Proceedings of ASME 2015 International Design Engineering Technical Conferences & Computers and Information in Engineering Conference
IDETC/CIE 2015
August 2-5, 2015, Boston, MA, USA

DETC2015-47535

CONSTRAINED MULTI-OBJECTIVE WIND FARM LAYOUT OPTIMIZATION: INTRODUCING A NOVEL CONSTRAINT HANDLING APPROACH BASED ON CONSTRAINT PROGRAMMING

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ABSTRACT

Recently, land has been exploited extensively for onshore wind farms and turbines are frequently located in proximity to human dwellings, natural habitats, and infrastructure. This proximity has made land use constraints and noise generation and propagation matters of increasing concern for all stakeholders. Hence, wind farm layout optimization approaches should be able to consider and address these concerns. In this study, we perform a constrained multi-objective wind farm layout optimization considering energy and noise as objective functions, and considering land use constraints arising from landowner participation, environmental setbacks and proximity to existing infrastructure. The optimization problem is solved with the NSGA-II algorithm, a multi-objective, continuous variable Genetic Algorithm. A novel hybrid constraint handling tool that uses penalty functions together with Constraint Programming algorithms is introduced. This constraint handling tool performs a combination of local and global searches to find feasible solutions. After verifying the performance of the proposed constraint handling approach with a suite of test functions, it is used together with NSGA-II to optimize a set of wind farm layout optimization test cases with different number of turbines and under different levels of land availability (constraint severity). The optimization results illustrate the potential of the new constraint handling approach to outperform existing constraint handling approaches, leading to better solutions with fewer evaluations of the objective functions and constraints.

Keywords: Wind farm layout, multi-objective optimization, Constraint Programming, penalty functions.

INTRODUCTION

Installed capacity for generating electricity from wind has seen a significant increase during the past decade [1–3]. In contrast to these growing trends, wind energy still faces resistance for being widely used onshore, due to health and environmental concerns. Although it is not proven that the

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noise production of turbines can have negative health impact, a number of jurisdictions have established regulations that limit noise emissions [4–6].

Wind farm design can be an iterative, lengthy process, in which designers have to check for compliance with land use constraints and environmental restrictions. Traditionally wind farm designers and researchers have considered energy or profit as the objective functions to be maximized [7, 8], while some added other constraints such as land use, set backs or noise limits in their optimization model [9–14].

Among optimization methods, stochastic meta-heuristics such as GA [15] and Particle Swarm Optimization (PSO) [16] are the most common approaches for the wind farm layout optimization problem [7, 8, 17, 18]. In addition, deterministic heuristics such as the Extended Pattern Search (EPS) approach of Du Pont and Cagan [19] are also used. Donovan [20, 21] and Fagerfjäll [9] introduced an alternative approach which uses mixed-integer programming (MIP) and solves the WFLO problem by the traditional branch-and-bound method. Although MIP solvers are widely available in operation research software packages, they all have limitations solving non-linear, non-convex problems such as WFLO. Thus, Donovan and Fagerfjäll made some approximations in their wake models and simplified the problem at the expense of accuracy in the solutions. To address this inaccuracy, Archer et al. [22] improved the simplified wake model by introducing a wind interference coefficient, while Turner et al. [23] suggested more accurate linear and quadratic wake model that can be solved by MIP solvers. The accuracy problem was resolved by Zhang et al. [24], who proposed the first Constraint Programming (CP) and MIP models that incorporated the non-linearity of the problem. Despite these advances in the solution of the WFLO with mathematical programming models, all of them use a discretized domain to solve the problem, a feature that can lead to suboptimal solutions. Moreover, these state-of-the-art MIP models [23, 24] still suffer limitations on problem size and turbine density, e.g., typically discretizing the wind farm into only $100-400$ potential turbine locations.

Using stochastic meta-heuristics for constrained WFLO problem requires developing a constraint handling approach to drive the search toward high-quality, *feasible* solutions. Penalty functions are perhaps the most widely used constraint handling approaches with the evolutionary algorithms due to their simplicity of implementation, general applicability, and strong theoretical basis [25]. The penalty function approaches consist of recasting the problem as an unconstrained one by incorporating a function of the constraint violations as a term in the objective function. Hence, penalty functions are generally applicable to constrained optimization problems, regardless of the underlying method used to solve the resulting unconstrained problem. The main problem with the penalty functions is that their setup parameters may vary from problem to problem. To address this issue, Debchoudhury et al. [26] proposed a modified penalty function, free from scaling parameters that finds the penalty terms based the constraint violation and the fitness function of the infeasible solutions. Datta et al. [27] introduced another penalty function approach, which is able to further improve the best solutions of by decreasing the level of constraint violation using a gradient free pattern search method. Montemurro et al. [28] proposed the automatic dynamic penalization method in which all the information needed for tuning the penalty parameters is extracted from the population members of the current generation. In addition, there are some other studies that choose the parameters of the penalty functions adaptively [29–31]. These penalty function approaches have performed well on the test functions. However, their main focus is to make the penalty functions independent of any external parameters. In this process, they may improve the local search of the penalty functions [27]; however, none of them introduces a strong local search that can be combined with the penalty functions. Here, we shall use the penalty approach; however, we improve its performance in constrained multi-objective optimization by hybridizing it with a powerful local search tool that complements the global search.

In this study, a computational approach is proposed for constrained multi-objective, continuous formulation of the WFLO problem. This approach addresses the growing health and environmental concerns of wind farms by not only maximizing energy generation but also minimizing noise production and avoiding natural habitats and human dwellings. To achieve this goal, the unconstrained multi-objective WFLO problem addressed by Kwong et al. [32, 33] is extended to include land use constraints. The resulting optimization problem is solved with NSGA-II [34] and without linearizing simplifications to retain accuracy. Since a GA is generally not able to handle constraints by its nature, penalty functions are used as the first constraint handling approach. Then, a new constraint handling approach is introduced, verified and applied to the WFLO problem. In the proposed approach, a Constraint Programming (CP) model is hybridized with penalty functions with the purpose of improving the intensification (i.e., local search) of penalty functions and avoiding the negative effects of pure diversification (i.e., global search).

WIND FARM MODELLING

Wake Modelling

An analytical closed-form wake model, suggested by Jensen [35], is used to evaluate the aerodynamic interactions between turbines. Further details on the calculation process of this model can be found in previous work [19, 36, 37]. Based on this model, an effective wind speed can be calculated from the sum of kinetic energy deficits for turbines under the influence of multiple wakes. The effective speed of a turbine inside n wake regions is expressed as

$$u = u_0 \left[1 - \sqrt{\sum_{i=1}^{n} \left(1 - \frac{u_i}{u_o} \right)^2} \right]. \quad (1)$$

Without loss of generality, the present work follows previous work [19, 36, 37] and uses a simplified expression for the power output of a turbine, in which power is a simple cubic function of the local effective speed at hub height. The cut-in and cut-off speeds are considered to be 4 m/s and 25 m/s respectively. The rated speed is 15 m/s, for which a power of 1.5 MW is generated. Thus, the annual energy production (AEP), which is an expected value of a random variable, because it is based on the probability distribution of wind speeds and directions, is calculated as

$$AEP = 8766 \sum_{i=1}^{k} \sum_{d \in \mathscr{D}} \frac{1}{3} u_{id}^3 p_d, \quad (2)$$

where u_{id} is the effective wind speed at turbine i at hub height for wind state d, i is an index over the number of turbines k, $d \in \mathscr{D}$ is the set of all possible wind states (i.e., the set of all possible wind speeds and directions), p_d is the probability of wind being at state d, and 8766 is the number of hours in a year.

Noise Modelling

In this work, locations where the sound level has to be measured or calculated are referred to as noise receptors. In wind farm layout design, all residences located within a wind farm or in its neighbourhood are considered to be noise receptors. According to the ISO-9613-2 standard [38], the equivalent continuous downwind octave-band sound pressure level (SPL) is calculated at each noise receptor for all point sources and eight octave bands with nominal mid-band frequencies from 63 Hz to 8 kHz [38], as

$$L_f = L_W + D_c - A, \quad (3)$$

where L_W is the octave-band sound power emitted by the source, D_c is the directivity correction for sources that are not omnidirectional, A is the octave-band attenuation, and f is a subscript indicating that this quantity is calculated for each octave-band frequency.

The sound pressure levels in Eq. 3 are converted to an effective SPL. Several octave-band weightings are available for this conversion; however, A-weighted sound pressure levels are customarily used in wind farm layout design and optimization [6]. The equivalent continuous A-weighted downwind sound pressure level at specific location can be calculated from summation of contributions of each sound source at each octave band,

$$L_{avg} = 10 \log \left(\sum_{i=1}^{n_s} \left(\sum_{j=1}^{8} 10^{0.1\left(L_f(i,j) + A_f(j)\right)} \right) \right), \quad (4)$$

where n_s is the number of point sound sources, j is the index representing one of the eight standard octave-band mid-band frequencies, and the $A_f(j)$ are the standard A-weighting coefficients.

The attenuation term (A) in Eq. 3 is the sum of different attenuation effects due to geometrical divergence, atmospheric absorption, ground effects, sound barriers, and miscellaneous effects. In this study, it is assumed that the attenuation caused by sound barriers and miscellaneous effects are negligible. Further details of the calculation procedure are available in the ISO 9613-2 document [38].

Constraint Modelling

In this work, we consider two types of constraints: proximity constraints and regulatory (land-use) constraints. The proximity constraint states that the distance between each pair of turbines must be at least five times their rotor diameter to avoid the strong turbulence and vibration effects. The proximity constraint is handled by calculating the Euclidean distance of turbines from each other in Cartesian coordinates. Thus, turbine i with coordinates (x_{t_i}, y_{t_i}) is feasible if its distances from each of the other turbines is greater than five times its diameter,

$$\sqrt{(x_{t_i} - x_{t_j})^2 + (y_{t_i} - y_{t_j})^2} > 5D, \quad \forall j \quad (5)$$

where D is the diameter of turbine i.

If we assume the proximity constraint as the first constraint, g_1 is the first constraint function and shows the amount of proximity constraint violation. This function can be defined as

$$g_1 = \sum_{i=1}^{n_{prox}-1} \sum_{j=i+1}^{n_{prox}} \left(5D - \sqrt{\left(x_{t_i} - x_{t_j}\right)^2 + \left(y_{t_i} - y_{t_j}\right)^2} \right), \quad (6)$$

where n_{prox} is the number of turbines, which violate the proximity constraint. Also, $\{(x_{t_i}, y_{t_i}), (x_{t_j}, y_{t_j})\}$ are the coordinates of each pair of turbines that violate the proximity constraint.

The regulatory constraints state that the turbines should not be located inside the non-feasible areas of the domain, as defined later in this section. To mathematically define the regulatory constraints, we assume that all the non-feasible areas of the domain can be modelled as convex polygons.

In general, there are several well-known approaches in the literature to determine if a point is inside a polygon [39–41]; however, they are not convenient for this application because they include many conditionals and/or inverse trigonometric functions. In this study, we used an approach based the area of the non-feasible polygon. In this approach, all the non-feasible polygons are considered to be convex and the non-convex polygons are divided into multiple convex polygons. The main idea is to draw lines from the location of turbine to the vertices of the polygon, such that each adjacent pair of vertices creates a triangle with the location of turbine. The summation of the areas of these triangles is compared to the area of the polygon and if they are the same, the turbine is inside the non-feasible polygon. Thus, turbine i with coordinates (x_{t_i}, y_{t_i}) is feasible if for any non-feasible polygon called P_k,

$$A_{i_k} > A_{P_k}, \quad \forall k \tag{7}$$

where A_{P_k} and A_i are the area of the non-feasible polygon and the summation of the areas of the aforementioned triangles, respectively. A_{P_k} and A_i are calculated in Eq. 8 and Eq. 9 using the so-called shoelace formula [42],

$$\begin{aligned} A_{P_k} = \frac{1}{2}\left[\sum_{j=1}^{n} |(x_{v_j} y_{v_{j+1}} - y_{v_j} x_{v_{j+1}})|\right] \\ + \frac{1}{2}|(x_{v_n} y_{v_1} - y_{v_n} x_{v_1})| \end{aligned} \tag{8}$$

$$\begin{aligned} A_{i_k} = \frac{1}{2}\left[\sum_{j=1}^{n} |x_{t_i}(y_{v_j} - y_{v_{j+1}}) + x_{v_j}(y_{v_{j+1}} - y_{t_i}) + x_{v_{j+1}}(y_{t_i} - y_{v_j})|\right] \\ + \frac{1}{2}|x_{t_i}(y_{v_n} - y_{v_1}) + x_{v_n}(y_{v_1} - y_{t_i}) + x_{v_1}(y_{t_i} - y_{v_n})| \end{aligned} \tag{9}$$

where $j \in \{1, 2, \cdots, n\}$, n is the number of non-feasible polygon's vertices and (x_{v_j}, y_{v_j}) are the coordinates of each vertex.

In a similar fashion to the proximity constraint, if we assume the regulatory constraint as the second constraint, g_2 shows the amount of regulatory constraint violation, defined as the summation of the minimum distances of the infeasible turbines to the sides of the non-feasible areas in which they are located. Hence, for a polygon with n sides the distance of turbine i from side j can be defined as the height of the triangle formed by the turbine's location point and two vertices of side j. We calculate this height by dividing the triangle's area by the base of the triangle, i.e., side j,

$$d_{i,j} = \frac{|x_{t_i}(y_{v_j} - y_{v_{j+1}}) + x_{v_j}(y_{v_{j+1}} - y_{t_i}) + x_{v_{j+1}}(y_{t_i} - y_{v_j})|}{\sqrt{(x_{v_j} - x_{v_{j+1}})^2 + (y_{v_j} - y_{v_{j+1}})^2}} \tag{10}$$

where $j \in \{1, 2, \cdots, n\}$. Finally, g_2 can be defined as,

$$g_2 = \sum_{i=1}^{n_{reg}} \min\{d_{i,1}, d_{i,2}, \cdots, d_{i,n}\} \tag{11}$$

where n_{reg} is the number of turbines that violate the regulatory constraint.

OPTIMIZATION MODEL

For this non-linear, non-convex multi-objective optimization, no solution can maximize the energy generation and minimize noise production simultaneously. In this case, there exist a set of solutions for which neither of the objective functions can improve without degrading the other. These optimal solutions are also known as the Pareto optimal solutions and are considered equally desirable.

The problem formulation is carried out using the following notation. We define $\mathbb{T}$ as a set of pairs representing the coordinates of the turbines,i.e.,

$$\mathbb{T} = \{(x_{t_1}, y_{t_1}), (x_{t_2}, y_{t_2}), \cdots, (x_{t_{n_T}}, y_{t_{n_T}})\}, \tag{12}$$

where n_T is the number of turbines. In a similar fashion, $\mathbb{R}$ is defined to show the coordinates of the noise receptors as

$$\mathbb{R} = \{(x_{r_1}, y_{r_1}), (x_{r_2}, y_{r_2}), \cdots, (x_{r_{n_R}}, y_{r_{n_R}})\}, \tag{13}$$

where n_R is the number of noise receptors. Note that $\mathbb{T}$ is a set of pairs of decision variables while $\mathbb{R}$ is a set of pairs of parameters.

With the purpose of imposing the regulatory constraints, the whole wind farm terrain is divided into n_p convex polygons, which are the members of $\mathbb{P}$,

$$\mathbb{P} = \{P_1, P_2, \cdots, P_{n_p}\}, \tag{14}$$

where each P_i is a set of pairs including the coordinates of all the vertices of polygon i in counter clockwise order,

$$P_i = \{(x_{v_1}, y_{v_1}), (x_{v_2}, y_{v_2}) \cdots, (x_{v_n}, y_{v_n})\}. \tag{15}$$

From the above mentioned polygons, some of them are identified as non-feasible due to the regulatory constraints. These polygons are all included in a set called $\mathbb{S}$, $\mathbb{S} \subset \mathbb{P}$, defined as

$$\mathbb{S} = \{P_i | P_i \text{ is non-feasible}\}. \tag{16}$$

Now, we can define the WFLO problem as,

$$\underset{\mathbb{T}}{\text{minimize}} \quad \left\{-AEP(\mathbb{T}), \max_{\mathbb{R}}(SPL(\mathbb{T}, \mathbb{R}))\right\}, \tag{17}$$

subject to,

$$\sqrt{(x_{t_i} - x_{t_j})^2 + (y_{t_i} - y_{t_j})^2} \geq 5D, \tag{18}$$

$\forall\{(x_{t_i}, y_{t_i}), (x_{t_j}, y_{t_j})\} \subset \mathbb{T},\ i, j \in \{1, 2, \cdots, n_T\}, i \neq j$, and

$$A_{i_k} - A_{P_k} > 0, \tag{19}$$

$\forall i \in \{1, 2, \cdots, n_T\}$, $\forall P_k \in \mathbb{S}$, where A_{P_k} and A_{i_k} are as defined in Eq. 8 and Eq. 9, respectively.

The objective functions of the problem can be defined as,

$$AEP(\mathbb{T}) = \sum_{i=1}^{n_T} \sum_{d \in \mathscr{D}} \frac{1}{3} \left(u_{id,\infty} \left(1 - \sqrt{\sum_{j \in \mathscr{U}_{id}} \left(1 - \frac{u_{ijd}}{u_{id,\infty}} \right)} \right) \right)^3 p_d, \tag{20}$$

and

$$SPL(\mathbb{T}, \mathbb{R}) = 10 \log \left(\sum_{i=1}^{n_T} \sum_{j=1}^{8} 10^{0.1\left(L_f^{(i,j)}(\mathbb{T},\mathbb{R}) + A_f^{(j)}\right)} \right), \tag{21}$$

where $\mathscr{U}_{id}$ is the set of turbines upstream of turbine i for wind state d, $u_{id,\infty}$ is the undisturbed wind speed at turbine i for wind state d, and u_{ijd} is the wind speed at turbine i due to a single wake caused by upstream turbine j for wind state d.

Multi-objective Genetic Algorithm

In this study, the NSGA-II algorithm [34] is implemented to solve the multi-objective wind farm layout optimization. The cross-over and mutation probabilities are considered to be 0.95 and 0.05 respectively, while n_{pop} (i.e., the population of each generation) and n_{gen} (i.e., the number of generations for which the GA is run) are selected based on computer experiments with typical instances of the WFLO, as described in the next sections. After evaluating the objective function values of the offspring population, the amount of constraint violation is calculated for each individual and passed to the constraint handling approach. To evaluate whether the algorithm has converged, this work implements the approach of Deb et al. [34], which determines convergence by monitoring the change in crowding distances across a certain number of generations. A test case is considered to be converged if the variance of the crowding distances of rank 1 solutions is less than 0.005 over 100 generations. In addition, a limit of 80,000 objective function evaluations has been imposed as a termination criterion. These convergence and termination criteria are uniformly applied when generating the results. The readers are referred to [34] for more details on the NSGA-II algorithm and its implementation.

Constraint Handling

In this section, two approaches that we used to handle the constraints are discussed. First, we handled them using dynamic penalty functions and second, a CP model is hybridized with dynamic penalty functions. In the following paragraphs these two approaches are discussed in depth.

Penalty Functions Approach Dynamic penalty functions [25] are implemented to penalize the objective functions of the infeasible layouts. The dynamic penalty coefficients increase as the optimization progresses through generations. Thus, the penalized objective functions are defined as,

$$AEP^P(\mathbb{T}) = AEP(\mathbb{T}) + \sum_{i=1}^{n_c} (\max(0, g_i))^2 \left(\frac{t}{C_{gen}}\right)^2 R_{AEP,i} \quad (22)$$

and

$$SPL^P(\mathbb{T},\mathbb{R}) = SPL(\mathbb{T},\mathbb{R}) + \sum_{i=1}^{n_c} (\max(0, g_i))^2 \left(\frac{t}{C_{gen}}\right)^2 R_{SPL,i}, \quad (23)$$

where AEP^P and SPL^P are the penalized objective functions, n_c is the number of constraints, g_i is the i-th constraint function, $R_{AEP,i}$ is the penalty coefficient for constraint i and the energy objective function, $R_{SPL,i}$ is the penalty coefficient for constraint i and the noise objective function, t is the current generation number and C_{gen} is a constant providing a relative generation parameter, which will be defined later. The generation parameter is squared according to the approach suggested in [43]. Since we have two constraints, i.e., proximity and regulatory constraints, n_c is equal to 2.

Based on the experiments carried out in previous study [36], the value of the penalty coefficients is set to 10^4, while two different values are assigned to the C_{gen} parameter, namely $C_{gen} = n_{gen}$ and $C_{gen} = n_{gen}/2$. With these parameter choices, the effective penalty coefficients are in the range $0-10^4$ and $0-4\times10^4$. When appropriate, the solutions found by these two formulations are merged together and the best solutions found overall are reported.

Hybrid Constraint Programming Approach Before introducing the hybrid Constraint Programming approach, it is necessary to get an insight about the effects of the penalty function approach on the iteration-level behavior of GA. As soon as an infeasible solution is penalized with penalty functions, the chance for that solution to be chosen to participate in the recombination process decreases drastically. The GA is forced to forget that solution and look for new feasible solutions in the domain. This characteristic of the penalty functions is known as exploration, global search, or diversification. For highly constrained problems, the probability of finding feasible solutions is relatively low; thus, the penalty functions can result in finding a small number of solutions (i.e., having empty slots in the Pareto set) and premature convergence [25]. Although the dynamic penalty approach performs a combination of global and local searches due to lower penalizations in the initial stages of the optimization [44–46], in this study we use CP to reinforce the local search behavior of the constraint handling approach.

The idea behind the CP model is to find feasible solutions that are as close as possible to the corresponding infeasible solutions. Since this model only searches the neighbourhood of the infeasible solutions, it can be considered as exploitation, local search or intensification. The advantage of repairing the infeasible solutions is that GA does not have to search for feasible solutions with that small probability discussed above. However, the drawback is that it prevents GA from exploring the feasible area of the domain and keeps searching close to the infeasible solutions. To avoid the drawbacks of either pure exploration or exploitation the hybrid approach, which uses the penalty functions and the CP model together is introduced. An infeasible layout is first passed to the CP model. This model strives to find a feasible layout which is as close as possible to the infeasible layout. If the CP model cannot find a feasible layout which is close enough to the infeasible layout in a certain amount of time, the infeasible layout will be penalized using dynamic penalty functions.

Based on the proposed approach, the CP model can be formulated as,

$$\text{minimize} \quad \sum_{i=1}^{n_{nf}} \left(\left(x_{t_i} - x^*_{t_i}\right)^2 + \left(y_{t_i} - y^*_{t_i}\right)^2 \right), \quad (24)$$

subject to,

$$\sqrt{(x_{t_j} - x^*_{t_i})^2 + (y_{t_j} - y^*_{t_i})^2} \geq 5D, \quad (25)$$

$\forall j \in \{1,2,\cdots,n_T\}, j \neq i$, and

$$A^*_{i_k} - A_{P_k} > 0 \quad \forall P_k \in \mathbb{S}, \quad (26)$$

where n_{nf} is the number of infeasible turbines in an infeasible layout (i.e., the number of turbines that violate either the proximity or the regulatory constraint in an infeasible layout) and $(x^*_{t_i}, y^*_{t_i})$ are the corrected coordinates of the ith infeasible turbine.

The CP code is developed using IBM ILOG CP Optimizer [47]. Since it is more common to use integer variables in a CP solver, domains of the coordinate variables are discretized for the sole purpose of this optimization sub-problem.

This model has three different parameters that can be tuned. The first parameter is the discretization resolution. If we make the discretization finer, the hybrid approach provides a better resolution. However, it is clear that the computational cost will increase. The second parameter is the time limit in which the CP model has to correct an infeasible layout. Longer time limits for the CP model result in better solutions; however, an increase in the overall run time. The last parameter is the maximum objective function value that a solution of the CP model can have in order to be accepted as a close enough feasible layout to the infeasible layout. In a similar way, decreasing this value results in longer run times and a decrease in the number of infeasible layouts that are corrected with the CP model in each generation of the evolutionary algorithm.

Based on a set of experiments with the hybrid CP and static penalty function model, the wind farm terrain is discretized in 20 meters intervals. These experiments show that a finer discretization increases the computational cost, while the optimization results will not be changed significantly. The time limit per call for the CP model is 10 seconds. The experiments on this parameter show that time limit does not have an effect on the number of solutions that are corrected by the hybrid approach. However, it is shown that the important parameter in this case is the objective function of the CP model. This objective function is defined as the sum of the squared Euclidean distances of the corrected feasible turbines from their corresponding infeasible turbines. The maximum objective function value (i.e., the maximum value of Eq. 24) for which the solution found by the CP solver is accepted is set to $10,000$ m^2. Considering the fact that this value is the sum of squared values, it is assumed as a reasonable value in wind farm with characteristics that are explained in the following section.

TEST CASES

In this study, the hybrid CP approach is first verified with sample test functions and then applied to the WFLO problem.

Verification Test Cases

The novel hybrid constraint handling algorithm is verified with a sample constrained multi-objective optimization problem that is previously used by Deb et al. [34] for testing constraint handling approaches with NSGA-II. This problem is called SRN and is formulated as,

$$\begin{aligned} \underset{X}{\text{minimize}} \quad & f_1(X) = (x_1-2)^2 + (x_2-1)^2 + 2, \\ & f_2(X) = 9x_1 - (x_2-1)^2, \\ \text{subject to} \quad & g_1(x) = x_1^2 + x_2^2 \leq 225,\ g_2(x) = x_1 - 3x_2 \leq -10, \end{aligned}$$

where $X = \{x_1, x_2\}$ and $x_1, x_2 \in [-20, 20]$.

We followed Deb et al. [34] to set the NSGA-II parameters for solving this problem. Each test case has a population of 100 and is run for 500 generations. Also, the test case is considered to be converged if the variance of the crowding distances of rank 1 solutions is less than 0.005 over 100 generations. Both constraint handling approaches are tested on these problems. In addition, different setups of the hybrid approach are tested on this problem to investigate the effect of the parameters of the hybrid approach on the results. To avoid the potential negative impacts of randomness, 20 different random test cases with two different penalty coefficient are solved for the SRN problem and are compared using box plots.

WFLO Test Cases

As described in [36], random wind farm test cases are generated with pre-defined feasibility percentage and uniformity. Following the standard test cases used in the literature, a 3 km $\times$ 3 km square is considered as the wind farm domain. The feasibility of wind farm domain is defined to specify the percentage area available in the farm for placing the turbines. In addition, a uniformity parameter is defined to characterize the spatial distribution of the non-feasible areas in the domain. To generate a test case, the wind farm domain is divided into 225 random convex polygons with areas of the same order of magnitude.

Based on industrial wind farm design experience, cases with 70%, 80%, and 90% feasibility percentages are considered and the uniformity parameter is kept constant for all the test cases. Figure 1 shows a WFLO test case with 80% feasible area. The noise receptors are shown in asterisks. The shaded polygons are the members of $\mathbb{S}$, which are chosen randomly and their total area satisfies the pre-defined domain feasibility. A noise receptor is located randomly inside each non-feasible polygon. As a result, the more constrained the domain becomes, the more noise receptors exist in it. For each feasibility percentage, five random test cases are created in order to avoid the effects of randomness in GA. Then, the optimization is carried out for each test case with 5, 10, and 15 turbines. The hub height and the rotor diameter of the turbines used in this study are 80 m and 77 m respectively.

In order to determine the population size and the number of generations for the GA, a set of computational experiments is carried out on sample test cases with the land availability percentages mentioned above and penalty functions as the constraint handling approach. Population sizes of 100, 150, and 200 individuals are tested and the corresponding number of generations is set to keep the number of objective function evaluations constant. For 70% of land availability, a population size of 200 results in the best solutions, no matter how many turbines are included in the wind farm. Similarly, for 80% and 90% of land availabilities the population sizes of 150 and 100 perform the best, respectively.

The wind regime is as defined by Kusiak et al. [37], with 24

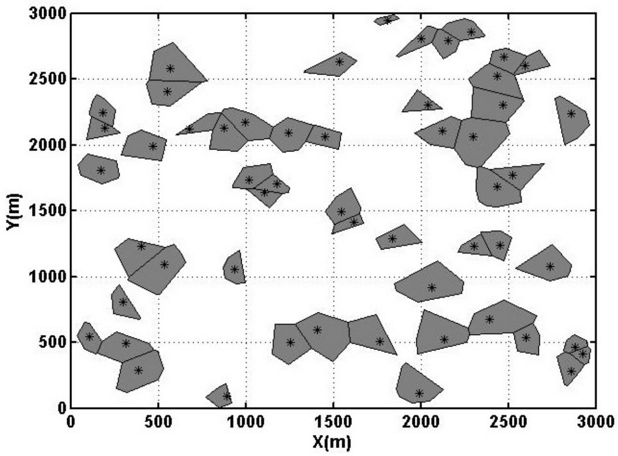


FIGURE 1. SAMPLE WIND FARM DOMAIN.

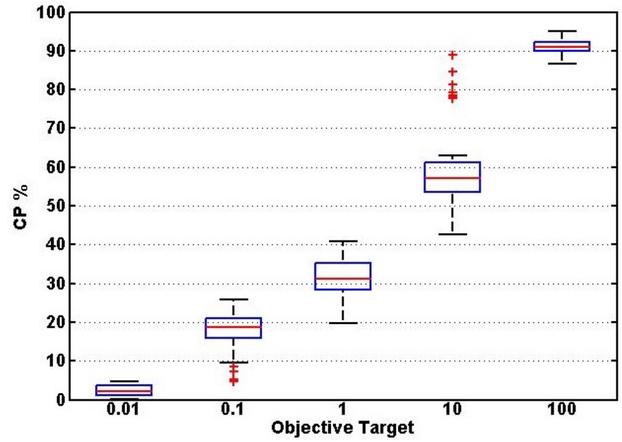


FIGURE 2. CP PERCENTAGE WITH DIFFERENT OBJECTIVE TARGETS FOR SRN PROBLEM.

wind directions in 15° intervals. The wind speed ranges from 4 m/s to 25 m/s in intervals of 0.5 m/s. For each direction-speed bin a probability is assigned based on the industrial data and they are used to calculate AEP in Eq. 20.

RESULTS AND DISCUSSION

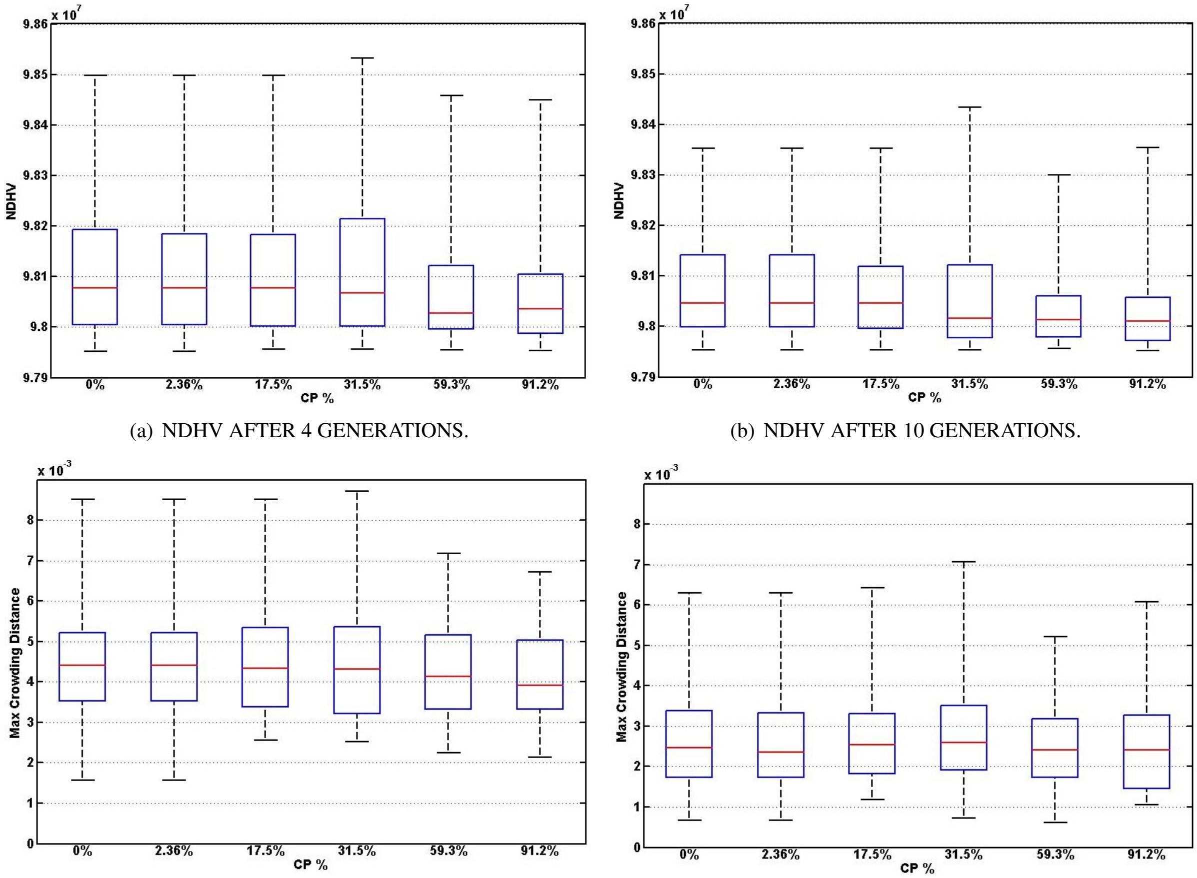
In this section, the verification results for the test problem are first discussed and then the application of the above mentioned constraint handling approaches in the WFLO problem is investigated.

Verification of Constraint Handling Approaches

In this section, we verify the functionality of the hybrid approach and find the setup for which the hybrid approach performs better than the dynamic penalty approach and other hybrid setups. The important characteristic of the hybrid approach is the percentage of infeasible solutions that are corrected by the CP model. Thus, we tune the parameters of the hybrid approach in such a way that different percentages of using the CP model can be compared. We use two metrics for this comparison. The first metric is the non-dominated hyper volume (NDHV). This metric shows how close the Pareto set is to the utopia point (i.e., the infimum of the objective functions vector) and the closer the Pareto set is to the utopia point the smaller the NDHV is. However, this metric is not sufficient for comparing the optimization results especially at the early stages of the optimization, where it is very likely that a Pareto set has a smaller value of NDHV due to incomplete exploration of the objective space. The second comparison metric is the maximum crowding distance in the Pareto set which provides a measure how well the objective space is explored by the optimization algorithm. A small value for the maximum crowding distance shows that the optimization algorithm has been successful in having a uniform coverage over all the areas of the Pareto set.

The SRN problem is a bi-objective optimization with non-linear objective functions, a linear, and a non-linear constraint, which can be solved analytically. As stated before, the maximum acceptable objective function for the CP model affects the number of infeasible solutions that are corrected by it. Thus, we run test cases with different maximum acceptable objective function values. For all these experiments the time limit of the CP model is 10 seconds per call and the decision variable space is discretized to 22500 available set of values for the variables of the problem, (the same discretization as the WFLO problem). For simplicity, we call the percentage of invocations to the CP solver that successfully return a feasible solution, CP percentage. Similarly, the maximum acceptable objective function value is called objective target. The objective target shows the maximum squared Euclidean distance of a corrected feasible solution from its corresponding infeasible solution in the variable space. Figure 2 shows the variation of CP percentage for different objective targets for SRN problem. As the objective target of the CP model decreases, it is forced to find feasible solutions closer to the infeasible solutions within the same time limit. Thus, CP percentage decreases with decreasing the objective target.

Figure 3 shows the comparison of the above mentioned metrics for the dynamic penalty approach (CP percentage of 0%) and the hybrid approach with different CP percentages and for two different number of generations. For each CP percentage, the problem is solved with 20 random test cases and two different penalty coefficients for each random test case. To compare the constraint handling approaches and the effect of different CP percentages with the hybrid approach, the Pareto set found by them at the 4th and 10th generation are evaluated. Based on our experiments, after the 10th generation, there is no further difference between the performance of the two constraint handling approaches, except for those test cases using the hybrid approach that were lagging. The comparison of Fig. 3(a) to Fig. 3(b) and Fig.3(c) to Fig. 3(d) shows that the median NDHV is



(a) NDHV AFTER 4 GENERATIONS.

(b) NDHV AFTER 10 GENERATIONS.

(c) CROWDING DISTANCE AFTER 4 GENERATIONS.

(d) CROWDING DISTANCE AFTER 10 GENERATIONS.

FIGURE 3. NON-DOMINATED HYPER VOLUME AND MAXIMUM CROWDING DISTANCE WITH DIFFERENT CONSTRAINT HANDLING APPROACHES FOR THE SRN PROBLEM AFTER 4 AND 10 GENERATIONS.

smaller for the hybrid approach with different CP percentages after only 4 generations, so we can say that the hybrid approach converges faster. This trend can also be observed after 10 generations. In addition, the range of NDHV and maximum crowding distance for the hybrid approach across multiple runs is smaller compared to that of the penalty approach (smaller box plots and length of whiskers), which indicates less variability of results from run to run. Among different CP percentages for which the hybrid approach is tested, 59.3% and 91.2% have smaller NDHV and maximum crowding distance. The comparison of these two CP percentages shows that the 59.3% test cases perform better due to lower maximum crowding distance and NDHV. The 91.2% test cases are not able to explore the objective space to the extent of the 59.3% test cases due to extreme local search, thus having a higher maximum crowding distance.

Constraint Handling Performance for WFLO Problem

The experiments for the WFLO problem are carried out with one objective target for the CP model and hence relatively the same CP percentage. The objective target used for these experiments yields to extremely high CP percentages, a characteristic that makes the local search of the hybrid constraint handling approach much stronger than its global search. Further experiments are in progress to investigate the effect of CP percentage on the performance of the hybrid approach in this constrained, multi-objective WFLO problem. However, there are some valuable points with these results that can be discussed.

Table 1 shows the average number of infeasible layouts for 10 runs of each test case. It is shown that extreme use of the CP model in the hybrid approach results in more infeasible layouts compared to the pure global search by penalty functions. Each time that an infeasible layout is corrected by the CP model, a feasible solution that is very close to the infeasible solution is created. Although this solution increases the chance to find an optimal layout that could not be found by the global search, it is likely that this corrected feasible solution leads to infeasible solutions after the recombination process of the GA.

Figures 4, 5, and 6 compare the optimal Pareto sets found

TABLE 1. CONSTRAINT HANDLING INFORMATION OF THE CONSTRAINT HANDLING APPROACHES FOR DIFFERENT NUMBER OF TURBINES AND LAND AVAILABILITIES.

Number of Turbines	Feasibility	Penalty Approach		Hybrid Approach	
		Average Infeasible Layouts	CP %	Average Infeasible Layouts	CP %
5	70%	1536.2	0	4789.9	99.35
5	80%	356.9	0	1375.3	99.28
5	90%	73.3	0	246.3	97.32
10	70%	3996.6	0	5435.7	98.39
10	80%	1582.2	0	4857.6	98.34
10	90%	1631.0	0	3162.3	93.02
15	70%	4158.2	0	9658.4	92.67
15	80%	3488.6	0	4183.6	97.07
15	90%	2084.7	0	4159.3	94.49

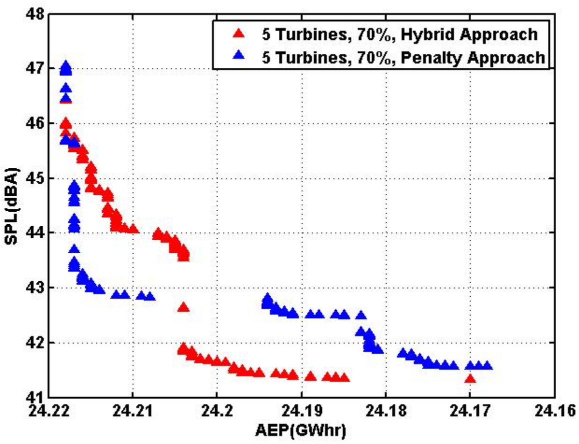
TABLE 2. RUN-TIME AND CONVERGENCE OF THE CONSTRAINT HANDLING APPROACHES FOR DIFFERENT NUMBER OF TURBINES AND LAND AVAILABILITIES.

Number of Turbines	Feasibility	Penalty Approach		Hybrid Approach	
		Average Run-time (hr)	Convergence (out of 10)	Average Run-time (hr)	Convergence (out of 10)
5	70%	15.76	3/10	15.54	5/10
5	80%	15.11	9/10	16.39	7/10
5	90%	18.40	4/10	14.01	9/10
10	70%	69.44	0/10	82.85	2/10
10	80%	77.89	1/10	88.22	0/10
10	90%	84.84	4/10	78.47	4/10
15	70%	150.42	0/10	168.73	0/10
15	80%	153.01	0/10	178.04	1/10
15	90%	183.09	0/10	220.71	1/10

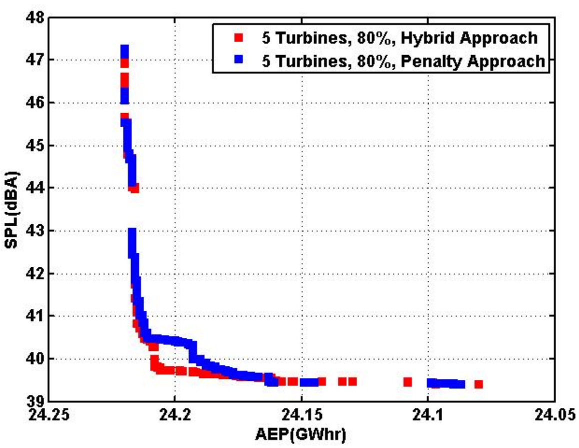
by the penalty and hybrid constraint handling approaches. It should be noted that the x axis is reversed in all the above mentioned figures, so that the utopia point is located in the bottom left corner of each figure. The performance of the constraint handling approaches are different for different test cases. However, by comparing Figures 5(c) and 6(a) with Table 1, it can be claimed that the hybrid approach is performing better (i.e., higher energy generation and lower noise production) than the penalty approach due to its lower percentage of repaired solutions compared to the other test cases. Thus, there should be a certain CP percentage for which the hybrid approach performs better than the penalty approach. For the other test cases the hybrid approach is not as effective as it is expected, since its constraint handling is mostly based on local search and the hybrid approach loses its hybrid nature.

For all the test cases, the Pareto sets found by the two constraint handling approaches are close to each other; however, for the test case with 10 turbines and 80% of land availability (Fig. 5(b)) the Pareto set found by the penalty approach is significantly better than the Pareto set found by the hybrid approach. Investigation of the optimal layouts found by the two constraint handling approaches shows that the optimal layouts found by the penalty approach have more turbines near the borders of the domain, where there is no concentration of non-feasible areas. However, the local search carried out by the hybrid approach, corrects the infeasible layouts by keeping the turbines close to the non-feasible areas and fails to explore the borders of the wind farm terrain to find feasible/optimal layouts.

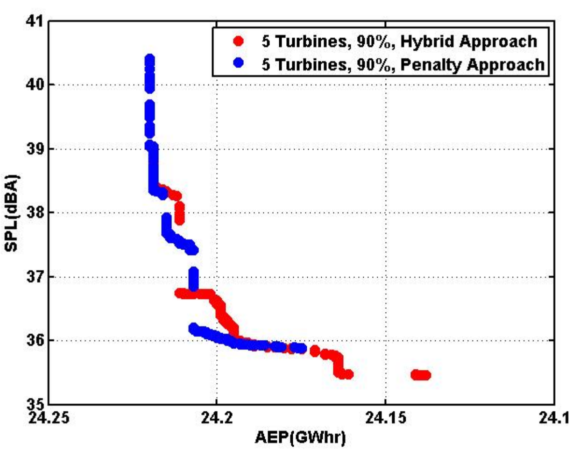
Finally, it is essential to discuss the run-time and convergence of the constraint handling approaches. The average run-time and the number of times that convergence is observed based on the aforementioned convergence criterion during the 10 runs for each test case are provided in Table 2. As the constraint severity increases, i.e., the number of turbines increases and the land availability decreases, the CP model requires more time to correct the infeasible solutions. Thus, the run-time of the hybrid approach increases. However, this increase can be controlled by decreasing the number of infeasible solutions that are corrected by the CP model. In other words, reducing the CP percentage may result in a smaller run-time difference between the two constraint handling approaches. Generally, the hybrid approach has a better convergence compared to the penalty approach. However, it is important to note that this higher convergence rate for the hybrid approach can be due to the high CP percentage. The local search of the hybrid approach is stronger and as mentioned before may prevent it from exploring the domain and result in a premature convergence.



(a) 70% FEASIBILITY.

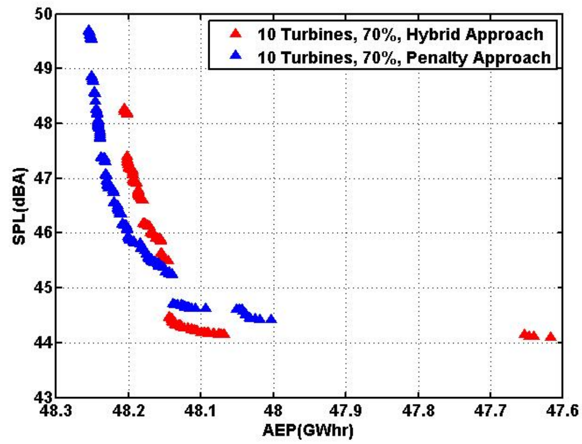


(b) 80% FEASIBILITY.

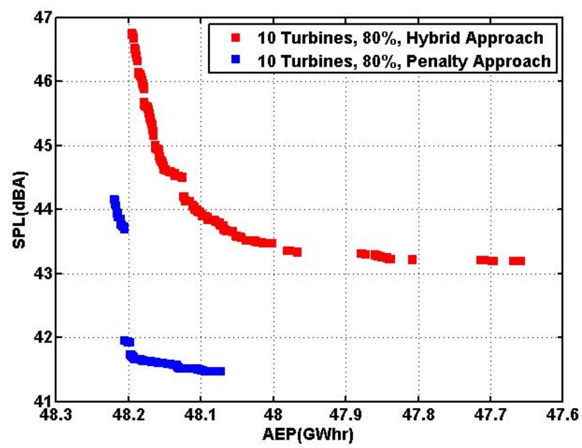


(c) 90% FEASIBILITY.

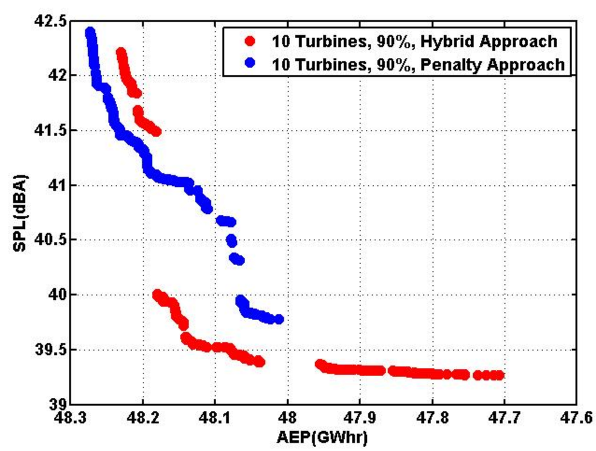
FIGURE 4. COMPARISON OF CONSTRAINT HANDLING APPROACHES FOR 5 TURBINES (X AXIS IS REVERSED).



(a) 70% FEASIBILITY.

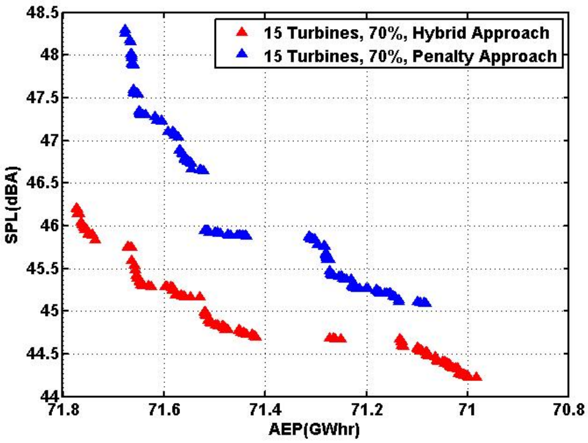


(b) 80% FEASIBILITY.

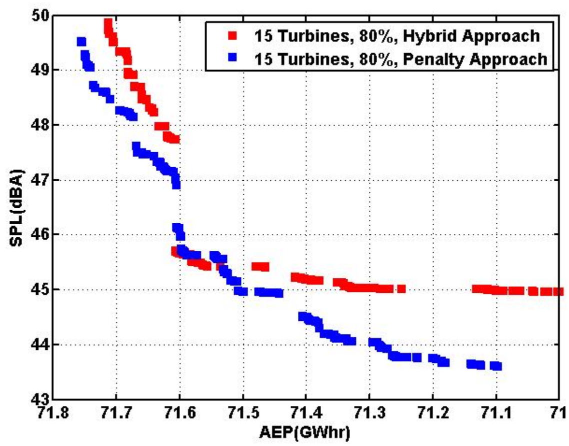


(c) 90% FEASIBILITY.

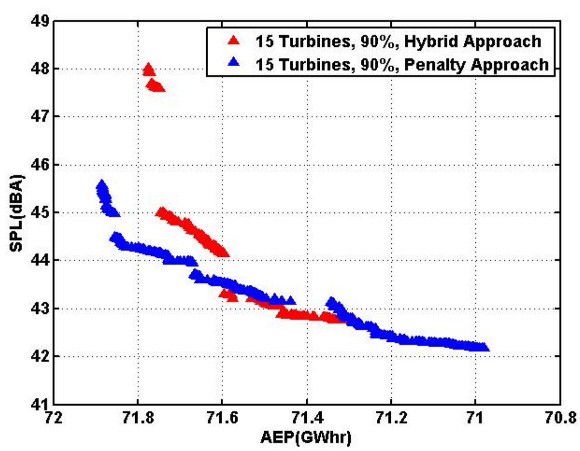
FIGURE 5. COMPARISON OF CONSTRAINT HANDLING APPROACHES FOR 10 TURBINES (X AXIS IS REVERSED).



(a) 70% FEASIBILITY.



(b) 80% FEASIBILITY.



(c) 90% FEASIBILITY.

FIGURE 6. COMPARISON OF CONSTRAINT HANDLING APPROACHES FOR 15 TURBINES (X AXIS IS REVERSED).

CONCLUSION

In this paper, the constrained multi-objective energy-noise wind farm layout optimization is solved with a continuous variable Genetic Algorithm, called NSGA-II. Primarily, dynamic penalty functions are used to handle the constraints. Then, a hybrid approach based on the combination of penalty functions and a Constraint Programming model is introduced to improve the objective function of the solutions (smaller objective functions in case of minimization and larger otherwise).

The local search associated with the dynamic penalty approach is confined to a smaller amount of penalization in the initial stages of the optimization. Thus, it is important for us to hybridize the penalty function approach with a powerful local search. The Constraint Programming model is designed for this purpose. This model tries to find the closest feasible solutions to the infeasible solutions. However, the penalty functions perform a global search by penalizing the objective functions of the infeasible solutions. The hybridization of the Constraint Programming model with the penalty function approach creates a constraint handling approach that is able to carry out a combination of local and global searches.

The results of the optimization with the test problems show that the hybrid approach has the potential to perform better than the penalty approach. However, its performance is dependent on the number of the infeasible solutions that are corrected by the Constraint Programming model. Our experiments for the WFLO problem also show that there is a certain amount of infeasible solution correction that will produce results with higher energy generation and lower noise production compared to the penalty function approach. In addition, a moderate use of both local and global searches will have a computational cost, which is close to that of the penalty function approach. It should not be neglected that extreme use of the Constraint Programming model can result in handling the constraints with pure local search. This pure local search converges to suboptimal solutions because it does not explore the optimization space comprehensively.

Future work will focus on documenting the impact of the percentage of solutions that are corrected by the Constraint Programming model on the results of the WFLO problem. Furthermore, the effect of discretization on the optimization results will be investigated. For this purpose, CP Optimizer [47] can be substituted by other CP solvers such as SCIP [48] that are capable of solving problems with continuous variables. In addition, a more complete set of test problems with non-linear objective functions and constraints can be solved to investigate the effect of non-linearity on the performance of the hybrid approach. WFLO problem can also be solved with more turbines to explore the performance the hybrid approach on large scale WFLO problems. Finally, the effect of the parameters of the Constraint Programming model on the number of infeasible solutions corrected by this model is an interesting area to explore.

ACKNOWLEDGMENT

This work was supported by the Natural Sciences and Engineering Research Council of Canada (NSERC), through the Collaborative Research and Development (CRD) and Discovery Grant programs. The authors would like to thank Hatch Ltd. for its financial support of this research.

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