Intruder Alert! Optimization Models for Solving the Mobile Robot Graph-Clear Problem

Michael Morin[†‡], Margarita P. Castro[⋆†], Kyle E. C. Booth[⋆†], Tony T. Tran[†], Chang Liu[†], and J. Christopher Beck[†]
{mmorin, mpcastro, kbooth, tran, cliu, jcb}@mie.utoronto.ca
michael.morin@osd.ulaval.ca

[†]Department of Mechanical and Industrial Engineering, University of Toronto, Toronto, ON, Canada
[‡]Department of Operations and Decision Support Systems, Université Laval, Québec, QC, Canada

We investigate optimization-based approaches and heuristic methods for the *graph-clear problem* (GCP), an $\mathcal{NP}$-Hard variant of the pursuit-evasion problem. The goal is to find a schedule that minimizes the total number of robots needed to "clear" possible intruders from a facility, represented as a graph. The team of robots can use *sweep* actions to remove intruders from *contaminated* nodes and *block* actions to prevent intruders from traveling between nodes. A solution to the GCP is a schedule of sweep and block actions that detects all potential intruders in the facility while minimizing the number of robots required. Solutions such that cleared vertices at each time step form a connected subgraph are termed *contiguous*, while those that prevent recontamination and, therefore, the need to sweep a node more than once, are called *progressive*.

We prove, via a counter-example, that enforcing contiguity may remove all optimal solutions and, conversely, that preventing two special forms of recontamination does not remove all optimal solutions. However, the completeness for the general case of progressive solutions remains open.

We then present *mixed-integer linear programming* (MILP) and *constraint programming* (CP) approaches, as well as new heuristic variants for solving the GCP and compare them to previously proposed heuristics. This is the first time that MILP and CP have been applied to the problem. Our experimental results indicate that our heuristic modifications improve upon the heuristics in the literature, that constraint programming finds better solutions than the heuristics in run-times reasonable for the application, and that mixed-integer linear programming is the superior approach for proving optimality. Nonetheless, for larger problem instances, the optimality gap for CP and MILP remains very large, indicating the need for future research and improvement. Given the performance of CP and MILP compared to the heuristic approaches, coupled with the appeal of the model-and-solve framework, we conclude that they are currently the most suitable approaches for the graph-clear problem.

⋆ Equally contributing authors