the current location, and a numeric variable t represents the current time. In addition, a numeric constant c^*_{ij} represents the shortest travel time from customer i to j, a set expression $R(U, i, t) = \{j \in U \mid t + c_{ij} \leq b_j\}$ represents the set of customers that can be visited next, and $t^j = \max\{t + c_{ij}, a_j\}$. The DP model is

$$\text{compute } V(N \setminus \{0\}, 0, 0)$$

$$V(U, i, t) = \infty \qquad \text{if } \exists j \in U, t + c^*_{ij} > b_j$$

$$V(U, i, t) = \begin{cases} c_{i0} & \text{if } U = \emptyset \\ \min\limits_{j \in R(U,i,t)} c_{ij} + V(U \setminus \{j\}, j, t^j) & \text{if } U \neq \emptyset \end{cases}$$

$$V(U, i, t) \leq V(U, i, t') \qquad \text{if } t \leq t'.$$

While they use the trivial lower bound $V(U, i, t) \geq 0$ with the above model, we use a more sophisticated lower bound following the DP model for a multi-commodity pick and delivery traveling salesperson problem. A numeric constant $c^{\text{in}}_j = \min_{k \in N \setminus \{j\}} c_{kj}$ is the shortest travel time to customer j, and a numeric constant $c^{\text{out}}_j = \min_{k \in N \setminus \{j\}} c_{jk}$ is the shortest travel time from customer j. Since all unvisited customers must be both arrived at and departed from,

$$V(U, i, t) \geq \max \left\{ \sum_{j \in (U \cup \{0\}) \setminus \{i\}} c^{\text{in}}_j, \sum_{j \in (U \cup \{i\}) \setminus \{0\}} c^{\text{out}}_j \right\}.$$

CVRP

In CVRP, a set of customers $N = \{0, .., n\}$ is given, where 0 is the depot. Traveling from customer i to j incurs the travel time c_{ij}, and the objective is to minimize the total travel time to visit all customers using m vehicles. Each vehicle has the capacity q and starts from and returns to the depot. Each customer has the demand d_i, and the sum of the demands of customers visited by a single vehicle must not exceed q.

Kuroiwa and Beck (2023a) used a DP model proposed by Gromicho et al. (2012). In the model, c_{ij}, m, q, and d_i are numeric constants. A set variable U represents the set of unvisited customers, an element variable i represents the current location of the vehicle, a numeric resource variable l represents the current load of the vehicle, and a numeric resource variable k represents the number of used vehicles so far. In addition, a numeric constant $c'_{ij} = c_{i0} + c_{0j}$ represents the travel time via the depot. A set expression $U' = \{j \in U \mid l + d_j \leq q\}$ represents the set of customers that can be visited by the current vehicle. The DP model is

$$\text{compute } V(N \setminus \{0\}, 0, 0, 1)$$

$$V(\emptyset, i, l, k) = c_{i0}$$

$$V(U, i, l, k) = \min \begin{cases} \min\limits_{j \in U'} c_{ij} + V(U \setminus \{j\}, j, l + d_j, k) \\ \min\limits_{j \in U} c'_{ij} + V(U \setminus \{j\}, j, d_j, k + 1) \end{cases} \qquad \text{if } k < m$$

$$V(U, i, l, k) = \min_{j \in U'} c_{ij} + V(U \setminus \{j\}, j, l + d_j, k) \text{ if } k = m$$

$$V(U, i, l, k) \leq V(U, i, l', k') \qquad \text{if } l \leq l' \wedge k \leq k'.$$

While they use the trivial lower bound $V(U, i, l, k) \geq 0$, the same lower bound as the above DP model for TSPTW,

$$V(U, i, l, k) \geq \max \left\{ \sum_{j \in (U \cup \{0\}) \setminus \{i\}} c^{\text{in}}_j, \sum_{j \in (U \cup \{i\}) \setminus \{0\}} c^{\text{out}}_j \right\}$$

is valid. In addition, we add a state constraint based on the capacity here. The sum of demands of unvisited customers

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$$V(U, i, l, k) = \infty \quad \text{if } (m - k + 1)q < l + \sum_{j \in U} d_j.$$

CP Models

We present the CP models for a multi-commodity pick and delivery traveling salesperson problem (m-PDTSP), talent scheduling, and single machine scheduling to minimize the total weighted tardiness ($1||\sum w_i T_i$).

Multi-Commodity Pick and Delivery TSP

We use the CP model proposed by Castro, Cire, and Beck (2020) with an improved implementation. Let $N = \{0, ..., n+1\}$ be the set of customers, where the vehicle starts from 0 and stops at $n+1$, $A \subseteq N \times N$ be the set of edges, c_{ij} be the travel time from customer i to j, and q be the capacity of the vehicle. Using the 1-PDTSP reduction (Gouveia and Ruthmair 2015), let δ_i be the net change of the load at a customer i and P_i be the customers that must be visited before i. Let u be the upper bound of the optimal cost. The CP model is

$$\begin{aligned}
\min\ & \mathsf{StartOf}(x_{n+1}) - n \\
\text{s.t. } & \mathsf{Nolap}(\pi, \{c_{ij} \mid (i,j) \in A\}) \\
& \mathsf{Before}(\pi, x_i, x_j) && j \in N, i \in P_j \\
& \sum_{i \in N} \mathsf{StepAtStart}(x_i, \delta_i) \leq q \\
& \mathsf{First}(\pi, x_0) \\
& \mathsf{Last}(\pi, x_{n+1}) \\
& x_i : \mathsf{intervalVar}(1, [0, u+n+1]) && i \in N \\
& \pi : \mathsf{sequenceVar}(\{x_0, ..., x_{n+1}\})
\end{aligned}$$

In the implementation, $u = \sum_{i=0}^{n} \max_{j \in N:(i,j) \in A} c_{ij}$ is used. Our implementation has several improvements from the original one.[1] We eliminate unnecessary edges from A using the preprocessing method (Letchford and Salazar-González 2016) while the original implementation does not use it. In the Nolap global constraint, x_j must be distant from x_i by at least c_{ij} when x_j comes after x_i in the sequence π. While c_{ij} is specified for $(i,j) \in A$ in the above model, to implement Nolap in CP optimizer, c_{ij} must be defined for $(i,j) \in N \times N$. Furthermore, if the argument is_direct of Nolap is active, the constraint is applied only when x_j comes directly after x_i in the sequence π. Otherwise, the constraint is applied when x_j comes indirectly after x_i in π. In the original implementation, $c_{ij} = 9999999$ is used for $(i,j) \notin A$, and is_direct is always activated. In our implementation, we use $c_{ij} = u + n + 1$, and is_direct is activated only for the class1 instances, where the triangle inequality may not hold. For the class2 and class3 instances, since the distance is Euclidean, we do not activate is_direct and use the Euclidean distance for all $(i,j) \in N \times N$ in Nolap.

[1] https://github.com/MargaritaCastro/mdd_mpdtsp

Talent Scheduling

We extend the CP model used by Chu and Stuckey (2015), which was originally implemented with MiniZinc (Nethercote et al. 2007), with the AllDifferent global constraint. While AllDifferent is redundant in the model, we observed that it slightly improves the performance in a preliminary experiment. Let $N = \{0, ..., n-1\}$ be the set of scenes, $A = \{0, ..., m-1\}$ be the set of actors, $A_s \subseteq A$ be the set of actors playing in scene s, and $N_a \subseteq N$ be the set of scenes where actor a plays. Let d_s be the duration of a scene s, and let c_a be the cost of an actor a per day. Let x_i be a variable representing the i th scene in the schedule, b_{si} be a variable representing if scene s is shot before the i th scene, o_{ai} be a variable representing if any scene in N_a is shot by the i th scene, and f_{ai} be a variable representing if all scenes in N_a finish before the i th scene. We use an indicator function $\mathbb{1} : \{\top, \bot\} \to \{0, 1\}$, where $\mathbb{1}(\top) = 1$ and $\mathbb{1}(\bot) = 0$. Let $N_+ = \{1, ..., n-1\}$. The CP model is

$$\begin{aligned}
\min\ & \sum_{i \in N} d_{x_i} \sum_{a \in A} c_a o_{ai} (1 - f_{ai}) \\
\text{s.t. } & \mathsf{AllDifferent}(\{x_i \mid i \in N\}) \\
& b_{s0} = 0 && s \in N \\
& b_{si} = b_{s,i-1} + \mathbb{1}(x_{i-1} = s) && i \in N_+, s \in N \\
& b_{si} = 1 \to x_i \neq s && i \in N_+, s \in N \\
& o_{a0} = \mathbb{1}\Big(\bigvee_{s \in N_a} x_0 = s\Big) && a \in A \\
& o_{ai} = \mathbb{1}\Big(o_{a,i-1} = 1 \vee \bigvee_{s \in N_a} x_i = s\Big) && i \in N_+, a \in A \\
& f_{ai} = \prod_{s \in N_a} b_{si} && i \in N, a \in A \\
& x_i \in N && i \in N \\
& b_{si} \in \{0, 1\} && s, i \in N \\
& f_{si} \in \{0, 1\} && s, i \in N.
\end{aligned}$$

We implement $\mathbb{1}$ in CP Optimizer by constraint reification.

Single Machine Total Weighted Tardiness

We use a CP model based on interval variables. Let $N = \{0, ..., n-1\}$ be a set of jobs, p_i be the processing time of job i, d_i be the deadline for i, w_i be the weight for i, and P_i be the set of jobs processed before i, extracted by precedence theorems of Kanet (2007). We use an interval variable x_i with the duration p_i and within $[0, \sum_{j \in N} p_j]$, which represents the interval of time when job i is processed. We use a sequence variable π to sequence the interval variables.

$$\begin{aligned}
\min\ & \sum_{i \in N} w_i \max\{\mathsf{EndOf}(x_i) - d_i, 0\} \\
\text{s.t. } & \mathsf{Nolap}(\pi) \\
& \mathsf{Before}(\pi, x_i, x_j) && j \in N, i \in P_j \\
& x_i : \mathsf{intervalVar}\left(p_i, \left[0, \sum_{j \in N} p_j\right]\right) && i \in N \\
& \pi : \mathsf{seqeunceVar}(\{x_0, ..., x_{n-1}\}).
\end{aligned}$$

Experimental Results

Table 1 shows instances where a new best solution is found or the infeasibility is proved for the first time. For m-PDTSP, while optimally solved instances were reported by previous work, the solution costs were not reported (Castro, Cire, and Beck 2020). We present the ratio of the coverage over the number of instances against time, the ratio of instances against the primal gap at the time limit, and the ratio of instances against the primal gap for each problem class in Figures 1-9. We show the number of optimally solved instances, the average primal gap, and the average primal integral for each instance set in Table 2, Table 3, and Table 4, respectively. In addition, we show the number of instances unsolved by the memory limit and by the time limit separately in Table 5.

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Instance	Found	Best-Known	Proved	Methods
TSPTW				
rbg193.2	12139	12142		CBFS, ACPS, CABS
m-PDTSP				
p43.3Q4max1	-	-	infeasible	CAASDy, DFBnB, CBFS, ACPS, APPS, DBDFS, CABS
p43.3Q6max1	56600	-	optimal	CAASDy, DFBnB, CBFS, ACPS, APPS, DBDFS, CABS
p43.3Q7max1	56120	-	optimal	CABS
p43.3Q8max1	29450	-	optimal	CP
p43.3Q10max5	-	-	infeasible	CAASDy, DFBnB, CBFS, ACPS, APPS, DBDFS, CABS
p43.3Q15max5	56325	-	optimal	CAASDy, DFBnB, CBFS, ACPS, APPS, DBDFS, CABS
p43.3Q20max5	29475	-	optimal	CP

Table 1: Instances where a new best solution is found or the infeasibility is proved for the first time.

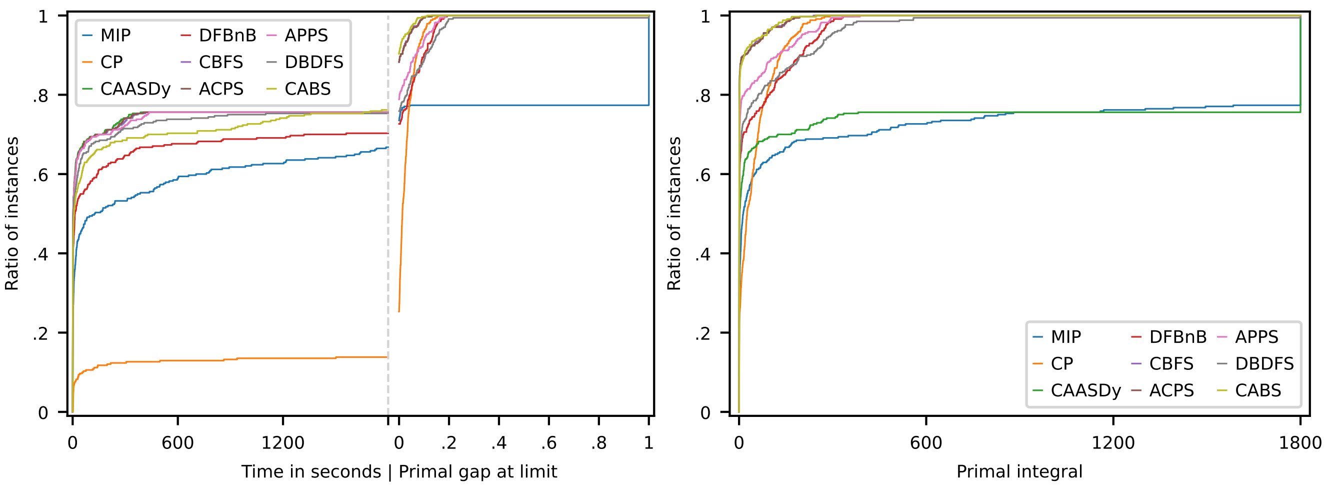

Figure 1: The ratio of the coverage over the number of instances against time, the ratio of instances against the primal gap at the time limit (left), and the ratio of instances against the primal integral (right) on **TSPTW**.

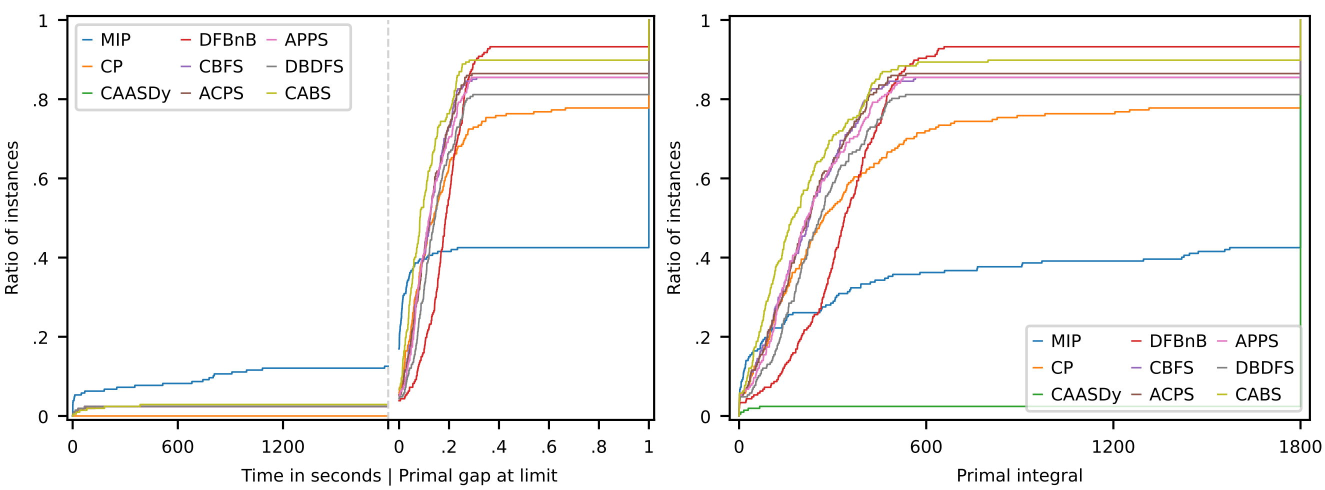

Figure 2: The ratio of the coverage over the number of instances against time, the ratio of instances against the primal gap at the time limit (left), and the ratio of instances against the primal integral (right) on **CVRP**.

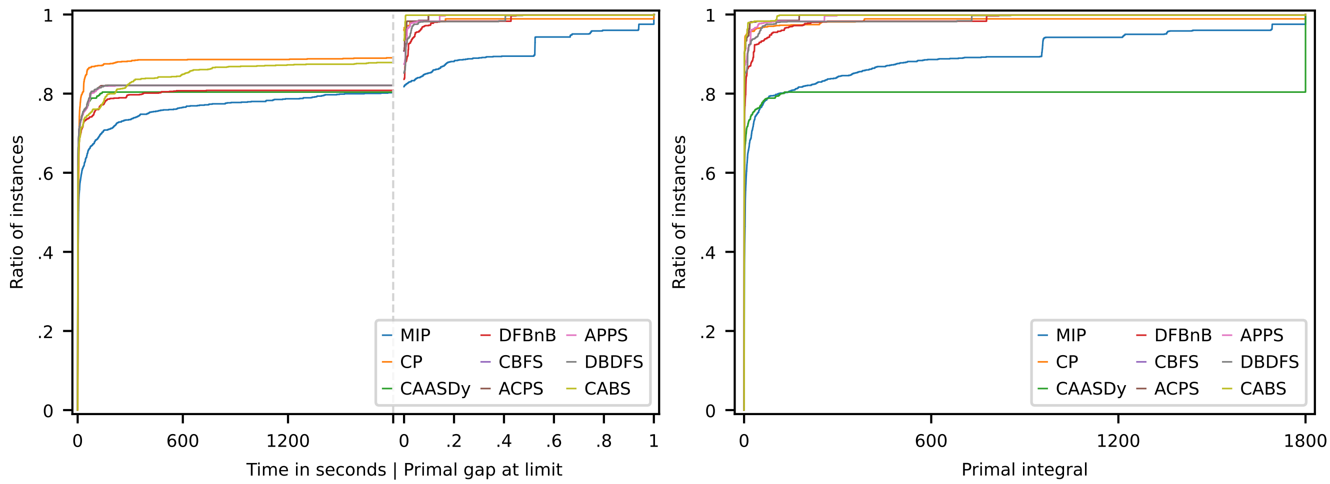

Figure 3: The ratio of the coverage over the number of instances against time, the ratio of instances against the primal gap at the time limit (left), and the ratio of instances against the primal integral (right) on **m-PDTSP**.

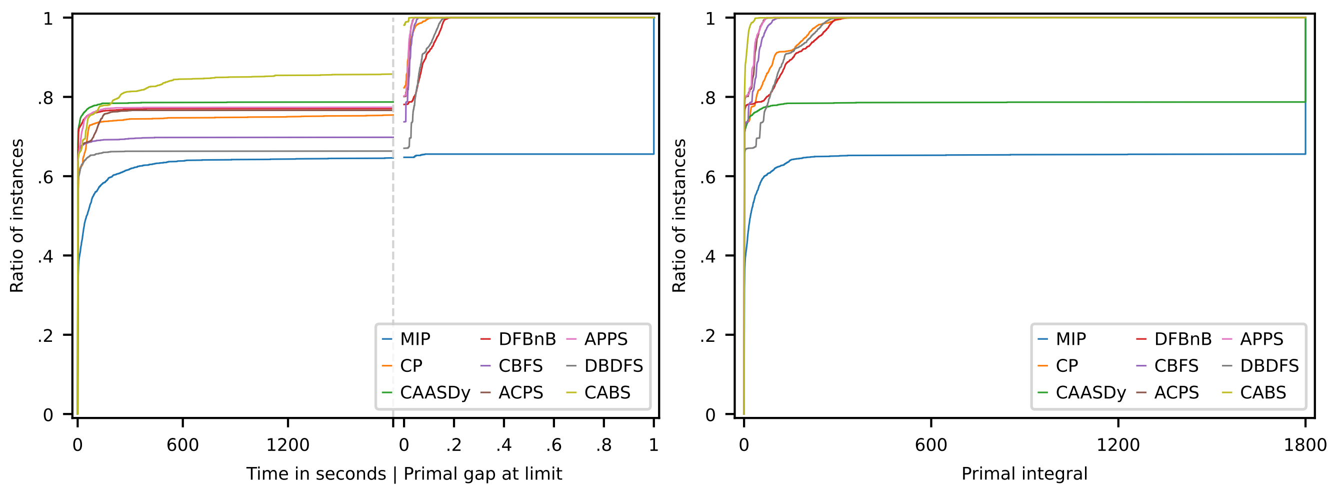

Figure 4: The ratio of the coverage over the number of instances against time, the ratio of instances against the primal gap at the time limit (left), and the ratio of instances against the primal integral (right) on **SALBP-1**.

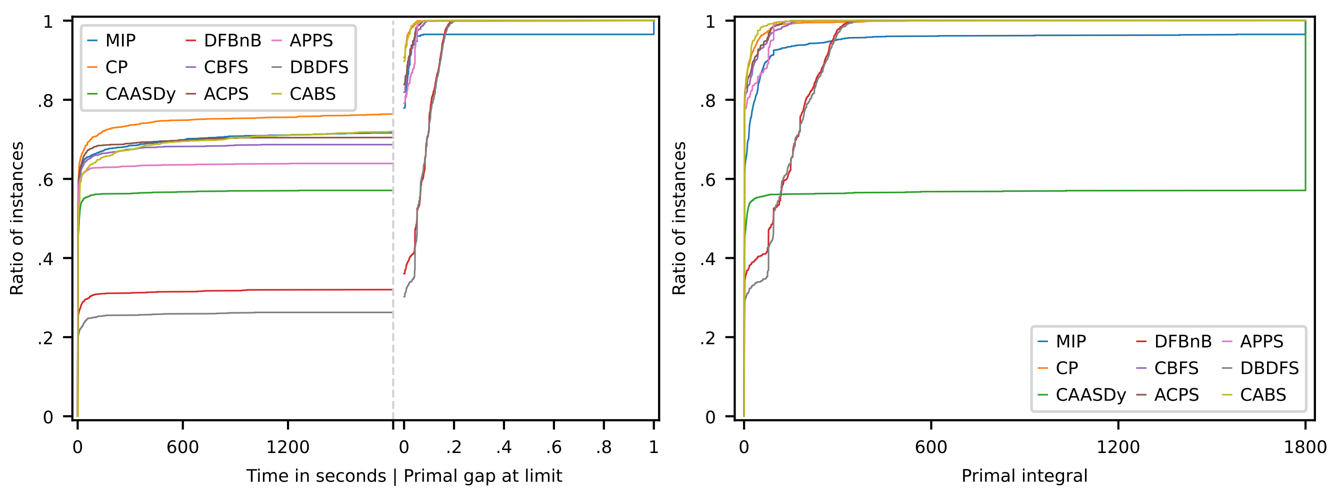

Figure 5: The ratio of the coverage over the number of instances against time, the ratio of instances against the primal gap at the time limit (left), and the ratio of instances against the primal integral (right) on **bin packing**.

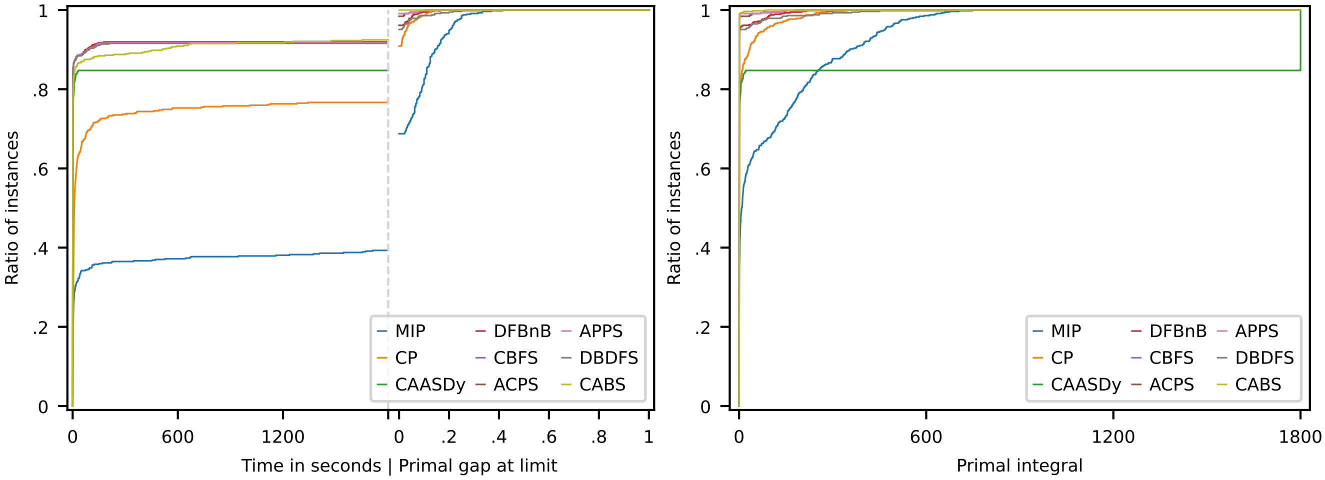

Figure 6: The ratio of the coverage over the number of instances against time, the ratio of instances against the primal gap at the time limit (left), and the ratio of instances against the primal integral (right) on **MOSP**.

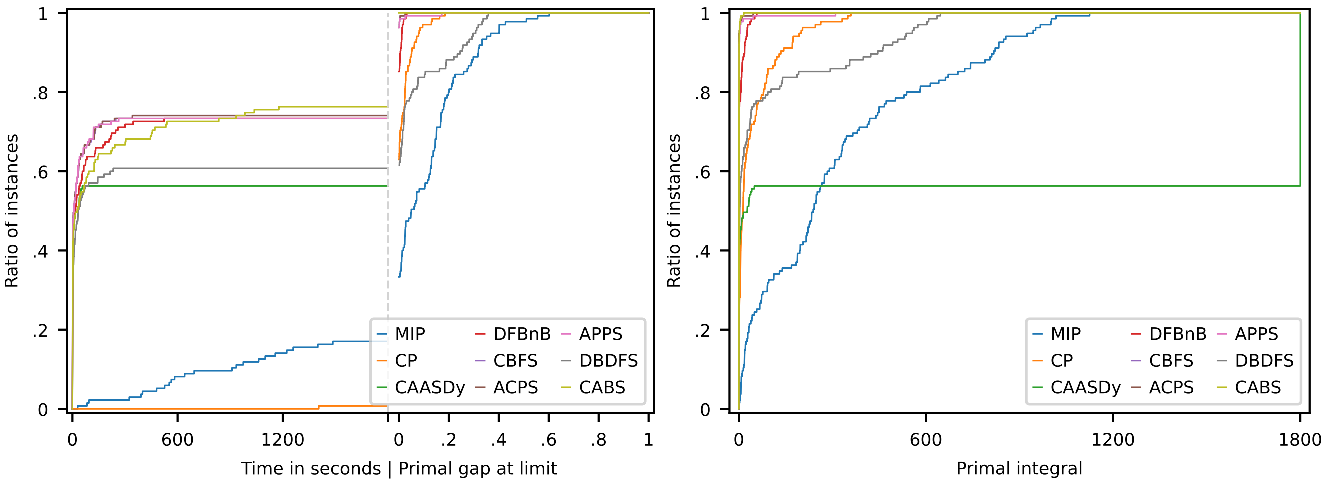

Figure 7: The ratio of the coverage over the number of instances against time, the ratio of instances against the primal gap at the time limit (left), and the ratio of instances against the primal integral (right) on **graph-clear**.

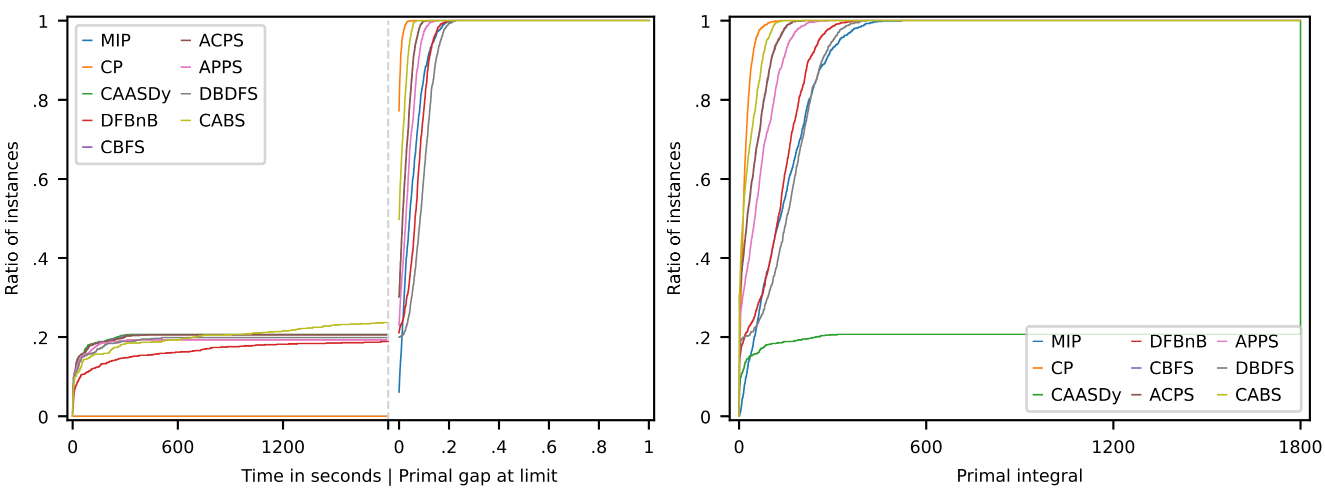

Figure 8: The ratio of the coverage over the number of instances against time, the ratio of instances against the primal gap at the time limit (left), and the ratio of instances against the primal integral (right) on **talent scheduling**.

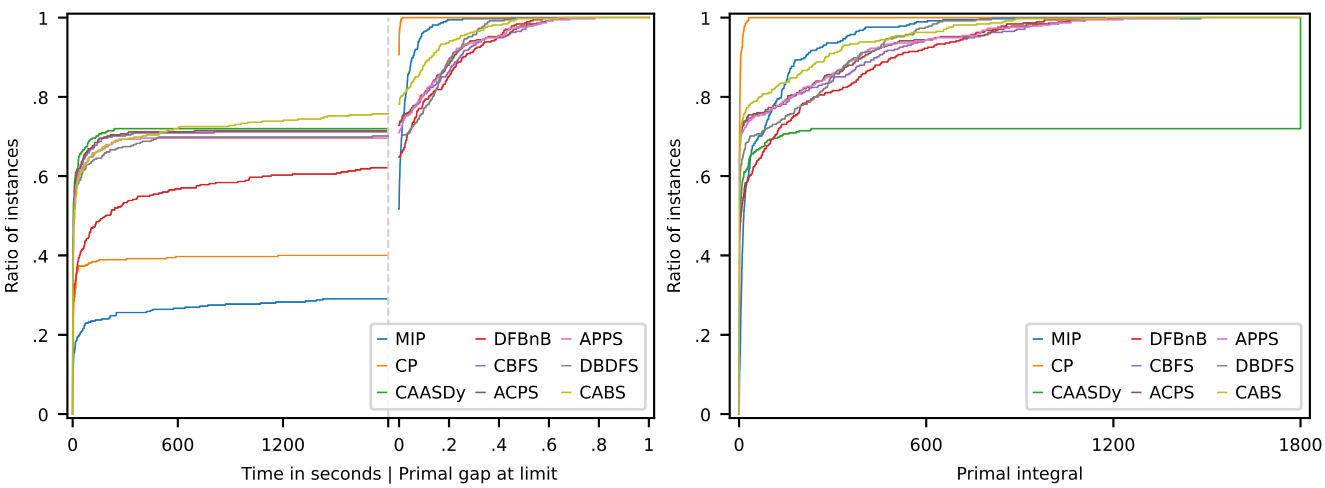

Figure 9: The ratio of the coverage over the number of instances against time, the ratio of instances against the primal gap at the time limit (left), and the ratio of instances against the primal integral (right) on $1||\sum w_iT_i$.

	MIP	CP	CAASDy	DFBnB	CBFS	ACPS	APPS	DBDFS	CABS
TSPTW (340)	227	47	257	239	257	257	257	256	**259**
AFG (50)	36	7	**45**	38	**45**	**45**	**45**	44	**45**
Dumas (135)	121	36	**135**	**135**	**135**	**135**	**135**	**135**	**135**
GDE (130)	70	4	77	66	77	77	77	77	**79**
OT (25)	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
CVRP (207)	**26**	0	5	5	5	5	5	5	6
A (27)	**5**	0	0	0	0	0	0	0	0
B (23)	**4**	0	0	0	0	0	0	0	0
DIMACS (12)	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
E (13)	**6**	0	1	1	1	1	1	1	1
F (3)	**2**	0	0	0	0	0	0	0	0
M (5)	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
P (24)	**9**	0	4	4	4	4	4	4	5
X (100)	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
m-PDTSP (1178)	945	**1049**	947	952	967	967	967	967	1035
class1 (248)	129	128	128	129	144	144	144	144	**145**
class2 (720)	666	**720**	667	671	671	671	671	671	**720**
class3 (210)	150	**201**	152	152	152	152	152	152	170
SALBP-1 (2100)	1357	1584	1653	1618	1466	1609	1625	1393	**1801**
Small (525)	**525**	**525**	**525**	**525**	**525**	**525**	**525**	**525**	**525**
Medium (525)	517	501	509	508	510	510	510	510	**523**
Large (525)	315	404	414	411	418	420	416	358	**432**
Very Large (525)	0	154	205	174	13	154	174	0	**321**
Bin Packing (1615)	1157	**1234**	922	517	1109	1138	1032	424	1163
Falkenauer U (80)	25	36	33	0	41	54	**56**	0	42
Falkenauer T (80)	37	**56**	27	20	24	25	27	18	24
Hard28 (28)	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
Scholl 1 (720)	**605**	529	517	343	547	549	541	301	537
Scholl 2 (480)	354	**445**	335	151	343	348	347	105	391
Scholl 3 (10)	0	**1**	0	0	0	0	**1**	0	**1**
Schwerin 1 (100)	80	**96**	0	0	**96**	**96**	39	0	**96**
Schwerin 2 (100)	54	61	0	0	56	58	11	0	**62**
Wäscher (17)	2	**10**	**10**	3	2	8	**10**	0	**10**
MOSP (570)	224	437	483	524	523	524	523	522	**527**
Challenge (46)	41	44	44	**45**	**45**	**45**	**45**	**45**	**45**
SCOOP (24)	9	**23**	16	21	20	21	20	20	21
Faggioli and Bentivoglio (300)	130	**300**	298	**300**	**300**	**300**	**300**	**300**	**300**
Chu and Stuckey (200)	44	70	125	158	158	158	158	157	**161**
Graph-Clear (135)	23	1	76	99	100	100	99	82	**103**
Planar 20 (20)	16	1	**20**	**20**	**20**	**20**	**20**	**20**	**20**
Planar 30 (20)	0	0	**20**	**20**	**20**	**20**	**20**	**20**	**20**
Planar 40 (20)	0	0	5	18	18	18	18	2	**19**
Random 20 (25)	7	0	**25**	**25**	**25**	**25**	**25**	**25**	**25**
Random 30 (25)	0	0	6	15	15	15	15	15	**17**
Random 40 (25)	0	0	0	1	**2**	**2**	1	0	**2**
Talent Scheduling (1000)	0	0	207	189	206	206	193	199	**237**
$1\|\|\sum w_i T_i$ (375)	109	150	270	233	267	268	261	263	**284**
wt040 (125)	45	61	**125**	118	124	124	123	124	**125**
wt050 (125)	37	51	110	85	108	109	105	105	**118**
wt100 (125)	27	38	35	30	35	35	33	34	**41**

Table 2: Coverage in each instance set of each problem class.

	MIP	CP	CAASDy	DFBnB	CBFS	ACPS	APPS	DBDFS	CABS
TSPTW (340)	0.2268	0.0259	0.2441	0.0224	0.0048	0.0049	0.0151	0.0275	**0.0033**
AFG (50)	0.2000	0.0118	0.1000	0.0052	0.0004	0.0004	0.0005	0.0227	**0.0003**
Dumas (135)	0.0371	0.0077	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**
GDE (130)	0.2855	0.0431	0.4077	0.0314	0.0024	0.0024	0.0162	0.0270	**0.0009**
OT (25)	1.0000	0.0638	1.0000	0.1310	0.0524	0.0540	0.1208	0.1882	**0.0395**
CVRP (207)	0.5845	0.3174	0.9758	0.2318	0.2399	0.2329	0.2440	0.2909	**0.1851**
A (27)	**0.0226**	0.0546	1.0000	0.1802	0.0749	0.0749	0.0748	0.0999	0.0422
B (23)	**0.0279**	0.0452	1.0000	0.1648	0.0728	0.0733	0.0739	0.0708	0.0466
DIMACS (12)	1.0000	0.3328	1.0000	0.1837	**0.1629**	0.1638	0.1718	0.2339	0.1674
E (13)	0.1747	0.0683	0.9231	0.1665	0.0611	0.0615	0.0749	0.1105	**0.0417**
F (3)	0.3333	**0.0920**	1.0000	0.2547	0.1638	0.1528	0.1522	0.1506	0.1053
M (5)	0.8163	0.2536	1.0000	0.2655	0.1794	0.1944	0.1904	0.2107	**0.1568**
P (24)	0.0580	0.0761	0.8333	0.1241	0.0498	0.0482	0.0558	0.1184	**0.0306**
X (100)	0.9900	0.5493	1.0000	**0.2989**	0.4063	0.3916	0.4100	0.4729	0.3173
m-PDTSP (1178)	0.0858	0.0125	0.1961	0.0126	0.0036	0.0036	0.0052	0.0103	**0.0019**
class1 (248)	0.3301	0.0588	0.4839	0.0427	0.0168	0.0168	0.0214	0.0420	**0.0091**
class2 (720)	0.0156	**0.0000**	0.0736	0.0010	**0.0000**	**0.0000**	0.0004	0.0005	**0.0000**
class3 (210)	0.0382	0.0006	0.2762	0.0166	0.0002	0.0001	0.0025	0.0065	**0.0000**
SALBP-1 (2100)	0.3447	0.0046	0.2129	0.0198	0.0059	0.0039	0.0037	0.0215	**0.0002**
Small (525)	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**
Medium (525)	0.0007	**0.0000**	0.0305	0.0002	**0.0000**	**0.0000**	**0.0000**	0.0003	**0.0000**
Large (525)	0.3783	0.0010	0.2114	0.0144	0.0015	0.0019	0.0027	0.0169	**0.0007**
Very Large (525)	1.0000	0.0174	0.6095	0.0648	0.0223	0.0138	0.0121	0.0689	**0.0002**
Bin Packing (1615)	0.0387	**0.0016**	0.4291	0.0582	0.0054	0.0044	0.0075	0.0620	0.0018
Falkenauer U (80)	0.0671	0.0039	0.5875	0.1450	0.0036	0.0007	**0.0005**	0.1455	0.0015
Falkenauer T (80)	0.0132	0.0116	0.6625	0.0593	0.0290	0.0226	0.0263	0.0768	**0.0072**
Hard28 (28)	**0.0000**	**0.0000**	1.0000	0.1008	**0.0000**	**0.0000**	**0.0000**	0.0919	**0.0000**
Scholl 1 (720)	0.0507	0.0006	0.2819	0.0393	0.0005	0.0005	0.0005	0.0443	**0.0004**
Scholl 2 (480)	0.0360	**0.0009**	0.3021	0.0643	0.0084	0.0072	0.0067	0.0727	0.0032
Scholl 3 (10)	0.0192	0.0070	1.0000	0.1259	0.0241	0.0309	0.0070	0.1219	**0.0035**
Schwerin 1 (100)	0.0105	**0.0021**	1.0000	0.0782	**0.0021**	**0.0021**	0.0321	0.0583	**0.0021**
Schwerin 2 (100)	0.0041	0.0009	1.0000	0.0585	0.0061	0.0078	0.0296	0.0478	**0.0005**
Wäscher (17)	0.0362	**0.0052**	0.4118	0.0487	0.0362	0.0114	**0.0052**	0.0665	0.0070
MOSP (570)	0.0394	0.0044	0.1526	0.0022	0.0007	0.0008	0.0007	0.0042	**0.0000**
Challenge (46)	0.0143	0.0006	0.0435	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**
Chu and Stuckey (200)	0.0778	0.0123	0.3750	0.0061	0.0012	0.0013	0.0010	0.0103	**0.0000**
Faggioli and Bentivoglio (300)	0.0159	**0.0000**	0.0067	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**
SCOOP (24)	0.0606	**0.0000**	0.3333	0.0022	0.0076	0.0074	0.0086	0.0154	**0.0000**
Graph-Clear (135)	0.1102	0.0151	0.4370	0.0017	0.0003	0.0003	0.0015	0.0479	**0.0000**
Planar 20 (20)	**0.0000**	0.0014	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**
Planar 30 (20)	0.0216	0.0147	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**
Planar 40 (20)	0.2436	0.0600	0.7500	**0.0000**	**0.0000**	**0.0000**	0.0086	0.2526	**0.0000**
Random 20 (25)	0.0052	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**
Random 30 (25)	0.1392	0.0052	0.7600	0.0002	**0.0000**	**0.0000**	**0.0000**	0.0041	**0.0000**
Random 40 (25)	0.2386	0.0153	1.0000	0.0090	0.0016	0.0016	0.0013	0.0523	**0.0000**
Talent Scheduling (1000)	0.0515	**0.0018**	0.7930	0.0617	0.0222	0.0220	0.0328	0.0797	0.0106
$1\|\|\sum w_i T_i$ (375)	0.0182	**0.0002**	0.2800	0.0669	0.0614	0.0572	0.0577	0.0542	0.0340
wt040 (125)	0.0016	**0.0000**	**0.0000**	0.0020	0.0002	0.0001	0.0007	0.0010	**0.0000**
wt050 (125)	0.0066	**0.0003**	0.1200	0.0270	0.0130	0.0118	0.0143	0.0273	0.0008
wt100 (125)	0.0464	**0.0003**	0.7200	0.1717	0.1709	0.1597	0.1582	0.1344	0.1012

Table 3: Average primal gap at the time limit for each instance set.

	MIP	CP	CAASDy	DFBnB	CBFS	ACPS	APPS	DBDFS	CABS
TSPTW (340)	484.05	48.97	458.61	46.25	9.54	9.79	29.23	56.77	**8.97**
AFG (50)	406.62	22.43	204.80	11.27	1.06	0.98	1.36	49.29	**0.86**
Dumas (135)	141.84	14.77	1.92	0.80	**0.13**	**0.13**	0.34	1.24	0.16
GDE (130)	616.13	80.96	772.51	68.53	5.25	5.25	32.12	54.53	**4.83**
OT (25)	1800.00	120.33	1800.00	245.84	99.61	103.18	225.97	383.19	**94.23**
CVRP (207)	1157.43	601.15	1757.09	421.58	433.51	420.95	440.53	524.25	**351.21**
A (27)	318.19	112.69	1800.00	329.84	135.22	135.16	135.07	180.16	**90.75**
B (23)	276.60	110.64	1800.00	301.03	131.53	132.44	133.52	127.67	**97.58**
DIMACS (12)	1800.00	658.20	1800.00	333.33	**296.35**	298.13	312.06	423.29	306.13
E (13)	549.43	137.62	1661.60	304.94	110.29	111.06	135.19	199.17	**87.34**
F (3)	606.87	**185.15**	1800.00	461.06	296.03	275.65	274.52	271.72	212.25
M (5)	1754.63	477.52	1800.00	480.72	323.33	350.43	343.37	379.70	**307.77**
P (24)	293.29	146.73	1504.83	226.94	89.89	87.16	100.72	213.35	**62.89**
X (100)	1782.59	1026.98	1800.00	**542.41**	734.09	707.48	740.13	852.16	595.12
m-PDTSP (1178)	180.00	26.04	357.79	24.12	6.59	6.57	9.55	18.77	**5.33**
class1 (248)	617.01	117.37	872.58	79.12	30.69	30.69	39.16	76.24	**24.71**
class2 (720)	46.41	1.58	138.97	3.06	**0.10**	**0.10**	0.81	0.96	0.18
class3 (210)	121.92	2.03	500.05	31.35	0.36	0.28	4.52	11.92	**0.14**
SALBP-1 (2100)	634.64	28.48	387.52	35.99	11.40	7.47	6.88	38.99	**1.92**
Small (525)	0.14	0.09	0.04	0.02	0.08	0.02	0.02	0.02	**0.01**
Medium (525)	13.07	0.26	56.88	0.52	0.06	0.06	**0.05**	0.60	0.06
Large (525)	725.35	2.90	384.40	26.44	2.83	3.45	4.95	30.81	**1.83**
Very Large (525)	1800.00	110.65	1108.77	116.96	42.62	26.34	22.52	124.53	**5.80**
Bin Packing (1615)	86.19	8.04	779.41	105.46	10.42	8.64	13.94	112.18	**5.26**
Falkenauer U (80)	214.99	64.42	1058.89	263.36	9.51	4.60	**3.25**	264.36	7.80
Falkenauer T (80)	35.33	31.83	1194.75	107.52	54.54	42.14	48.20	138.99	**20.15**
Hard28 (28)	17.66	2.00	1800.00	181.90	0.25	0.25	0.23	166.70	**0.21**
Scholl 1 (720)	108.22	3.25	521.22	71.21	1.51	1.43	**1.31**	80.36	1.47
Scholl 2 (480)	75.30	**3.63**	546.14	116.45	15.66	13.41	12.54	131.30	8.48
Scholl 3 (10)	40.94	15.20	1800.00	226.78	43.69	56.42	12.84	219.50	**6.83**
Schwerin 1 (100)	22.10	6.13	1800.00	140.83	4.04	**3.88**	57.84	105.05	4.38
Schwerin 2 (100)	7.90	**1.99**	1800.00	105.38	11.27	14.24	53.50	86.05	2.25
Wäscher (17)	71.15	10.94	743.11	88.47	65.76	21.74	**9.81**	120.21	23.58
MOSP (570)	100.41	13.01	275.54	4.19	1.40	1.44	1.33	7.72	**0.36**
Challenge (46)	34.83	2.39	78.37	0.04	0.04	0.04	0.04	0.04	**0.02**
Chu and Stuckey (200)	190.56	36.09	676.19	11.34	2.28	2.44	1.83	18.59	**0.84**
Faggioli and Bentivoglio (300)	46.55	0.25	12.66	0.03	0.03	0.03	0.03	0.03	**0.01**
SCOOP (24)	148.19	**0.66**	600.73	4.53	13.87	13.51	15.71	27.96	1.29
Graph-Clear (135)	311.83	44.27	789.92	4.37	0.70	0.72	3.07	87.81	**0.49**
Planar 20 (20)	15.72	5.65	0.18	0.03	0.04	0.03	0.03	0.04	**0.02**
Planar 30 (20)	208.42	44.65	6.40	0.09	0.07	**0.06**	0.15	2.86	0.08
Planar 40 (20)	823.00	160.65	1355.79	3.01	0.74	**0.71**	17.07	459.21	1.51
Random 20 (25)	51.73	1.53	1.05	0.19	**0.08**	0.16	0.11	0.43	0.43
Random 30 (25)	330.42	17.68	1374.60	0.92	**0.07**	**0.07**	**0.07**	9.44	0.09
Random 40 (25)	464.03	51.08	1800.00	20.00	2.98	2.99	2.61	94.62	**0.81**
Talent Scheduling (1000)	142.69	**18.14**	1435.53	119.44	40.63	40.15	59.60	144.27	26.36
$1\|\|\sum w_i T_i$ (375)	74.56	**2.26**	513.70	138.85	112.23	104.54	105.27	100.09	73.60
wt040 (125)	10.18	0.76	7.26	13.74	0.93	0.67	1.79	3.36	**0.65**
wt050 (125)	25.88	**1.30**	234.28	73.98	25.18	22.92	27.25	52.79	12.38
wt100 (125)	187.62	**4.73**	1299.57	328.83	310.59	290.03	286.75	244.13	207.76

Table 4: Average primal integral for each instance set.

	MIP	CP	CAASDy	DFBnB	CBFS	ACPS	APPS	DBDFS	CABS
TSPTW (340)	0/113	0/293	83/0	49/52	82/1	82/1	83/0	83/1	1/80
AFG (50)	0/14	0/43	5/0	4/8	4/1	4/1	5/0	5/1	1/4
Dumas (135)	0/14	0/99	0/0	0/0	0/0	0/0	0/0	0/0	0/0
GDE (130)	0/60	0/126	53/0	34/30	53/0	53/0	53/0	53/0	0/51
OT (25)	0/25	0/25	25/0	11/14	25/0	25/0	25/0	25/0	0/25
CVRP (207)	5/176	0/207	202/0	200/2	202/0	202/0	202/0	202/0	3/198
A (27)	0/22	0/27	27/0	27/0	27/0	27/0	27/0	27/0	0/27
B (23)	0/19	0/23	23/0	23/0	23/0	23/0	23/0	23/0	0/23
DIMACS (12)	1/11	0/12	12/0	11/1	12/0	12/0	12/0	12/0	0/12
E (13)	0/7	0/13	12/0	12/0	12/0	12/0	12/0	12/0	1/11
F (3)	0/1	0/3	3/0	3/0	3/0	3/0	3/0	3/0	0/3
M (5)	0/5	0/5	5/0	5/0	5/0	5/0	5/0	5/0	0/5
P (24)	0/15	0/24	20/0	20/0	20/0	20/0	20/0	20/0	2/17
X (100)	4/96	0/100	100/0	99/1	100/0	100/0	100/0	100/0	0/100
m-PDTSP (1178)	0/233	0/129	231/0	226/0	211/0	211/0	211/0	211/0	9/134
class1 (248)	13/119	12/120	142/0	141/0	126/0	126/0	126/0	126/0	23/102
class2 (720)	79/54	86/0	139/0	135/0	135/0	135/0	135/0	135/0	86/0
class3 (210)	0/60	0/9	58/0	58/0	58/0	58/0	58/0	58/0	8/32
SALBP-1 (2100)	250/493	0/516	432/15	482/0	634/0	491/0	475/0	707/0	7/292
Small (525)	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0
Medium (525)	0/8	0/24	16/0	17/0	15/0	15/0	15/0	15/0	2/0
Large (525)	0/210	0/121	111/0	114/0	107/0	105/0	109/0	167/0	5/88
Very Large (525)	250/275	0/371	305/15	351/0	512/0	371/0	351/0	525/0	0/204
Bin Packing (1615)	0/458	0/381	628/65	1060/38	436/70	407/70	532/51	1118/73	10/442
Falkenauer U (80)	0/55	0/44	47/0	80/0	39/0	26/0	24/0	80/0	0/38
Falkenauer T (80)	0/43	0/24	53/0	60/0	56/0	55/0	53/0	62/0	0/56
Hard28 (28)	0/28	0/28	20/8	28/0	28/0	28/0	27/1	28/0	0/28
Scholl 1 (720)	0/115	0/191	146/57	344/33	103/70	101/70	129/50	346/73	5/178
Scholl 2 (480)	0/126	0/35	145/0	324/5	137/0	132/0	133/0	375/0	5/84
Scholl 3 (10)	0/10	0/9	10/0	10/0	10/0	10/0	9/0	10/0	0/9
Schwerin 1 (100)	0/20	0/4	100/0	100/0	4/0	4/0	61/0	100/0	0/4
Schwerin 2 (100)	0/46	0/39	100/0	100/0	44/0	42/0	89/0	100/0	0/38
Wäscher (17)	0/15	0/7	7/0	14/0	15/0	9/0	7/0	17/0	0/7
MOSP (570)	2/344	0/133	87/0	46/0	47/0	46/0	47/0	48/0	0/43
Challenge (46)	0/5	0/2	2/0	1/0	1/0	1/0	1/0	1/0	0/1
Chu and Stuckey (200)	2/154	0/130	75/0	42/0	42/0	42/0	42/0	43/0	0/39
Faggioli and Bentivoglio (300)	0/170	0/0	2/0	0/0	0/0	0/0	0/0	0/0	0/0
SCOOP (24)	0/15	0/1	8/0	3/0	4/0	3/0	4/0	4/0	0/3
Graph-Clear (135)	0/112	0/134	59/0	36/0	35/0	35/0	36/0	53/0	13/19
Planar 20 (20)	0/4	0/19	0/0	0/0	0/0	0/0	0/0	0/0	0/0
Planar 30 (20)	0/20	0/20	0/0	0/0	0/0	0/0	0/0	0/0	0/0
Planar 40 (20)	0/20	0/20	15/0	2/0	2/0	2/0	2/0	18/0	0/1
Random 20 (25)	0/18	0/25	0/0	0/0	0/0	0/0	0/0	0/0	0/0
Random 30 (25)	0/25	0/25	19/0	10/0	10/0	10/0	10/0	10/0	8/0
Random 40 (25)	0/25	0/25	25/0	24/0	23/0	23/0	24/0	25/0	5/18
Talent Scheduling (1000)	0/1000	0/1000	793/0	509/302	794/0	794/0	807/0	801/0	2/761
$1\|\|\sum w_i T_i$ (375)	0/266	0/225	105/0	7/135	108/0	107/0	114/0	112/0	3/88
wt040 (125)	0/80	0/64	0/0	0/7	1/0	1/0	2/0	1/0	0/0
wt050 (125)	0/88	0/74	15/0	3/37	17/0	16/0	20/0	20/0	0/7
wt100 (125)	0/98	0/87	90/0	4/91	90/0	90/0	92/0	91/0	3/81

Table 5: Number of instances unsolved by the memory/time limit.